Systematic investigation of the Hoyle-analog states in light nuclei.

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Outline

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   - Three-cluster AMHHB Model
2. **Theoretical analysis**
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   - Hoyle-analog states in ^{11}B and ^{11}C
   - Hoyle states in ^{10}B
   - Hoyle states in ^{9}Be and ^{9}B
   - Shape of triangles
   - Hoyle-analog states in light nuclei

Motivations

- Synthesis of light nuclei in stellar environment
  - Resonance states in three-cluster continuum
  - Nature of resonance states in three-cluster continuum
  - **Hoyle-analog states**

Inhomogeneous BBN reactions

Inhomogeneous BBN reactions involve reaction chains such as

$$^1H(n,\gamma)^2H(n,\gamma)^3H(d,n)^4He(t,\gamma)^7Li(n,\gamma)^8Li,$$
$$^8Li(\alpha,n)^{11}B(n,\gamma)^{12}B(\beta^-)^{12}C(n,\gamma)^{13}C,$$

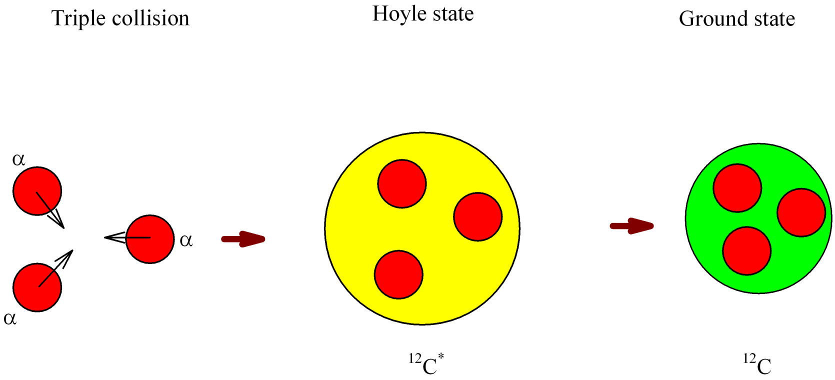
and

$$^7Li(n,\gamma)^8Li(n,\gamma)^9Li(\beta^-)^9Be,^7Li(^3He,p)^9Be,$$
$$^7Li(t,n)^9Be(t,n)^{11}B,$$

C.A. Bertulani, T. Kajino. “Frontiers in nuclear astrophysics”, Progr. Part. Nucl. Phys., **89**, 56, 2016.

Fred Hoyle and Hoyle state

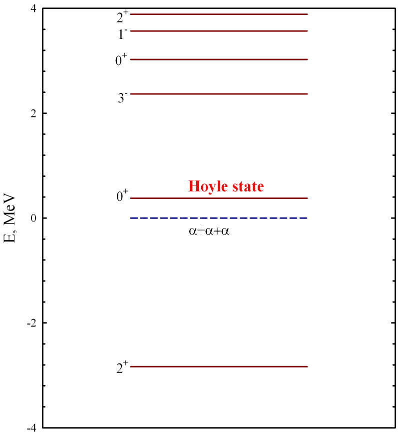
Synthesis of ^{12}C. Hoyle state



$$3\alpha \Rightarrow ^{12}C^{*} \Rightarrow ^{12}C + \gamma$$

F. Hoyle. "On Nuclear Reactions Occurring in Very Hot Stars. I. The Synthesis of Elements from Carbon to Nickel." Astrophys. J. Suppl., **1**, 121, 1954.

^{12}C. Hoyle state



- Hoyle state is the 0^+ resonance state in ^{12}C
- Energy of the Hoyle state is E = 0.4 MeV above the $\alpha+\alpha+\alpha$ threshold or E = 7.65 MeV above the ^{12}C ground state
- Very small width of the Hoyle state Γ = 8.5 eV, compare with width of the 0^+ resonance state in ^{8}Be Γ = 5.57 eV

- F. Hoyle. “On Nuclear Reactions Occurring in Very Hot Stars. I. The Synthesis of Elements from Carbon to Nickel.” Astrophysical Journal Supplement, vol. 1, p.121 (1954).
- M. Freer, H. O. U. Fynbo, “The Hoyle state in ^{12}C”, Prog. Part. Nucl. Phys., **78**, pp. 1-23, 2014

^{12}C. Hoyle state

Two quotations from F. Hoyle paper [1].

1. It was pointed out some years ago by Bethe [2] that effective element-building inside starts must proceed, in the absence of hydrogen, by triple collisions as a starting point:

$$3\alpha \rightarrow ^{12}C + \gamma. \quad (1)$$

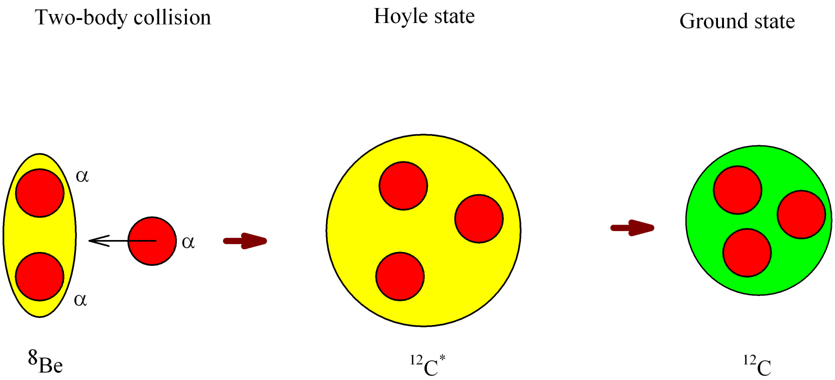
2. It is convenient to replace reaction (1) by

$$\alpha + \alpha \leftrightarrow ^{8}Be, \quad ^{8}Be + \alpha \rightarrow ^{12}C + \gamma. \quad (2)$$

This is a permissible step, since the lifetime of the unstable ^{8}Be is appreciably longer than the time required for nuclear collision of two α particles; that is, longer than the α particle radius divided by the relative velocity.

1. F. Hoyle. "On Nuclear Reactions Occurring in Very Hot Stars. I. The Synthesis of Elements from Carbon to Nickel." Astrophys. J. Suppl., **1**, 121, 1954.
2. H.A. Bethe. Phys. Rev., "Energy Production in Stars." **55**, 434, 1939.

