Design of TAMUTRAP and Testing of RFQ Pressure Control System

By: Mike Mehlman Advisor: Dr. Dan Melconian Degree Sought: M. Sc. Physics



Outline

- Motivation
 - Beta-Neutrino Correlation Measurements
 - Precision Mass Measurements
- TAMUTRAP
 - What is a Penning Trap
 - Why a Penning Trap
 - What is TAMUTRAP
- Progress
 - Physics Simulations
 - Penning Trap and Beam Line Design
 - Pressure Control System Implementation/Testing
- Conclusions and Outlook



Motivation

•Beta-Neutrino Correlation Measurements

•Precision Mass Measurements



- β-v correlation looks at the relationship between the directions of the beta and neutrino emitted during a beta decay
- $a_{\beta\nu}$ is the parameter that quantifies this affect
 - Decay cross-section proportional to^[1]

$$1 + \frac{p}{E} a_{\beta\nu} \cos\left(\theta_{\beta\nu}\right)$$

- Different for different decays
 - For ³²Ar a[~] = .9989 ± 0.0052 (stat) ± 0.0039 (syst) ^[2]
- From $a_{\beta\nu}$ we can infer details on the involved currents and the charged weak interaction

[1] J.D. Jackson, S.B. Treiman, and H.W. Wyld Jr., Phys. Rev. **106**, 517 (1957)
[2] EG Adelberger, et. al, Physical Review Letters **83**, 1299–1302 (1999).





- Use beta-delayed proton decays to measure $a_{\beta\nu}$ (T=2, 0⁺ \rightarrow 0⁺)
 - The proton contains information about the angle between beta and neutrino in the form of a momentum kick inherited through daughter
 - If beta and neutrino are ejected in same direction ($a_{\beta\nu} = 1$), proton will have greater energy spread around mean, with characteristic shape
 - If beta and neutrino are ejected in opposite directions $(a_{\beta\nu} = -1)$, proton will have smaller energy spread around mean, with characteristic shape



Beta-delayed Proton Decay



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Image: EG Adelberger, et. al, Physical Review Letters 83, 1299–1302 (1999).





Precision Mass Measurements

- Many applications
 - Astrophysics
 - Testing standard model predictions
 - Definition of constants/units





TAMUTRAP

- •What is a Penning Trap
- •Why a Penning Trap
- •What is TAMUTRAP



Penning Trap

- What is a penning Trap?
 - Charged particles
 - Traps ion radially with magnetic field
 - Traps ion axially with electric field



Image: G Gabrielse, et. al, Journal of Mass Spectrometry 88, 319-332 (1989).



Penning Trap

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- Use beta-delayed proton decays to measure $a_{\beta\nu}$ (T=2, 0⁺ \rightarrow 0⁺)
- Protons up to 4.75 MeV contained in proposed geometry (90mm radius)

Nuclide	Lifetime (ms)	Proton Energy (MeV)
²⁰ Mg	137.05	4.28
²⁴ Si	147.15	3.91
²⁸ S	180.33	3.70
³² Ar	141.38	3.36
³⁶ Ca	141.15	2.55
⁴⁰ Ti	72.13	3.73
⁴⁸ Fe	63.48	1.23



Orange: Proton of 4.2 MeV ~43mm radius Green: Beta of 10 MeV ≤~5mm radius



Why: Precision Mass Measurements

- Very precise mass measurements done with penning traps (uncertainties of 1 in 10¹¹)^[1]
- Measurement achieved by determining the (mass dependant) frequencies of ion motion in the trap^[2]
- Anharmonicity of electric field, mis-alignment, and imperfections result in lower precision

[1] G. Gabrielse, Physical Review Letters 102, 1-4 (2009).

[2] M. Saidur Rahaman, First On-line Mass Measurements at SHIPTRAP and Mass Determinations of Neutron-rich Fr and Ra Isotopes at ISOLTRAP, 2005. Image: M. Saidur Rahaman, First On-line Mass Measurements at SHIPTRAP and Mass Determinations of Neutron-rich Fr and Ra Isotopes at ISOLTRAP, 2005.





