# Recent Theoretical Advances in Nuclear Cluster Physics

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## CONTENT

The situation of alpha cluster states in 12C

The situation in 16O and heavier

Monopole transitions and alpha clustering in 16O, etc.

Extension of THSR to include valence neutrons

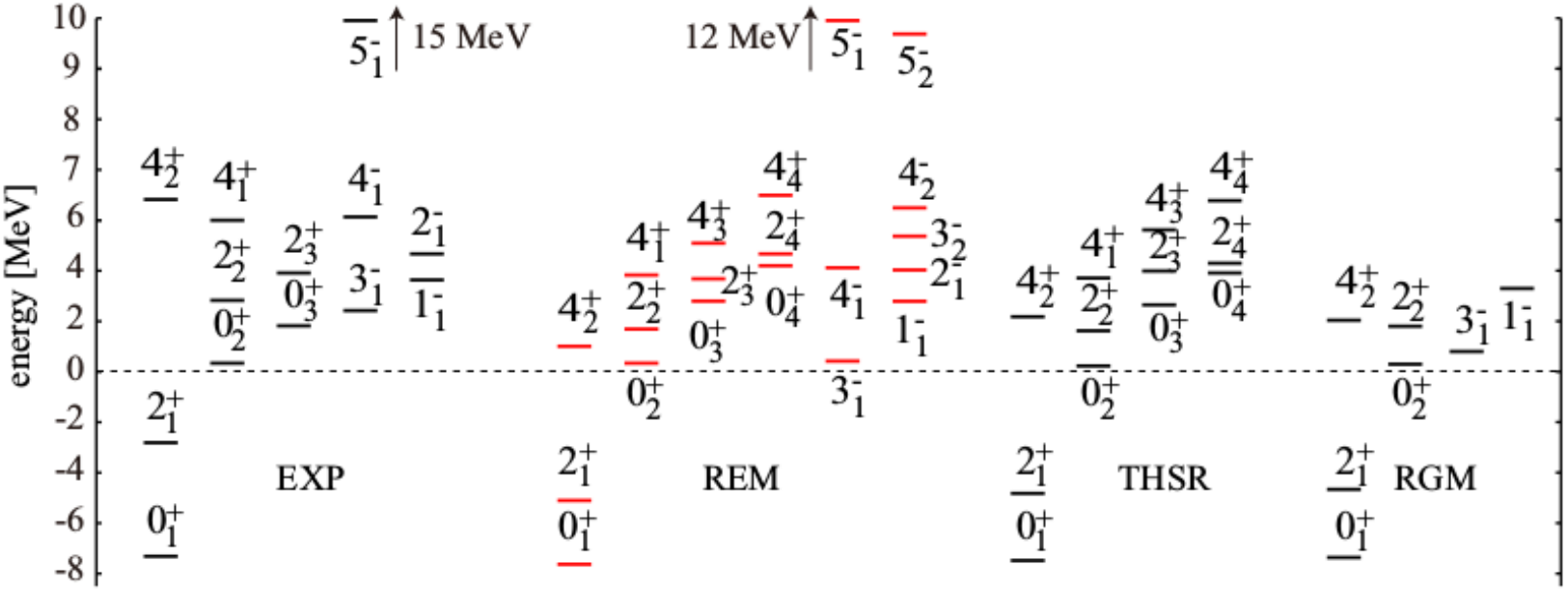
Alpha condensation in infinite matter

Alpha decay of 212Po

Alpha-rings

Conclusions

# Hoyle family




arXiv:1802.03523, to appearMaasaki KIMURA; Yasuro FUNAKI PRC 92, 021302; **C. Kurokawa, K. Kato: 3rd and 4th $0^+$ states**, NPA 792, 87