Synthesis of ^{12}C. Hoyle state



$$2\alpha \Rightarrow^{8} Be + \gamma, \quad \alpha +^{8} Be \Rightarrow^{12} C + \gamma$$

The 0^+ resonance in ^{8}Be: E=91.8 keV, Γ = 5.57$\pm$0.25 eV, $\tau \approx 10^{-16}$sec.

Synthesis of light nuclei in triple collision

General scheme

$$A_1 + A_2 + A_3 \Rightarrow A^* \Rightarrow A + \gamma,$$

where A^* denotes excited (resonance) state of a compound system.
First step, excitation of resonance state

$$\boxed{A_1 + A_2 + A_3 \Rightarrow A^*}.$$

Second step, transition from resonance to bound state $A^* \Rightarrow A + \gamma$.

- $^9Be = \alpha + \alpha + n$
- $^9B = \alpha + \alpha + p$
- $^{10}B = \alpha + \alpha + d$
- $^{11}B = \alpha + \alpha +^3 H$
- $^{11}C = \alpha + \alpha +^3 He$

The Algebraic Model for scattering with the Hyperspherical Harmonics Basis (AMHHB)

- V. Vasilevsky, A. V. Nesterov, F. Arickx, J. Broeckhove. *Phys. Rev. C*, vol. **63**, 034606 (16 pp), 2001.
- V. Vasilevsky, A. V. Nesterov, F. Arickx, J. Broeckhove. *Phys. Rev. C*, vol. **63**, 034607, 2001.
- V. Vasilevsky, A. V. Nesterov, F. Arickx, and J. Broeckhove. *Phys. Rev. C*, vol. **63**, 064604, 2001.
- J. Broeckhove, F. Arickx, P. Hellinckx, V. S. Vasilevsky, A. V. Nesterov. *J. Phys. G Nucl. Phys.*, vol. **34**, 1955, 2007.
- V. S. Vasilevsky, K. Katō, N. Zh. Takibayev. Phys. Rev. C, **96**, 034322, 2017.

Alternative methods

- Complex Scaling Method (K. Katō, et al)

  T. Myo, Y. Kikuchi, H. Masui and K. Katō, "Recent development of complex scaling method for many-body resonances and continua in light nuclei", Prog. Part. Nucl. Phys., 79, 1, 2014.

- Antisymmetrized Molecular Dynamics (H. Horiuchi, Y. Kanada-En'yo, et al)
- Fermionic Molecular Dynamics (H. Feldmeier, T. Neff)
- Microscopic R-Matrix Method (D. Baye, P. Descouvemont)
- No-Core Shell Model (P. Navrátil, S. Quaglioni)

Three-cluster model and Hyperspherical Harmonics Method

- V. Vasilevsky, A. V. Nesterov, F. Arickx, J. Broeckhove. *Phys. Rev. C*, vol. **63**, 034606, 2001.
- Korennov, S. and Descouvemont, P. *Nucl. Phys. A*, **740**, 249, 2004.
- S. Quaglioni, C. Romero-Redondo, and P. Navrátil, *Phys. Rev. C*, **88**, 034320, 2013.

Quest for the Hoyle-analog states

1. Kanada-En'yo, Y. "Negative parity states in ^{11}B and ^{11}C and the similarity with ^{12}C", *Phys. Rev. C*, **75**, 024302, 2007.
2. Kanada-En'yo, Y. and Suhara, T. and Kobayashi, F., "Cluster states in ^{11}B, ^{11}C and ^{8}He and their similarity to ^{12}C", *J. Phys. Confe. Ser.*, **321**, 012009, 2011.
3. Yamada, T. and Funaki, Y. "$\alpha + \alpha + t$ cluster structures and $^{12}C(0_2{}^+)$-analog states in ^{11}B", *Phys. Rev. C*, **82**, 064315, 2010.
4. Yamada, T. and Funaki, Y. "Three-Body Cluster Structures and Hoyle-Analogue States in ^{11}B", *Progr. Theor. Phys. Suppl.*, **196**, 388, 2012.
5. Y. Chiba, M. Kimura "Hoyle-analogue state in ^{13}C studied with Antisymmetrized Molecular Dynamics", arXiv:1801.00562, 2018.

Three-cluster wave function

Wave function for three *s*-shell clusters in the *LS* coupling scheme:

$$\Psi^J = \widehat{\mathcal{A}}\left\{\left[\Phi_1\left(A_1\right)\Phi_2\left(A_2\right)\Phi_3\left(A_3\right)\right]_S\ f_L\left(\mathbf{x},\mathbf{y}\right)\right\}_J$$

where

- $\Phi_\nu\left(A_\nu\right)$ is a shell-model wave function for the internal motion of α-particle ($\nu = 1, 2, 3$) (fixed)
- $f\left(\mathbf{x},\mathbf{y}\right)$ is a wave function of inter-cluster motion (to be determined)
- $\widehat{\mathcal{A}}$ is the antisymmetrization operator.

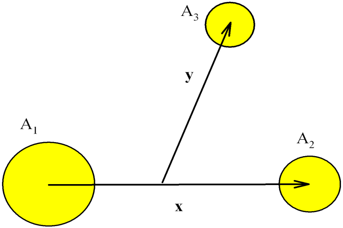
Hyperspherical coordinates

Hyperspherical coordinates: $\{\mathbf{x},\mathbf{y}\} \Rightarrow \{\rho,\Omega\}$, $\Omega = \{\theta,\widehat{\mathbf{x}},\widehat{\mathbf{y}}\}$

$$\rho = \sqrt{\mathbf{x}^2+\mathbf{y}^2} \quad \text{hyper-radius,}$$

$$\widehat{\mathbf{x}} = \mathbf{x}/\left|\mathbf{x}\right|,\ \widehat{\mathbf{y}} = \mathbf{y}/\left|\mathbf{y}\right| \quad \text{unit vectors}$$

$$\left|\mathbf{x}\right| = \rho\cos\theta, \quad \left|\mathbf{y}\right| = \rho\sin\theta$$



Expansion of wave function

$$f_L(\mathbf{x},\mathbf{y}) = \sum_{l_x,l_y} f_{l_x,l_y;L}(x,y)\left\{Y_{l_x}(\widehat{\mathbf{x}})\,Y_{l_y}(\widehat{\mathbf{y}})\right\}_L$$

$$= \sum_c \psi_c(\rho)\,\mathcal{Y}_c(\Omega) = \sum_c \sum_{n_\rho=0}^{\infty} C_{n_\rho,c} R_{n_\rho,K}(\rho,b)\,\mathcal{Y}_c(\Omega)$$

