Container evolution and dynamics of cluster formation

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Container picture and THSR ansatz

Hoyle band and other excited states in $^{12}$C

Container evolution for $^{16}$O
Container picture and THSR ansatz

Hoyle band and other excited states in $^{12}$C

Container evolution for $^{16}$O
First success of container picture for ordinary cluster state

Characterized by rel. distance parameter $R_z$. Localized clustering.

\[
\psi^{\text{Brink}}_{20\text{Ne}}(R_z, b) = \mathcal{A} \left\{ \exp \left( \frac{8(r_z - R_z)^2}{5b^2} \right) \phi_\alpha(b) \phi_{16O}(b) \right\}
\]

VS

Characterized by the size of container $B$. Non-localized clustering.

\[
\Phi^{\text{THSR}}_{20\text{Ne}}(\beta, b) = \mathcal{A} \left\{ \exp \left( \sum_k \frac{8r_k^2}{5(b^2 + 2\beta_k^2)} \right) \phi_\alpha(b) \phi_{16O}(b) \right\}
\]

\[
B_k^2 = b^2 + 2\beta_k^2 \quad (k = x, y, z)
\]

B. Zhou et al., PRC86, 014301 (2012); PRL 110, 262501(2013); PRC 89, 3319 (2014).
First success of container picture for ordinary cluster state

The energy levels of $\alpha^+{^{16}}\text{O}$ inversion doublet bands in $^{20}\text{Ne}$

- (0.9987) $^3_-$
- (0.9998) $^1_-$
- (0.9775) $^4_+$
- (0.9879) $^2_-$
- (0.9929) $^0_+$

In parentheses

\[
\left| \Phi_{^{20}\text{Ne}}^{\text{THSR}} \left| \sum_{R_z} f(R_z) \Psi_{^{20}\text{Ne}}^{\text{Brink}}(R_z, b) \right| \right|^2
\]

Localized clustering.

\[ B \]

Characterized by rel. distance parameter $R_z$.

Non-localized clustering.

\[ B \]

Characterized by the size parameter $B$.

Brink GCM

\[ B \]

Experiment

\[ B \]

B. Zhou et al., PRC86, 014301 (2012); PRL 110, 262501(2013); PRC 89, 3319 (2014).
Container picture succeeds in describing other non-gaslike cluster states.

\[ \text{Overlap between cont. w.f. and full sol.} \]

\[
\begin{align*}
J=0: & \quad 0.9873 & (\beta_x=\beta_y,\beta_z)=(0.1,5.1) \\
J=2: & \quad 0.9887 & (\beta_x=\beta_y,\beta_z)=(0.1,5.4) \\
J=4: & \quad 0.9806 & (\beta_x=\beta_y,\beta_z)=(0.1,6.6)
\end{align*}
\]

Strongly prolate

The superposition of 100 localized w.fs. coincides with one one-dim. cont. w.f.!

\[ \text{Overlap between cont. w.f. and full sol.} \]

\[
\begin{align*}
J=0: & \quad 0.9440 & (\beta_x=\beta_y,\beta_z)=(0.1,8.2) \\
J=2: & \quad 0.9417 & (\beta_x=\beta_y,\beta_z)=(0.1,8.4) \\
J=4: & \quad 0.9307 & (\beta_x=\beta_y,\beta_z)=(0.1,9.0)
\end{align*}
\]

Strongly prolate

The superposition of 300 localized w.fs. coincides with one one-dim. Cont. w.f.!

Overlap between cont. w.f. and full sol.

\[
\begin{align*}
J=0: & \quad 0.995 & (\beta_x=\beta_y,\beta_z)=(1.6,3.0) \\
J=2: & \quad 0.994 & (\beta_x=\beta_y,\beta_z)=(0.1,3.0) \\
J=4: & \quad 0.977 & (\beta_x=\beta_y,\beta_z)=(0.1,2.1)
\end{align*}
\]

Small size

Y. F. et al., PTEP (2014) 113D01.
This, in general, gives "container" picture of nuclear clustering

Nonlocalized clustering
Characterized by a size parameter $B$ of the container, corresponding to nuclear size.

$^8\text{Be}(0^+)$  Y. F. et al., PTP108, 297(2002);

$^{12}\text{C}(0^+_2)$  Y. F. et al., PRC67, 051306(R)(2003); PRC80, 064326(2009).

$^{20}\text{Ne} (^{16}\text{O}-\alpha)$  B. Zhou et al., PRC86, 014301 (2012); PRL 110, 262501 (2013); PRC 89, 034319 (2014).

$3\alpha$ and $4\alpha$ linear chain states  T. Suhara et al., PRL112, 062501 (2014)

$^{9-10}\text{Be}$  M. Lyu et al., PRC91, 014313 (2015); 93, 054308 (2016).