TAMUTRAP

- 2 cylindrical Penning traps within the bore of an Agilent 7T-210 magnet)
 - A gas filled, 7 electrode, cylindrical purification trap (optional)
 - A large-bore, novel, 5 electrode, cylindrical high precision penning trap





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Progress

Physics Simulations

- •Penning Trap and Beam Line Design
- •Pressure Control System Implementation/Testing



- Created a Monte Carlo simulation for the modeling of betadelayed proton decays
 - Features
 - Physics derived from the involved currents (based on J.D. Jackson, et al. Phys. Rev. 106, 517 (1957))
 - Can output any relevant information (energy, mass, velocity, etc.) for each particle involved
 - From this, time of flight to detector and detector acceptance can be calculated
 - Object-based framework allows for other reactions to be modeled





Each curve has 100,000,000 events







- Purification trap will use a copy of SHIPTRAP / ISOLTRAP
- Measurement trap needs to be a new design in order to have a large enough bore to contain all beta-delayed proton decay products (and be short enough to fit in magnet)
- Design constraints:
 - Large bore with small ion bunch size for beta-neutrino correlation measurements (beta-delayed protons decay products need to be fully contained)
 - "Tunable, orthogonalized" geometry for precision mass measurements



- Tunable
 - Electric field at trap center can be expanded as^[1]: $V = \frac{1}{2} V_0 \sum_{\substack{k=0\\k \in eeen}}^{\infty} C_k \left(\frac{r}{d}\right)^k P_k(\cos \theta)$
 - Term C₄ and higher order terms describe the anharmonicity of the potential (C₄ dominant)
 - To solve this problem, compensation electrodes are added to "tune out" the anharmonic terms

[1] G Gabrielse, et. Al. Journal of Mass Spectrometry 88, 319-332 (1989).



- Orthogonalized
 - C₂ may change when "tuning out" C₄ (adjusting compensation electrodes), and thereby change eigenfrequencies (which is what we measure)
 - Need to tune out C₄ during the course of the experiment, so look for a geometry where changing the potential on the compensation electrodes does not change C₂ (which affects measured frequency)

$$D_{2} = 0 = \sum_{n=0}^{\infty} \frac{2\left\{\frac{\sin(k_{n}(z_{0}-z_{c2})) - \sin(k_{n}(z_{0}-z_{c}))}{\pi J_{0}(ik_{n} \rho_{0})}\right\} d^{k} k_{n}^{k} (-1)^{k/2}}{k!}$$
 Orthogonality condition



- Search for a geometry that is orthogonalized with finite correction electrode size (tunable) at a reasonable voltage
 - Diameter determined by open space needed for measurements
 - Electrode spacing determined by need to avoid sparking, ease of installation/assembly/machining
 - Find a combination of ring, end, and compensation electrode lengths that best minimize anharmonic terms while being orthogonalized



- Calculated dimensions:
 - Ring: 1.15*2 cm
 - Compensation: 8.42257 cm
 - Endcap: 8 cm
 - Gaps: .05 cm
 - Radius: 9 cm
- Calculated tuning (C₄=0)
 condition for above geometry:
 - $V_c/V_o = -0.3708804$



COMPENSATION

RINC

- Model in SIMION and output electric field
- Fit electric field around trap center with Legendre polynomials compare to analytic solution
 - Even enlarging the geometry 10 times (further enlargement was prohibited by available RAM) did not allow SIMION to accurately reproduce the analytic result
 - However, analytic results reproduced results presented by Gabrielse in [1]



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Comparison of analytically determined coefficients to SIMION extracted coefficients for TAMU geometry

	Analytic	Expansion of SIMION
		output
Co	-0.5728	-0.5743
C1	-8.573*10 ⁻¹⁹	4.044*10-4
C ₂	0.4943	0.5189
C₃	6.400*10 ⁻¹⁷	-9.734*10-8
C ₄	2.908*10-6	0.03463
C₅	-3.644*10 ⁻¹⁸	-0.04818
C ₆	0.01998	0.1262
C ₇	-7.784*10 ⁻¹⁴	-0.1374
C ₈	-0.06823	0.03120

Comparison of analytic coefficients to coefficients presented in Gabrielse for Gabrielse geometry

	Gabrielse	Analytic
C ₂	0.5449	0.5448
C ₄	0	-5.806*10-5
C ₆	0	5.968*10-4
C ₈	0365	-0.03844







- When beam arrives at the Penning trap it must be bunched and have low emittance
- Employ a gas filled RFQ (Radio Frequency Quadrupole) Paul trap for bunching and cooling
- Employ other beam line elements to guide and focus the beam
- Best physically realizable geometry

















- Needed for RFQ and purification trap
 - Collisions with gas cool beam and allow bunching
- Maintains pressure of 10⁻² to 10⁻⁴ mbar Helium
- Capacitance manometer used as signal to control valve