Hyperspherical Harmonics

Here $c = \{K, l_x, l_y, L\}$, $\mathcal{Y}_c(\Omega)$ is a hyperspherical harmonic

$$\mathcal{Y}_c(\Omega) = \chi_{K,l_x,l_y}(\theta) \cdot \left\{ Y_{l_x}(\widehat{\mathbf{x}})\, Y_{l_y}(\widehat{\mathbf{y}}) \right\}_L$$

and $R_{n_\rho,K}(\rho, b)$ is a six-dimensional oscillator function

$$R_{n_\rho,K}(\rho, b) = (-1)^{n_\rho} \mathcal{N}_{n_\rho,K}\, r^K \exp\left\{-\frac{1}{2}r^2\right\} L_{n_\rho}^{K+3}\left(r^2\right),$$

$$r = \rho/b, \quad \mathcal{N}_{n_\rho,K} = b^{-3}\sqrt{\frac{2\Gamma(n_\rho+1)}{\Gamma(n_\rho+K+3)}},$$

and b is an oscillator length.

Hyperspherical Harmonics

Basis of the Hyperspherical Harmonics

$$\begin{aligned}\Psi_J &= \sum_c \widehat{\mathcal{A}}\left\{\Phi_1(A_1)\,\Phi_2(A_2)\,\Phi_3(A_3)\,\psi_c(\rho)\,\mathcal{Y}_c(\Omega)\right\} \\ &= \sum_c \sum_{n_\rho=0}^{\infty} C_{n_\rho,c}\left|n_\rho, c\right\rangle\end{aligned}$$

where $\left\{C_{n_\rho,c}\right\}$ is a wave function in the oscillator representation, $\left|n_\rho, c\right\rangle$ is a three-cluster oscillator function

$$\left|n_\rho, c\right\rangle = \widehat{\mathcal{A}}\left\{\Phi_1(A_1)\,\Phi_2(A_2)\,\Phi_3(A_3)\,R_{n_\rho,K}(\rho, b)\,\mathcal{Y}_c(\Omega)\right\}$$

Oscillator basis and linear algebraic equations

The many-particle Schrödinger equation

$$\left(\widehat{H}-E\right)\Psi=0$$

in oscillator representation

$$\sum_{\widetilde{c}}\sum_{\widetilde{n}_\rho=0}^{\infty}\left[\left\langle n_\rho,c\left|\widehat{H}\right|\widetilde{n}_\rho,\widetilde{c}\right\rangle-E\left\langle n_\rho,c|\widetilde{n}_\rho,\widetilde{c}\right\rangle\right]C_{\widetilde{n}_\rho,\widetilde{c}}=0$$

Numerical realization

$$\sum_{\widetilde{c}=1}^{N_c}\sum_{\widetilde{n}_\rho=0}^{N}\left[\left\langle n_\rho,c\left|\widehat{H}\right|\widetilde{n}_\rho,\widetilde{c}\right\rangle-E\left\langle n_\rho,c|\widetilde{n}_\rho,\widetilde{c}\right\rangle\right]C_{\widetilde{n}_\rho,\widetilde{c}}=0$$

plus proper boundary conditions (Details in V. Vasilevsky et al. *Phys. Rev. C*, vol. **63**, 034606, 2001).

Main stages of our calculations

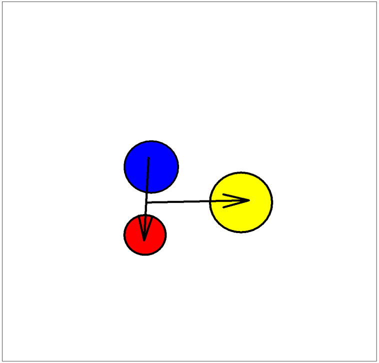
1. By solving the system of linear equations, we obtain the scattering S matrix
$$\left\| S_{c,\widetilde{c}} \right\|,$$
where $c, \widetilde{c} = 1, 2, \ldots, N_{ch}$
2. By analyzing behavior of the S matrix as a function of energy, we extract parameters of resonance states.
3. We study resonance wave functions in coordinate and oscillator representations

We are going to search for the Hoyle-analog states in light nuclei

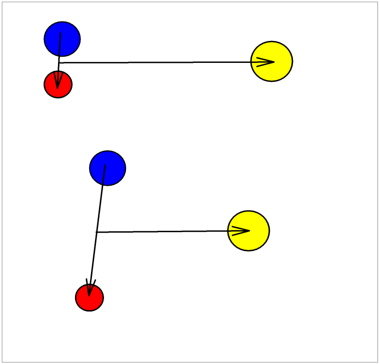
Reference	Nucleus	Configuration
Phys. Atom. Nucl., 77, 555, 2014	^{9}Be	$\alpha+\alpha+n$
	^{9}B	$\alpha+\alpha+p$
Phys. Rev., C96, 034322, 2017	^{9}Be	$\alpha+\alpha+n$
	^{9}B	$\alpha+\alpha+p$
Ukr. J. Phys., 59, 1065, 2014	^{10}B	$\alpha+\alpha+d$
Ukr. J. Phys., 58, 544, 2013	^{11}B	$\alpha+\alpha+^{3}H$
	^{11}C	$\alpha+\alpha+^{3}He$
Phys. Rev., C85, 4607, 2012	^{12}C	$\alpha+\alpha+\alpha$

Wave function in Oscillator representation.

Small N_{sh}



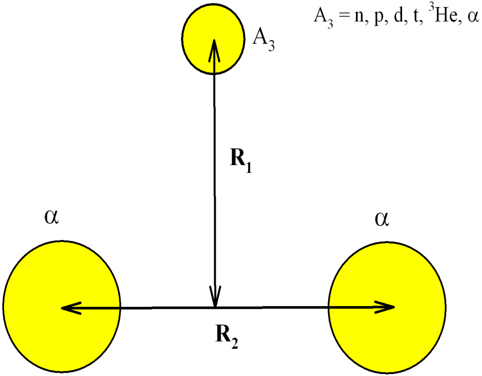
Large N_{sh}



Condensed information about wave function

$$W_{sh}\left(N_{sh}\right) = \sum_{n_{\rho},K \in N_{sh}} \left|C_{n_{\rho},K}\right|^{2}, \quad N_{sh} = 0, 1, 2, \ldots$$