$^9\text{B}$  Q. Zhao et al., arXive: 1801.05964.

$^9\Lambda\text{Be}(0^+)$  Y. F. et al., PTEP 2014, 113D01

$^{12}\text{C}(0_1^+)$  Y. F. et al., PRC80, 064326 (2009).  B. Zhou et al., PTEP 2014, 101D01.

Almost equivalent to the w.fs. obtained by the corresponding full cal. (RGM/GCM)

Providing basic understanding of the nuclear clustering

This, in general, gives "container" picture of nuclear clustering.

Nonlocalized clustering
Characterized by a size parameter $B$ of the container, corresponding to nuclear size.

New!
$^{12}\text{C}(3_{1}^{-})$ by B. Zhou

$^{8}\text{Be}(0^{+})$ Y. F. et al., PTP108, 297(2002);

$^{12}\text{C}(0_{2}^{+})$ Y. F. et al., PRC67, 051306(R)(2003); PRC80, 064326(2009).

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Almost equivalent to the w.fs. obtained by the corresponding full cal. (RGM/GCM)

Providing basic understanding of the nuclear clustering

Container picture and THSR ansatz

Hoyle band and other excited states in $^{12}\text{C}$

Container evolution for $^{16}\text{O}$
3α extended THSR wave function

\[ \Phi_\alpha(\beta, b) = \exp \left( -2 \sum_{k}^{x,y,z} \frac{r_k^2}{b^2 + 2\beta_k^2} \right) \phi_\alpha(b) \]

\[ \Phi^{e\text{THSR}}_{12C}(\beta_1, \beta_2, b) = \Psi_G^{-1} \mathcal{A}\{\Phi_\alpha(\beta_1, b)\Phi_\alpha(\beta_1, b)\Phi_\alpha(\beta_2, b)\} \]

\(\Psi_G\): Total center-of-mass w.f. to be eliminated

Internal w.f. of \(\alpha\) particle

\(b=1.35\text{ fm}:\text{ fixed}\)

\[ \phi_\alpha(b) := \]

\[ b \]

\(b\)
3α extended THSR wave function

\[
\Phi_\alpha(\beta, b) = \exp \left( -2 \sum_{k} \frac{r_k^2}{b^2 + 2\beta_k^2} \right) \Phi_\alpha(b)
\]

\[
\Phi_{12c}^{e\text{THSR}}(\beta_1, \beta_2, b) = \Psi_G^{-1} \mathcal{A}\{\Phi_\alpha(\beta_1, b) \Phi_\alpha(\beta_1, b) \Phi_\alpha(\beta_2, b)\}
\]

\(\Psi_G\): Total center-of-mass w.f. to be eliminated

Hill-Wheeler eq. or GCM (generator coordinate method)

\[
\sum_{\beta'_1, \beta'_2} \left\langle \hat{P}_{MK}^{J} \Phi_{12c}^{e\text{THSR}}(\beta_1, \beta_2, b) \right| \hat{H} - E \left| \hat{P}_{MK}^{J} \Phi_{12c}^{e\text{THSR}}(\beta'_1, \beta'_2, b) \right\rangle f(\beta'_1, \beta'_2) = 0
\]

\(\hat{P}_{MK}^{J}\): Angular momentum projection operator

Hamiltonian (NN force: Volkov No.2 force)

\[
\hat{H} = -\frac{\hbar^2}{2m} \sum_{i} \nabla_i^2 - T_G + \sum_{i<j} (V_{ij}^{(N)} + V_{ij}^{(C)})
\]

Spurious continuum components are effectively eliminated by \(r^2\) constraint method.

Results (of $^{12}$C)


Decay widths (unit: MeV)
- $\Gamma(0^+_3) = 1.1$ (Exp: 1.45)
- $\Gamma(0^+_4) = 0.6$ (Exp: 1.42)
- $\Gamma(2^+_2) = 1.1$ (Exp: 1.01)
- $\Gamma(4^+_2) = 2.4$ (Exp: 1.7)

Numbers with arrows: EM transition strengths (unit: $e^2fm^4$ for $B(E2)$, $efm^2$ for $M(E0)$)

New states recently observed

Hoyle state
$3\alpha$'s orbiting in an S-wave

3-\alpha breakup threshold
Results (of $^{12}\text{C}$)


Decay widths (unit: MeV)

- $\Gamma(0^+_3) = 1.1$ (Exp: 1.42)
- $\Gamma(0^+_4) = 0.6$ (Exp: 1.42)
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- $\Gamma(4^+_2) = 2.4$ (Exp: 1.7)

New observed states are consistently reproduced.

