Recent Theoretical Advances in Nuclear Cluster Physics

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CONTENT

The situation of alpha cluster states in 12C
The situation in 16O and heavier
Monopole transitions and alpha clustering in 16O, etc.
Extension of THSR to include valence neutrons
Alpha condensation in infinite matter
Alpha decay of 212Po
Alpha-rings
Conclusions
Hoyle family

Transition probabilities
Inelastic form factor to Hoyle state (Funaki et al.); very good agreement with data with no adjustable parameter! THSR and GFMC; Los Alamos, Pieper et al.

Very sensitive to size of Hoyle state! $\frac{V_{\text{Hoyle}}}{V_{\text{g.s.}}} \sim 4$

Kamimura, Hagaki, T. Neff et al.
Freer et al., also Cseh, but **P. O. Hess**: Pauli very important!!

**FIG. 4** (color online). Rotational band structure of the ground-state band, the Hoyle band, and the bending vibration in $^{12}\text{C}$.

**AMD, from Kanada-En’yo:**
T. Otsuka et al.: No core SM

- $0^+_1$: Concentrated in $14^{th}$ (3 clusters) & $15^{th}$ (compact shape) groups

- $0^+_2$: Scattered among all groups $\rightarrow$ Gas-like state?

also J. Draayer et al., K. Launey No inelastic form factor so far!
Thus, the 1 fm enhancement in the nuclear radius in $^{12}\text{C}(2^+_2)$ is exotic phenomena, which occurs in much lower excitation energy than the neutron-excess nucleus.

future studies. Since the experimental information of the differential cross section of the $2^+_2$ channel is still insufficient, the measurement of the differential cross section of the excited state at $E_x \sim 10$ MeV and careful MDA to separate the $2^+_2$ component should be extended over a wide angle and energy region.
Alpha condensation, 3-alpha decay out of Hoyle: S. Ishikawa

S-wave dominance: 80 percent
Meissner et al., PRL 109, 252501  Lattice QMC; lattice spacing: 2fm
Yasuro FUNAKI: extended THSR for 4 alpha’s in $^{16}O$, PRC 97, 021304.

experimental problem with 6-th $^0$ state around 15.1 MeV

\[ |\text{THSR} - 16O\rangle = \alpha^+ \alpha^+ \alpha^+ \alpha^+ |\text{vacuum}\rangle \]
Taiichi YAMADA: Monopole transitions in 16O:
THSR extension to alpha-clusters plus valence neutrons: M. Lyu et al.

New cluster approach on properties of $^{8-11}$Be isotopes with isospin-dependent spin-orbit potential

Neutron separation of $^{8-11}$Be

PRC 93, 054308 (2017)

M. Kimura: 14, 16C, chain states

FIG. 3. The one-neutron separation energies $S_n$ of $^{8-11}$Be isotopes. “THSR” denotes calculated results with the THSR wave function and “Exp” denotes the experimental values from Ref. [38].
Alpha condensation in infinite matter, Sogo et al., PRC 81, 064310

We also need the in-medium four-body Schrödinger equation for the order parameter, in analogy with the pairing case shown in Eq. (4) in the previous section. It is given by

\[ \varepsilon_{1234}(c_4c_3c_2c_1) + \sum_{1'2'3'4'} V_{1234;1'2'3'4'} (c_4'c_3'c_2'c_1') = 0, \]  

(12)

where

\[ V_{1234;1'2'3'4'} = (1 - \rho_1 - \rho_2) \frac{1}{2} \tilde{v}_{12;1'2'} \delta_{33'} \delta_{44'} + (1 - \rho_1 - \rho_3) \]

\[ \times \frac{1}{2} \tilde{v}_{13;1'3'} \delta_{22'} \delta_{44'} + \text{permutations}. \]  

(13)

FIG. 1. Graphic representation of the BCS mass operator in Eq. (2).

FIG. 4. Graphical representation of the approximate \( \alpha \)-BEC mass operator \( M_{\text{approx}} \) of Eq. (17).
Quantum Phase Transition in infinite matter and $^{16}$O

Lazauskas, Sogo et al., PRC 79, 051301

![Graph showing critical temperature of $\alpha$ and deuteron condensations as functions of chemical potential and density of free nucleon](image)

FIG. 2. Critical temperature of $\alpha$ and deuteron condensations as functions of (a) chemical potential and (b) density of free nucleon, derived from Eq. (4) for the $\alpha$ particle and Eq. (11) for the deuteron. Crosses ($\times$) correspond to calculations of Eq. (1) with the Malfliet-Tjon interaction (MT I-III) using the FY method.
QPT in finite nuclei
Gogny force D1S, M. Girod;

'RMF', JP. Ebran
Quartetting

α - Particles Only Exist in Low Density BCS Phase

No BCS phase (dense phase) of α-particles possible!

Bose-Einstein-Condensation of α-particles (dilute)
Alpha pocket at the surface of $^{208}\text{Pb}$
Chang Xu, G. Roepke et al., PRC 95;

Double folding: Adamian et al., PRC 94

Future challenge: alpha decay of deformed $^{226}\text{Ra}$

FIG. 13. The same as in Fig. 12, but for the asymmetric reactions indicated. The dotted line is obtained with the same parameters as the solid line, but with $a = 0.39$ fm for $^4\text{He}$. 
7-alpha ring measured?? Nature-article: Natowitz et al.

Figure 1. Left (right) panel shows the 6α (8α) ring structure in $^{24}\text{Mg}$ ($^{32}\text{S}$) with constrained HFB calculations [28].
Conclusions

Immense activity in nuclear cluster physics
Mostly for self-conjugate nuclei but also with valence neutrons (protons)
Progress with Hoyle family of states in $^{12}\text{C}$
Some progress with alpha gas states in $^{16}\text{O}$ but experimental ambiguities with sixth zero plus state at about 15.1 MeV

Alpha-condensation is a Quantum-Phase-transition

Alpha-decay $^{212}\text{Po}$

Alpha rings
All started with this....

THANK YOU!