Inter-cluster distances R_1 and R_2



$$R_1^2 = \int d\mathbf{x} d\mathbf{y} \left| f(\mathbf{x}, \mathbf{y}) \right|^2 \cdot \mathbf{y}^2$$

$$R_2^2 = \int d\mathbf{x} d\mathbf{y} \left| f(\mathbf{x}, \mathbf{y}) \right|^2 \cdot \mathbf{x}^2$$

Normalization condition:

$$\left\langle \Psi_{E,J} | \Psi_{\widetilde{E},J} \right\rangle = \sum_{n_\rho, c} C^{E,J}_{n_\rho, c} C^{\widetilde{E},J}_{n_\rho, c} = \delta\left(E - \widetilde{E}\right).$$

Theoretical set-up

Input parameters

NN potential

Minnesota potential, modified Hasegawa-Nagata potential

Basis

Hypermomentum: $K_{\max} = 14$ for even parity states
Hypermomentum: $K_{\max} = 13$ for odd parity states
Hyperradial excitations: $n_\rho \leq 100$ ($\sim 200\ \hbar\Omega$)

Adjustable parameters

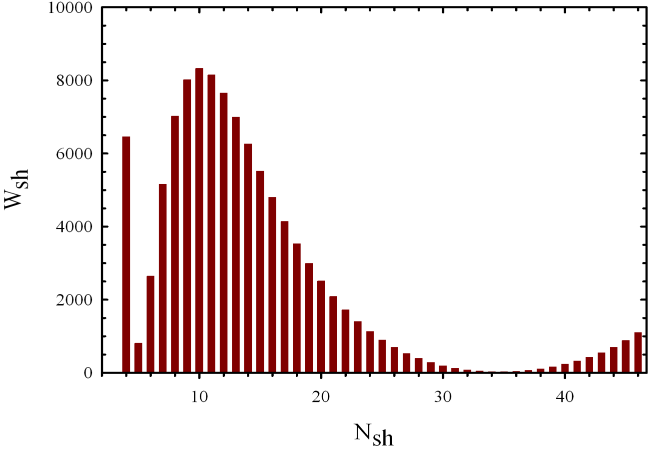
Oscillator length b: $E_{th}(b) = min\{E(A_1) + E(A_2) + E(A_3)\}$
Majorana parameter m of NN potential: $E_{GS}(m) = E_{GS}^{(Exp.)}$

Hoyle state and other resonance states in ^{12}C.

	CSM [1]		AMHHB [2]	
J^{π}	E, MeV	Γ, keV	E, MeV	Γ, keV
0^{+}	0.76	2.4	0.68	2.9
	1.66	1480	5.16	534
2^{+}	2.28	1100	2.78	10
	5.14	1900	3.17	280
	6.82	240	5.60	0.6
1^{-}	3.65	0.30	3.52	0.21

1. C. Kurokawa, K. Katō. *Nucl. Phys. A*, **792**, 87, 2007.
2. V. Vasilevsky, et al. *Phys. Rev. C*, **85**, 034318, 2012.

Wave function of the Hoyle state (E= 0.39 MeV, Γ= 1 eV) in ^{12}C.

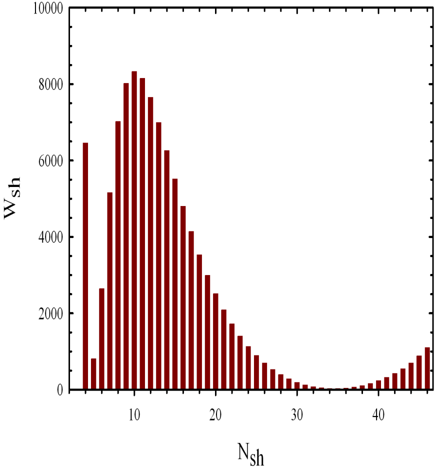


Similar results:

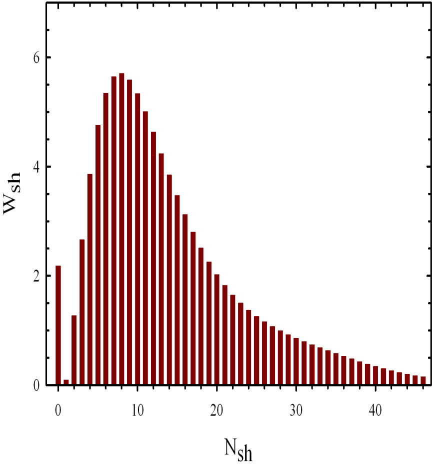
1. C. Kurokawa, K. Katō. *Nucl. Phys. A*, **792**, 87, 2007.
2. T. Neff, H. Feldmeier. *Few-Body Syst.*, **45**, 145, 2009.