Pressure as a Function of Time Diaphragm=1mm, Set Point=.01mbar, G=250, L=1.5





Error as a Function of Diaphragm Size



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37

Deviation as a Function of Diaphragm Size







- Accomplishments:
 - Simulated decays of interest
 - Geometrically optimized a new large-bore Penning
 Trap design (totally unique!)
 - Mechanically designed entire beam line, electrodes, mounts, and support structure
 - Performed tests on pressure control system
 needed for RFQ and purification trap



Beam Line

- Simulations
- Geometrical design
- Mechanical design
- Electronics design
- Fabrication
- Assembly 4
- Control system
- Testing

Trap

- Simulations
- Geometrical design



- Mechanical design
- Electronics design
- Detector design
- Fabrication
- Assembly
- Control system / Acquisition
- Testing



Thank you!

- Thanks to
 - Dan Melconian and Praveen Shidling
 - Dr. Greg Chubaryan
 - Spencer Behling, Ben Fenker, and Yakup Boran
 - Ryan Mueller and Levi Clark
 - Greg Derrig, Steve Molitor, and Bob Olsen

And my committee: Dr. Sherry Yennello, Dr. John Hardy, and Dr. Bhaskar Dutta



Backup Slides



Decay Cross Sections

$$\begin{split} \frac{d^4 W(p_e, p_v, l)}{d\Omega_e d\Omega_{v_e}} &\sim 1 + a_{\beta v} \frac{p_e \cdot p_{v_e}}{E_e E_{v_e}} + b_{fierz} \frac{m_e}{E_e} + \frac{\langle l \rangle}{l} \cdot \left[A_\beta \frac{p_e}{E_e} + B_v \frac{p_v}{E_v} + D \frac{p_e \times p_{v_e}}{E_e E_v} \right] \\ &+ c_{align} \left[\frac{p_e \cdot p_{v_e}}{3E_e E_{v_e}} - \frac{(p_e \cdot \hat{\imath})(p_{v_e} \cdot \hat{\imath})}{E_e E_{v_e}} \right] \left[\frac{I(I+1) - 3\langle (I \cdot \hat{\imath})^2 \rangle}{I(2I-1)} \right] \end{split}$$

$$\xi = |M_F|^2 (|C_S|^2 + |C_V|^2 + |C_{S'}|^2 + |C_{V'}|^2) + |M_{GT}|^2 (|C_T|^2 + |C_A|^2 + |C_{T'}|^2 + |C_{A'}|^2)$$

$$a\xi = |M_F|^2 (-|C_S|^2 + |C_V|^2 - |C_{S'}|^2 + |C_{V'}|^2) + \frac{|M_{GT}|^2}{3} (|C_T|^2 - |C_A|^2 + |C_{T'}|^2 - |C_{A'}|^2)$$

$$b\xi = \pm 2Re[|M_F|^2(C_S C_V^* + C_{S'} C_{V'}^*) + |M_{GT}|^2(C_T C_A^* + C_{T'} C_{A'}^*)]$$

$$\tilde{a} = \frac{a}{1 + 0.1913 * b}$$

J.D. Jackson, S.B. Treiman, and H.W. Wyld Jr., Phys. Rev. **106**, 517 (1957) EG Adelberger, et. al, Physical Review Letters **83**, 1299–1302 (1999).



Buffer Gas Cooling

- Buffer gas purification procedure
 - Uses gas filled 7 electrode purification of same design as SHIPTRAP and ISOLTRAP
 - Procedure:
 - Ions enter trap and exhibit the 3 motions
 - In presence of gas (~10-4 mbar), reduced cyclotron motion quickly damped and magnetron orbit increases (left image)
 - Magnetron and reduced cyclotron motions are coupled by exciting at pure cyclotron frequency. Ions of mass corresponding to this pure cyclotron excitation (desired ions) are centered (right image) while impurities continue to increase magnetron orbit
 - This continues until the desired ions are centered and have largely magnetron motion (but small amplitude)
 - Other ions either lost at electrodes or have large orbit
 - The remaining large radius impurities are lost upon ejection through a small diaphragm (which also acts to limit gas pressure in the measurement trap)
 - Results in purified bunch of ions exhibiting mostly magnetron motion



M. Saidur Rahaman, First On-line Mass Measurements at SHIPTRAP and Mass Determinations of Neutron-rich Fr and Ra Isotopes at ISOLTRAP, 2005.