Large spatial size
3.7 fm $\sim$ 4.7 fm

except for the shell-model-like states
$(0^+_1, 2^+_1, 4^+_1 : \sim 2.4$ fm)

All excited states above the threshold are governed by cluster dynamics.
New observed states are consistently reproduced.

Rich alpha cluster dynamics built on the Hoyle state, as if the Hoyle state were the g.s. of cluster excitations

All excited states above the threshold are governed by cluster dynamics
$0_3^+$ state: higher nodal excitation of the Hoyle state

Overlap functions of the $0_1^+$, $0_2^+$, $0_3^+$ states for $^8\text{Be}(0^+)+\alpha(S)$ channel

$0_1^+$ state: 2 nodes
$0_2^+$ state: 3 nodal oscillation (nodes disappear due to the dissolution of $^8\text{Be}$ core)
$0_3^+$ state: 4 nodes (higher nodal structure)

\[ M(E0; \ 0_3^+ \rightarrow 0_2^+) = 34.5 \]

Very large monopole transition strength between the $0_2^+$ and $0_3^+$ states
c.f. $M(E0; \ 0_2^+ \rightarrow 0_1^+) = 6.4$

$r(S^2 \sim 1.8)$
$r(S^2 \sim 1.1)$
Squared overlap with single THSR config. for the $0_4^+$ state of $^{12}$C

For the $0_4^+$ state

Clear linear-chain structure

For the $0_4^+$ state

$\beta_z > \beta_x = \beta_y$

Prolate shape

$\beta_z = \beta_x$

Spherical

$\beta_z < \beta_x = \beta_y$

Oblate shape
Rotational structure of the Hoyle band

**Fragmented into the Hoyle state and $0_3^+$ state**

The Hoyle state is not a simple bandhead of $\alpha + ^8\text{Be}$ rotation

Specificity of the Hoyle state as the 3-alpha condensate

$\alpha$ rotates outside the core $\rightarrow \alpha$ rotates inside the core ($\alpha$ cond.)

$B(E2; 4_2^+ \rightarrow 2_2^+) = 591$

$B(E2; 2_2^+ \rightarrow 0_2^+) = 295$

$B(E2; 2_2^+ \rightarrow 0_3^+) = 104$

$B(E2; 2_2^+ \rightarrow 0_4^+) = 27$

$\Gamma(0_4^+) = 0.6$ (Exp: 1.42)

$\Gamma(0_3^+) = 1.1$ (Exp: 1.45)
Possible interpretations of these bands

Rich alpha cluster dynamics built on the Hoyle state, as if the Hoyle state were the g.s. of cluster excitations

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4\alpha extended THSR wave function

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\[ \Phi_{16O}^{e\text{THSR}}(\beta_1, \beta_2, b) = \Psi_G^{-1} \mathcal{A}\{\Phi_\alpha(\beta_1, b)\Phi_\alpha(\beta_1, b)\Phi_\alpha(\beta_1, b)\Phi_\alpha(\beta_2, b)\} \]

\Psi_G\): Total center-of-mass w.f. to be eliminated

Internal w.f. of \alpha particle

b=1.44 fm: fixed

\[ \phi_\alpha(b) := \]

\[ b \]

\[ \beta_1 \]

\[ \beta_2 \]
4$\alpha$ extended THSR wave function

$$
\Phi_\alpha(\beta, b) = \exp \left( -2 \sum_{k} \frac{r_k^2}{b^2 + 2\beta_k^2} \right) \Phi_\alpha(b)
$$

$$
\Phi^{\text{THSR}}_{16O}(\beta_1, \beta_2, b) = \Psi_G^{-1} \mathcal{A} \{ \Phi_\alpha(\beta_1, b) \Phi_\alpha(\beta_1, b) \Phi_\alpha(\beta_1, b) \Phi_\alpha(\beta_2, b) \}
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Hill-Wheeler eq. or GCM (generator coordinate method)

$$
\sum_{\beta_1', \beta_2'} \left\langle \hat{P}^J_{MK} \Phi_{16O}^{\text{THSR}}(\beta_1, \beta_2, b) \right| \hat{H} - E \left| \hat{P}^J_{MK} \Phi_{16O}^{\text{THSR}}(\beta_1', \beta_2', b) \right\rangle f(\beta_1', \beta_2') = 0
$$

$\hat{P}^J_{MK}$: Angular momentum projection operator

Hamiltonian (NN force: F1 force)

$$
\hat{H} = -\frac{\hbar^2}{2m} \sum_i V_i^2 - T_G + \sum_{i<j} (V_{ij}^{(N)} + V_{ij}^{(C)}) + \sum_{i<j<k} V_{ijk}^{(N)}
$$

A. Tohsaki, PRC 49, 1814 (1994).

Spurious continuum components are effectively eliminated by $r^2$ constraint method.