Wave functions of the 0^+ resonance states in ^{12}C

First resonance, Γ=1 eV

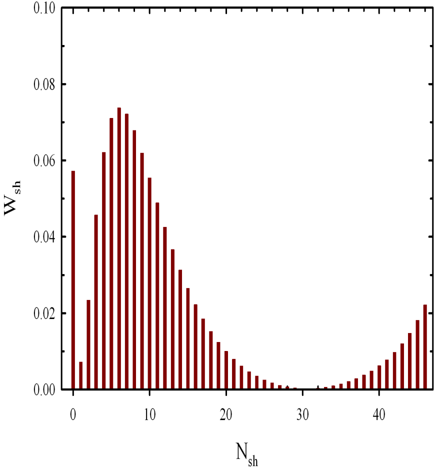


Second resonance, Γ=692 keV

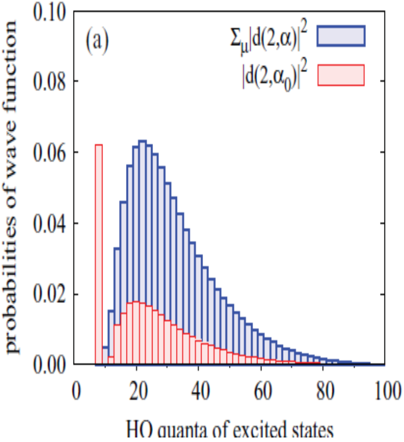


Wave functions of the 0^+ resonance state in ^{12}C

AMHHB



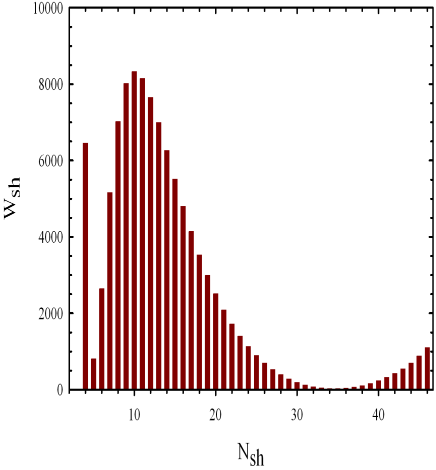
CSM



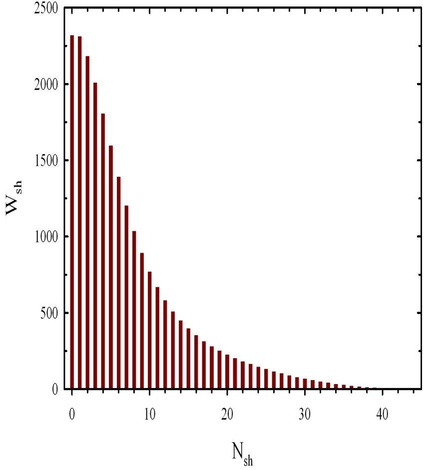
CSM: C. Kurokawa, K. Katō. *Nucl. Phys. A*, **792**, 87, 2007.

Wave functions of the 0^+ and 1^- resonance states in ^{12}C

0^+ state, Γ=1 eV



1^- state, Γ=979 eV



Candidates for the Hoyle state

The Hoyle state

Criteria

1. Resonance state which is close to three-cluster threshold (E is small)
2. Resonance state with very small width (Γ is small)
3. Resonance state with zero values of the total orbital momentum (L=0)
4. Large values of W_{sh} in the internal region $0 \leq N_{sh} \leq 45$

Candidates for the Hoyle state in ^{11}B

J^{π}	E, MeV	Γ, keV	J^{π}	E, MeV	Γ, keV
$\frac{3}{2}^{-}$	0.755	0.58	$\frac{1}{2}^{+}$	0.437	15.26
	1.402	185.18		0.702	12.30
	1.756	143.72		1.597	15.95
$\frac{1}{2}^{-}$	1.436	374.64	$\frac{3}{2}^{+}$	1.147	1.50
	1.895	100.95		1.367	8.58
	2.404	450.07		1.715	41.24
$\frac{5}{2}^{-}$	0.583	$5.14\cdot10^{-4}$	$\frac{5}{2}^{+}$	1.047	1.54
	1.990	32.63		1.951	40.20
	2.251	138.87		2.265	54.73
	2.905	120.46		2.748	167.61

V. S. Vasilevsky. “Microscopic three-cluster description of ^{11}B and ^{11}C nuclei”, Ukr. J Phys., 58, pp. 544-553, 2013.

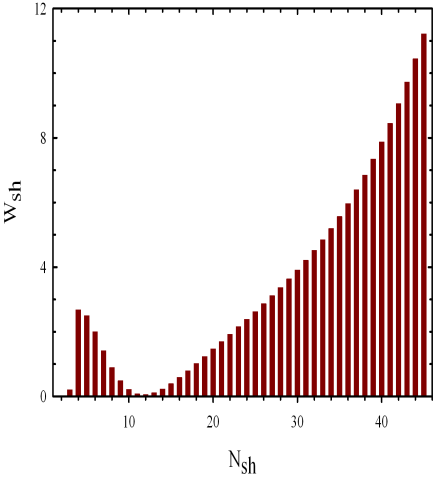
Candidates for the Hoyle state in ^{11}C

J^{π}	E, MeV	Γ, keV	J^{π}	E, MeV	Γ, keV
$\frac{3}{2}^{-}$	0.805	$9.93{\cdot}10^{-3}$	$\frac{1}{2}^{+}$	0.906	162.94
	1.920	105.08		1.930	59.88
	2.324	619.76		2.679	86.69
$\frac{1}{2}^{-}$	1.142	0.71	$\frac{3}{2}^{+}$	2.268	34.25
	2.266	790.98		2.478	159.28
	3.014	366.15		2.850	115.19
$\frac{5}{2}^{-}$	0.783	$9.65{\cdot}10^{-5}$	$\frac{5}{2}^{+}$	1.460	0.90
	1.897	5.77		2.346	82.72
	3.026	182.69		3.179	122.75

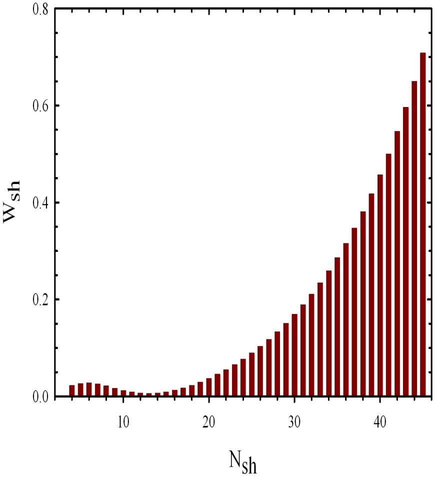
V. S. Vasilevsky. “Microscopic three-cluster description of ^{11}B and ^{11}C nuclei”, Ukr. J Phys., 58, pp. 544-553, 2013.