Precision Mass Measurements

- Time of flight technique (measure pure cyclotron frequency)
 - Quadrupole RF applied at pure cyclotron frequency, ω_c , which couples magnetron and reduced cyclotron motion
 - Periodic conversion between ω_{+} and ω_{-} with period T_{conv}
 - Ion start out in magnetron motion (from purification) or can be excited into magnetron motion)
 - For a set conversion time, excitation frequency is scanned over
 - Most ion motion will be converted from magnetron to reduced cyclotron when pure cyclotron frequency applied
 - Ions are ejected from the trap. Passing through a negative gradient field (leaving the magnet), ions in the reduced cyclotron motion experience an axial force: F_z
 - Largest force is felt by the bunch for ions most exhibiting reduced cyclotron motion, yielding the greatest acceleration for the bunch, and shortest time of flight to detector
 - Shortest time of flight corresponds to the pure cyclotron frequency, which yields the mass





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Analytic Solution

- Follow a procedure similar to that in [1], but account for short endcaps and gaps between electrodes
 - Superpose potentials for each electrode
 - Expand each in Legendre polynomials
 - Expand potentials in Bessel functions
 - Equate two formulations and solve for Legendre coefficients
 - Apply boundary conditions (the proper V at its own surface and 0 elsewhere in addition to periodic boundary conditions) and orthogonality rules to solve for the Bessel coefficients
- Results in a complete description of the electric field around the trap center

 $V = V_0 \phi_0 + V_1 \phi_1 + V_2 \phi_2$

$$\begin{split} V &= \frac{1}{2} V_0 \sum_{\substack{k=0 \\ \text{even}}}^{\infty} B_k \left(\frac{r}{d}\right)^k P_k(\cos\theta) \\ \phi_0 &= \frac{1}{2} \sum_{\substack{k=0 \\ \text{even}}}^{\infty} C_k \left(\frac{r}{d}\right)^k P_k(\cos\theta) \\ B_k &= C_k + D_k \left(\frac{V_1}{V_0}\right) + E_k \left(\frac{V_2}{V_0}\right) \end{split}$$

$$V = V_i \sum_{n=0}^{\infty} A_n J_0(i\mathbf{k}_n \rho) \cos(k_n z)$$

$$C_{k} = \sum_{n=0}^{\infty} \frac{2 A_{n}^{C} d^{k} k_{n}^{k} (-1)^{k/2}}{k!}$$

$$A_n^{\ C} = \frac{(-1)^n - \sin(k_n(z_r + z_g + z_c + z_g)) - \sin(k_n z_r)}{k_n(z_{\text{tot}}) J_0(ik_n \rho_0)}$$

[1] G Gabrielse, L Haarsma, and S L Rolston, Journal of Mass Spectrometry 88, 319-332 (1989).



Larmour Precession

90 mm free Radius corresponds to a maximum proton energy of 4.75 MeV

Lifetime (ms)	E BD UVIEV I IIIIAX I
137.0560288845	4.2823691866
147.1548941707	3.9138712687
180.3368801111	3.6961394127
141.3841140071	3.3551881729
147.1548941707	2.5496376376
72.1347520444	3.7322422345
63.4785817991	1.2295907704
	137.0560288845 147.1548941707 180.3368801111 141.3841140071 147.1548941707 72.1347520444 63.4785817991



Why: Precision Mass Measurements

- Time of flight technique (measure pure cyclotron frequency)
 - Convert magnetron to reduced cyclotron motion by coupling with pure cyclotron frequency excitation
 - Ions in reduced cyclotron motion feel an accelerating force upon ejection due to B-field gradient
 - Scan over frequencies to find pure cyclotron, which is mass dependent



$$\overrightarrow{F_z} = -\overrightarrow{\bigtriangledown}(\overrightarrow{\mu} \cdot \overrightarrow{B}) = \overrightarrow{\mu} \frac{\delta B}{\delta z}$$

M. Saidur Rahaman, First On-line Mass Measurements at SHIPTRAP and Mass Determinations of Neutron-rich Fr and Ra Isotopes at ISOLTRAP, 2005.