$J^\pi = 0^+$ spectra

Y. Suzuki, PTP 55, 1751 (1976); 56, 111 (1976).

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<th>$e$THSR</th>
<th>$\alpha^{+12}$ C OCM</th>
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<td>$M(E0: 0_1^+ \rightarrow 0_2^+)$</td>
<td>5.9</td>
<td>3.88</td>
<td>3.66 ± 0.55</td>
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<td>$M(E0: 0_1^+ \rightarrow 0_3^+)$</td>
<td>5.7</td>
<td>3.50</td>
<td>4.40 ± 0.44</td>
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$J^\pi = 0^+$ spectra

$^{12}$C(1$^-$) + $\alpha$ (P) structure is difficult to describe by the present eTHSR but extension is possible.

Talk by B. Zhou in this morning.

$^{12}$C(0$^+$) + $\alpha$ (S)

$^{12}$C(2$^+$) + $\alpha$ (D)

$^{12}$C(1$^-$) + $\alpha$ (P)

4$\alpha$ cond.

$\alpha^{+12}$C OCM

Y. Suzuki, PTP 55, 1751 (1976); 56, 111 (1976).

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Squared overlap surface with single config. of eTHSR

\[
(\beta_{1x} = \beta_{1y}, \beta_{1z}): \text{fixed at } \otimes
\]

Container for 3 \(\alpha\)

\(\times\) : maximum

For the fourth \(\alpha\)

\[
|\langle \Phi(\beta_1, \beta_2)|0^+_I\rangle|^2
\]

Prolate shape \((\beta_z > \beta_x = \beta_y)\)

Spherical \((\beta_z = \beta_x)\)

Oblate shape \((\beta_z < \beta_x = \beta_y)\)

tetrahedral

\(\beta_z \perp \)
Squared overlap surface with single config. of eTHSR

$0^+_II$

Spherical

$^{12}\text{C}(0^+_1)+\alpha$

(\(\beta_{1x} = \beta_{1y}, \beta_{1z}\)): fixed at \(\otimes\)

Container for 3 \(\alpha\)

\(\times\) : maximum

For the fourth \(\alpha\)

\[ |\langle \Phi(\beta_1, \beta_2)|0^+_\lambda\rangle|^2 \]

\(\beta_1\)  \(\beta_2\)  THSR+GCM

<table>
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<th>Sq. overlap</th>
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<td>$^{12}\text{C}(0^+_1)$</td>
<td>0.93</td>
<td>(1.9, 1.8fm)</td>
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<td>$^{12}\text{C}(2^+_1)$</td>
<td>0.90</td>
<td>(1.9, 0.5fm)</td>
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<td>$^{12}\text{C}(0^+_2)$</td>
<td>0.99</td>
<td>(5.6, 1.4fm)</td>
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Squared overlap surface with single config. of eTHSR

Deformed $^{12}\text{C}(2^+_1)+\alpha$

$(\beta_{1x} = \beta_{1y}, \beta_{1z})$: fixed at $\otimes$

Container for 3 $\alpha$

$\times$: maximum

For the fourth $\alpha$

$|\langle \Phi(\beta_1, \beta_2)|0^+_\lambda\rangle|^2$

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Squared overlap surface with single config. of eTHSR

- Prolate shape: \( \beta_z > \beta_x = \beta_y \)
- Spherical shape: \( \beta_z = \beta_x = \beta_y \)
- Oblate shape: \( \beta_z < \beta_x = \beta_y \)

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<th>( ^{12}\text{C}(0_1^+) + \alpha )</th>
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\( \beta_{1x} = \beta_{1y}, \beta_{1z} \): fixed at \( \bigotimes \)

Container for 3 \( \alpha \)

\( \times \): maximum

For the fourth \( \alpha \)

\[ |\langle \Phi(\beta_1, \beta_2) | 0_1^+ \rangle|^2 \]
Squared overlap surface with single config. of eTHSR

4α in a common container

\(0^+_V\)

\(0.01\) step

\(0.1\) step

Prolate shape
\((\beta_z > \beta_x = \beta_y)\)

Spherical
\((\beta_x = \beta_y)\)

Oblate shape
\((\beta_z < \beta_x = \beta_y)\)

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<tr>
<th>(\text{Atomic Mass} )</th>
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\((\beta_{1x} = \beta_{1y}, \beta_{1z})\): fixed at \(\oplus\)

Container for 3α

\(\times\) : maximum

For the fourth \(\alpha\)

\(|\langle \Phi(\beta_1, \beta_2)|0^+_\alpha \rangle|^2\)
50th anniversary of Ikeda diagram (1968)

Be

C

O

Ne

Mg

C

C

C

O

O

Ne

Mg
“Container” evolution
Thanks to my Collaborators

Bo Zhou (Nanjing U.)
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