Wave function of $1/2^+$ state

^{11}B, E=0.44 MeV, Γ=15 keV

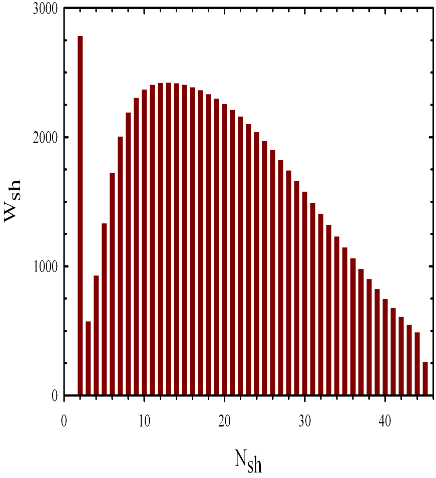


^{11}C, E=0.91 MeV, Γ=163 keV

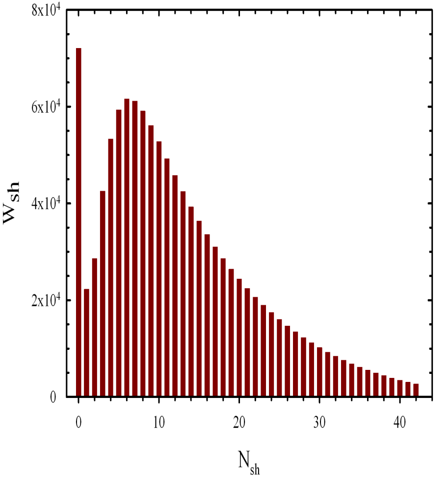


Wave function of $3/2^-$ state

^{11}B, E=0.76 MeV, Γ=580 eV

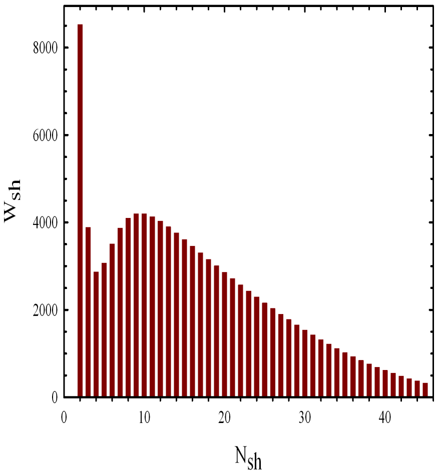


^{11}C, E=0.81 MeV, Γ=9 eV

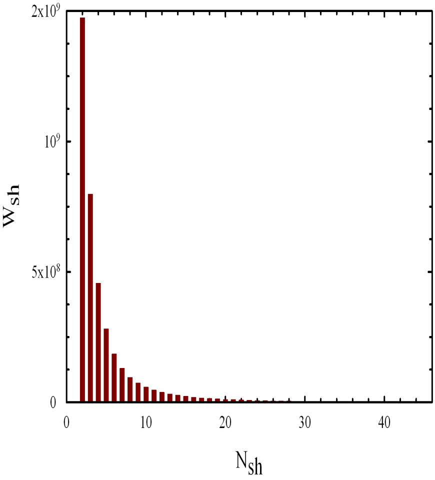


Wave function of $5/2^-$ states

^{11}B, E=0.58 MeV, Γ=0.5 eV

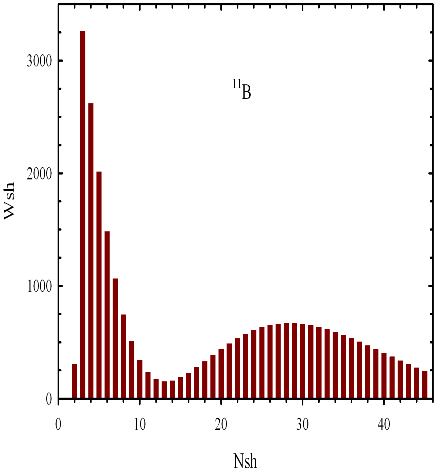


^{11}C, E=0.78 MeV, Γ=0.1 eV

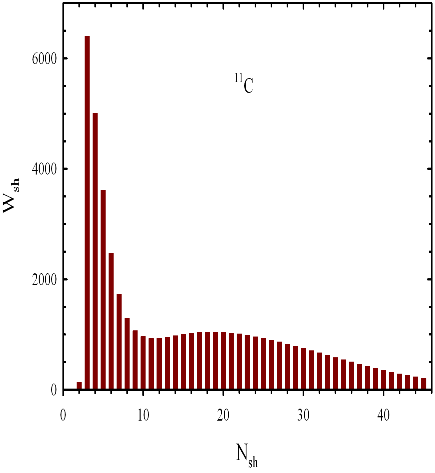


Wave function of $5/2^+$ states

^{11}B, E=1.05 MeV, Γ=1.5 keV



^{11}C, E=1.46 MeV, Γ=0.9 keV



OCM+CSM versus AMHHB: ^{11}B

	OCM+CSM [1,2]			AMHHB		
J^{π}	E, MeV	Γ, keV	HAS	E, MeV	Γ, keV	HAS
$1/2^{+}$	0.75	190	Y	0.44	15	N
$3/2_{3}^{-}$	-2.90	-	-	-0.59	-	-
$3/2_{4}^{-}$				0.76	0.58	Y

1. Yamada, T. and Funaki, Y. "$\alpha + \alpha + t$ cluster structures and $^{12}C(0_2{}^{+})$-analog states in ^{11}B", *Phys. Rev. C*, **82**, 064315, 2010.
2. Yamada, T. and Funaki, Y. "Three-Body Cluster Structures and Hoyle-Analogue States in ^{11}B", *Progr. Theor. Phys. Suppl.*, **196**, 388, 2012.

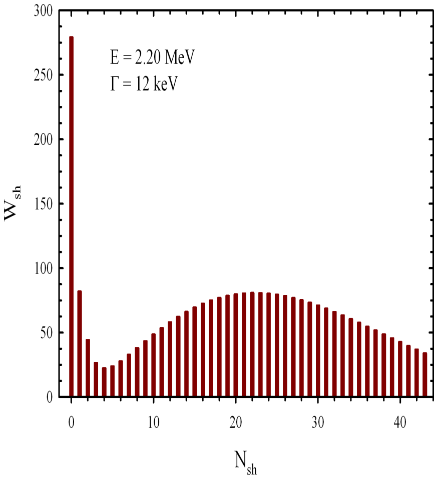
Candidates for the Hoyle state in ^{10}B

J^{π}	E, MeV	Γ, keV	Γ/E	R_1, fm	R_2, fm
1^+	0.604	232.30	0.384		
	0.987	7.08	7.17×10^{-3}	6.67	10.67
	1.536	196.36	0.128		
2^+	1.055	12.06	11.43×10^{-3}	6.64	10.83
	2.810	170.74	0.061		
3^+	1.062	11.73	11.05×10^{-3}	6.43	10.35
	2.202	526.47	0.239		
1^-	1.100	76.75	0.070	9.31	10.84
	1.820	562.71	0.309		

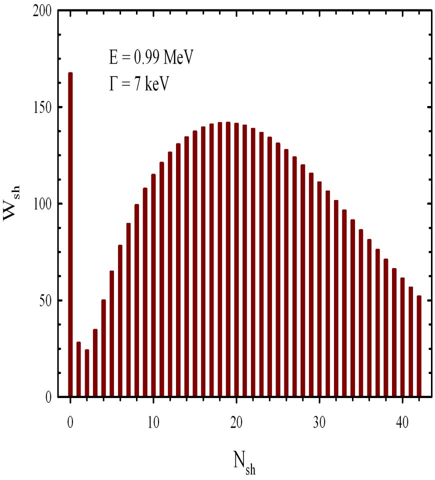
A. V. Nesterov, V. S. Vasilevsky, T. P. Kovalenko. “Spectrum of bound states of nucleus ^{10}B in a three-cluster microscopic model.” Ukr. J. Phys., **59**, 1065, 2014.