• Tunable

- Electric field at trap center can be expanded as [1]:

$$V = \frac{1}{2} V_0 \sum_{\substack{k=0\\ \text{even}}}^{\infty} C_k \left(\frac{r}{d}\right)^k P_k(\cos \theta)$$

- Term C_4 and higher order terms describe the anharmonicity of the potential (C_4 dominant)
- These affect the eigenmotions in the following way [2]:

$$\begin{split} \Delta(\omega_{+}+\omega_{-})^{(4)} &= \frac{3C_4}{4z_0^2} \frac{\omega_z^2}{(\omega_{+}+\omega_{-})} (\rho_{+}^2 - \rho_{-}^2) \\ \Delta(\omega_{+}+\omega_{-})^{(6)} &= \frac{15C_6}{8z_0^4} \frac{\omega_z^2}{(\omega_{+}+\omega_{-})} [3z^2(\rho_{+}^2 - \rho_{-}^2) + (\rho_{+}^4 - \rho_{-}^4)] \end{split}$$

 To solve this problem, correction electrodes are added to "tune out" the anharmonic terms

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Diaphragm Size	% Error at .005	Deviation at .005	Gain at .005	% Error at .01	Deviation at .01		% Error at .05	Deviation at .05	
(mm)	mBar	mBar	mBar	mBar	mBar	Gain at .01 mBar	mBar	mBar	Gain at .05 mBar
1	3.16E-03	9.58E-06	G=1000	5.64E-03	9.49E-06	G=250	NA	NA	NA
2	1.24E-02	6.44E-06	G=100	4.02E-03	5.66E-06	G=100	1.90E-04	6.02E-06	G=250
4	4.52E-04	5.65E-06	G=25	2.57E-03	5.24E-06	G=50	7.24E-04	5.91E-06	G=100
6	2.22E-02	3.14E-05	G=10	9.32E-03	1.92E-05	G=25	2.48E-03	1.74E-05	G=50





Capacitance Manometer







TAMUTRAP

Dimensions (cm):

- Ring (/2): 1.15
- Compensation: 8.42257
- Endcap: 8
- Gaps: .05
- Inner Radius: 9
- Vc/Vo: -0.3708804
- Total Length (inner): 35.44514 (no spacing between caps and end electrodes)
- Characteristic distance: 8.1576

Comparison of Numerically Determined Expansion Coefficients (SIMION and Mathematica):

	Analytic	Expansion of SIMION output
C ₀	-0.5728	-0.5743
C ₁	-8.573*10 ⁻¹⁹	0.0004044
C ₂	0.4943	0.5189
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- Applied Physics PhD
 - No courses remaining after this semester
 - Project: characterize beam transport from K-150 cyclotron to entrance of magnet
 - Plan to do prelim before by end of Fall semester
 - Plan to defend in 2014



- β-v correlation looks at the relationship ۲ between the directions of the beta and neutrino emitted during a beta decay
- $a_{\beta\nu}$ is the parameter that quantifies this affect
 - Decay cross-section proportional to^[1]
 - $1 + \frac{p}{F} a_{\beta\nu} \cos (\theta_{\beta\nu})$ Different for different decays
 - For ³²Ar a[~] = .9989 ± 0.0052 (stat) ± 0.0039 (syst) ^[2]
- From $a_{\beta\nu}$ we can infer details on the involved currents and the charged weak interaction

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$$\xi = |M_{\rm F}|^2 (|C_{\rm S}|^2 + |C_{\rm V}|^2 + |C'_{\rm S}|^2 + |C'_{\rm V}|^2) + |M_{\rm GT}|^2 (|C_{\rm T}|^2 + |C_{\rm A}|^2 + |C'_{\rm T}|^2 + |C'_{\rm A}|^2)$$
(A.3)
$$a\xi = |M_{\rm F}|^2 \left\{ [-|C_{\rm S}|^2 + |C_{\rm V}|^2 - |C'_{\rm S}|^2 + |C_{\rm V}|^2] \pm \frac{\alpha Zm}{2} 2 \operatorname{Im} (C_{\rm S}C_{\rm V}^* + C'_{\rm S}C'_{\rm V}^*) \right\}$$

$$a\xi = |M_{\rm F}|^2 \left\{ [-|C_{\rm S}|^2 + |C_{\rm V}|^2 - |C_{\rm S}'|^2 + |C_{\rm V}|^2] \mp \frac{\alpha Zm}{p_{\rm e}} 2 \operatorname{Im} (C_{\rm S}C_{\rm V}^* + C_{\rm S}'C_{\rm V}^*) \right\} \\ + \frac{|M_{\rm GT}|^2}{3} \left\{ [|C_{\rm T}|^2 - |C_{\rm A}|^2 + |C_{\rm T}'|^2 - |C_{\rm A}'|^2] \pm \frac{\alpha Zm}{p_{\rm e}} 2 \operatorname{Im} (C_{\rm T}C_{\rm A}^* + C_{\rm T}'C_{\rm A}^*) \right\} (A.4)$$

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