Wave function of 3^+ and 1^+ resonance states in ^{10}B.

^{10}B, $J^\pi = 3^+$



^{10}B, $J^\pi = 1^+$

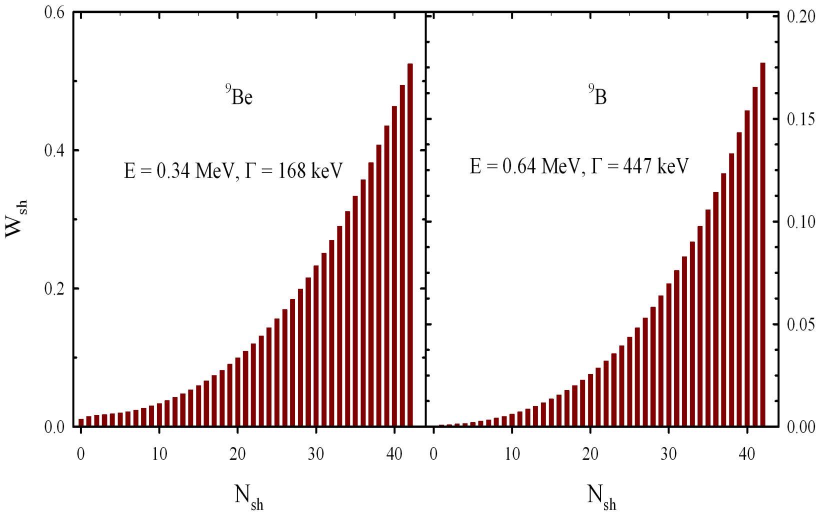


Candidates for the Hoyle state in ^9Be and ^9B

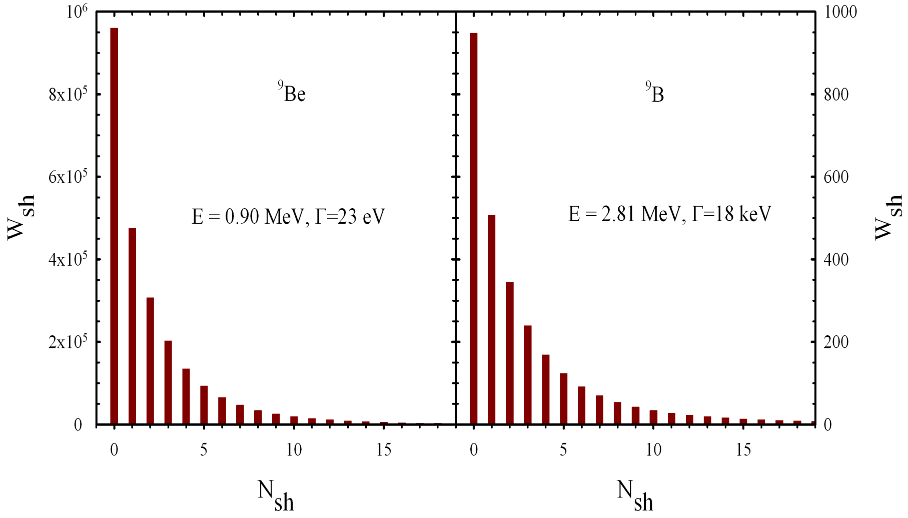
^{9}Be			^{9}B		
J^π	E, MeV	Γ, MeV	J^π	E, MeV	Γ, MeV
$3/2^-$	-1.574	-	$3/2^-$	0.379	1.076×10^{-6}
$1/2^+$	0.338	0.168	$1/2^+$	0.636	0.477
$5/2^-$	0.897	2.36×10^{-5}	$5/2^-$	2.805	0.018
$5/2^+$	2.086	0.112	$3/2^+$	2.338	2.796
$3/2_2^-$	2.704	2.534	$1/2^-$	3.398	3.428
$1/2^-$	2.866	1.597	$5/2^+$	3.670	0.415
$3/2^+$	4.062	1.224	$3/2_2^-$	3.420	3.361
$7/2^-$	4.766	4.041	$5/2_2^-$	5.697	5.146

V. S. Vasilevsky, K. Katō, N. Takibayev. “Formation and decay of resonance states in ^{9}Be and ^{9}B nuclei. Microscopic three-cluster model investigations”, Phys. Rev. C, **96**, 034322, 2017.

Wave function of the $1/2^+$ resonance states in ^9Be and ^9B

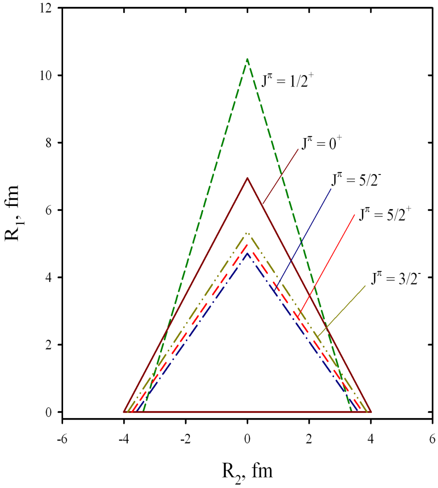


Wave function of the $5/2^-$ resonance state in ^9Be and ^9B.

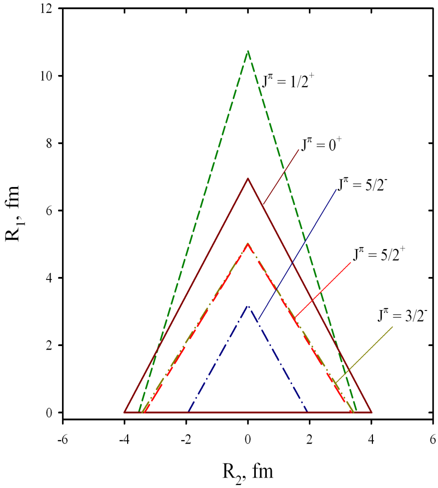


Shape of triangles. Resonance states in ^{11}B and ^{11}C.

^{11}B

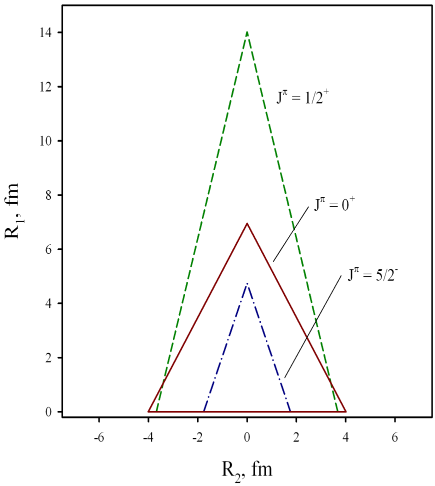


^{11}C

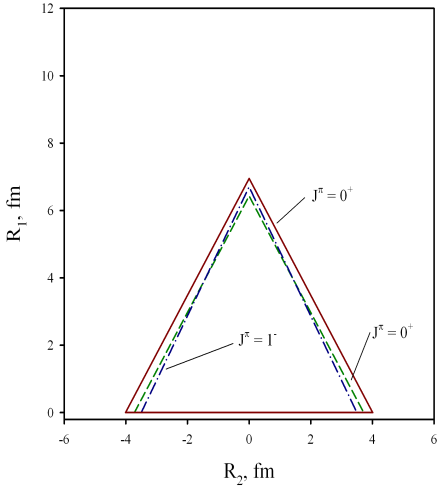


Shape of triangles. Resonance states in ^{9}Be and ^{12}C.

^{9}Be

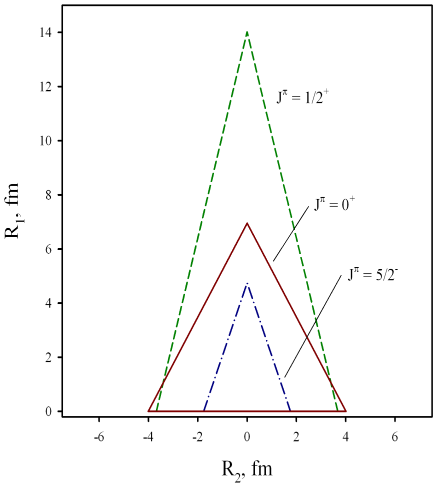


^{12}C

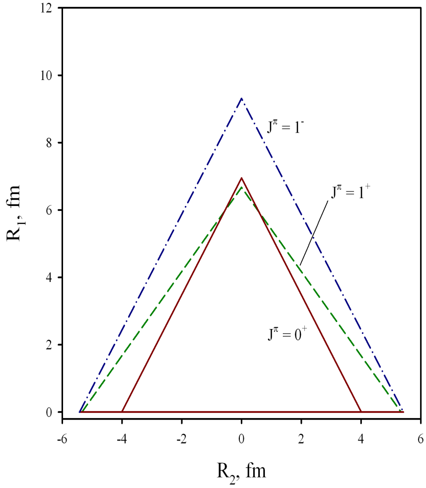


Shape of triangles. Resonance states in ^{9}Be and ^{10}B.

^{9}Be



^{10}B

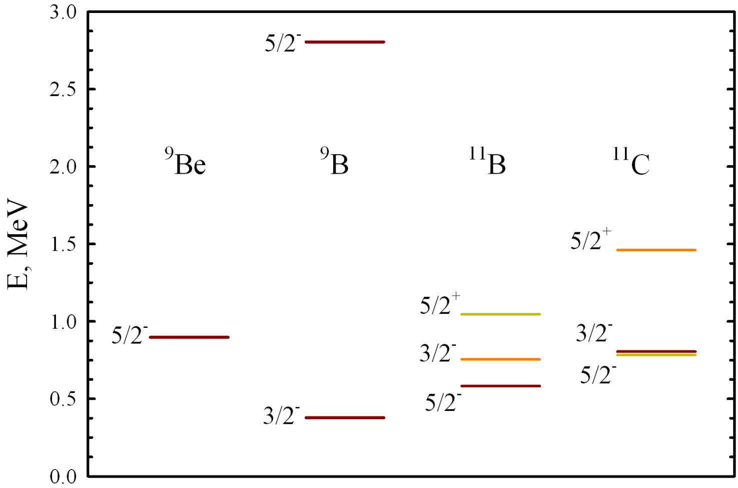


Energy and width of the Hoyle-analog states

Nucleus	Partition	J^{π}	E, MeV	Γ, keV	Γ/E
^{9}Be	$\alpha+\alpha+n$	$5/2^{-}$	0.897	$2.36{\cdot}10^{-2}$	$2.63{\cdot}10^{-5}$
^{9}B	$\alpha+\alpha+n$	$3/2^{-}$	0.379	$1.08{\cdot}10^{-3}$	$2.84{\cdot}10^{-6}$
		$5/2^{-}$	2.805	$18.0{\cdot}10^{-3}$	$6.42{\cdot}10^{-6}$
^{11}B	$\alpha+\alpha+^{3}H$	$5/2^{-}$	0.583	$5.14{\cdot}10^{-4}$	$8.87{\cdot}10^{-7}$
		$3/2^{-}$	0.755	0.58	7.70×10^{-4}
		$5/2^{+}$	1.047	1.54	1.47×10^{-3}
^{11}C	$\alpha+\alpha+^{3}He$	$3/2^{-}$	0.805	$9.93{\cdot}10^{-3}$	$1.23{\cdot}10^{-5}$
		$5/2^{-}$	0.783	$9.64{\cdot}10^{-5}$	$1.23{\cdot}10^{-7}$
		$5/2^{+}$	1.460	0.90	6.16×10^{-4}
^{12}C	$\alpha+\alpha+\alpha$	0^{+}	0.395	$1.14{\cdot}10^{-3}$	$2.88{\cdot}10^{-6}$

New criterion for the Hoyle-analog states: $\Gamma/E < 2\cdot10^{-3}$

Energy and width of the Hoyle-analog states



Summary

- Hoyle-analog states in light nuclei:
  - $5/2^-$ state in ^9Be
  - $3/2^-$ $5/2^-$ states in ^9B
  - $3/2^-$, $5/2^-$ and $5/2^+$ states in ^{11}B
  - $3/2^-$, $5/2^-$ and $5/2^+$ states in ^{11}C
- We did not find the Hoyle-analog state in ^{10}B
- Hoyle-analog state is a very narrow resonance state with a fairly compact three-cluster configuration
- Synthesis of light nuclei ^9Be, ^{11}B and ^{11}C is possible in a triple collision of $\alpha + \alpha + n$, $\alpha + \alpha +^3 H$ and $\alpha + \alpha +^3 He$, respectively

THANK YOU VERY MUCH!