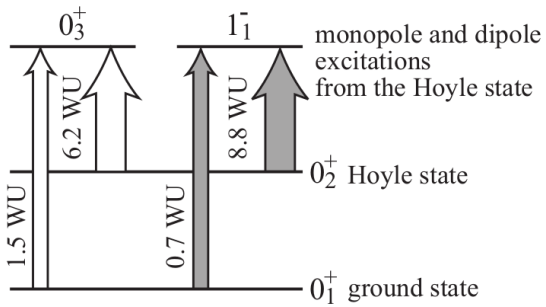
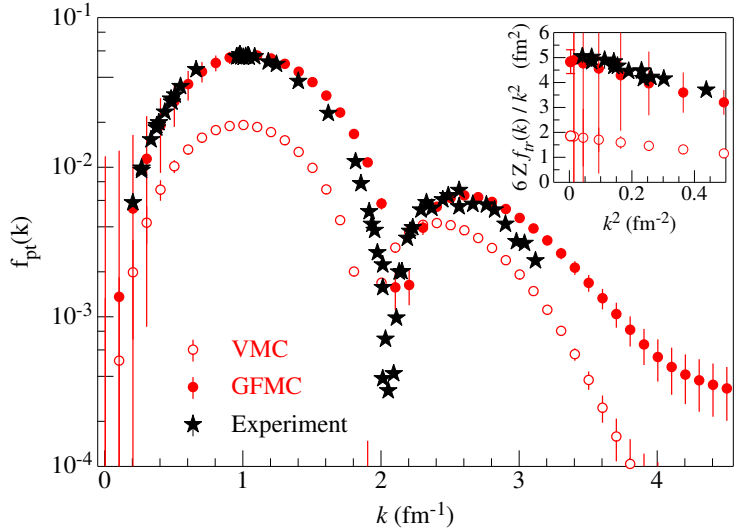
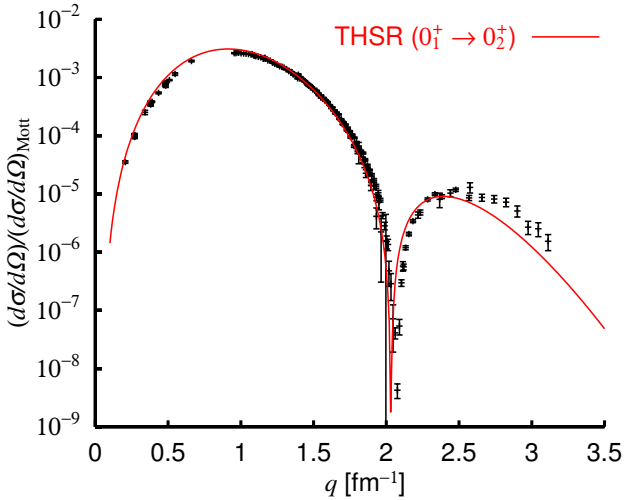



FIG. 9. Excitation modes of the Hoyle state are schematically shown. Arrows show the monopole and dipole transitions.

Transition probabilities

**Inelastic form factor to Hoyle state (Funaki et al.); very good agreement with data with no adjustable parameter! THSR** and **GFMC; Los Alamos, Pieper et al.**




**Very sensitive to size of Hoyle state!** $\frac{V_{\rm Hoyle}}{V_{\rm g.s.}} \sim 4$

Yasuro FUNAKI et al., Phys. Scripta 91, 123001 (2016)

Freer et al., also Cseh, but P. O. Hess: Pauli very important!!

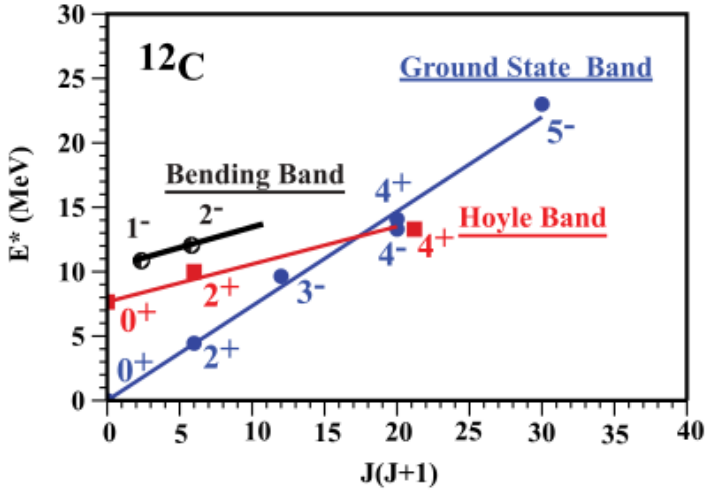
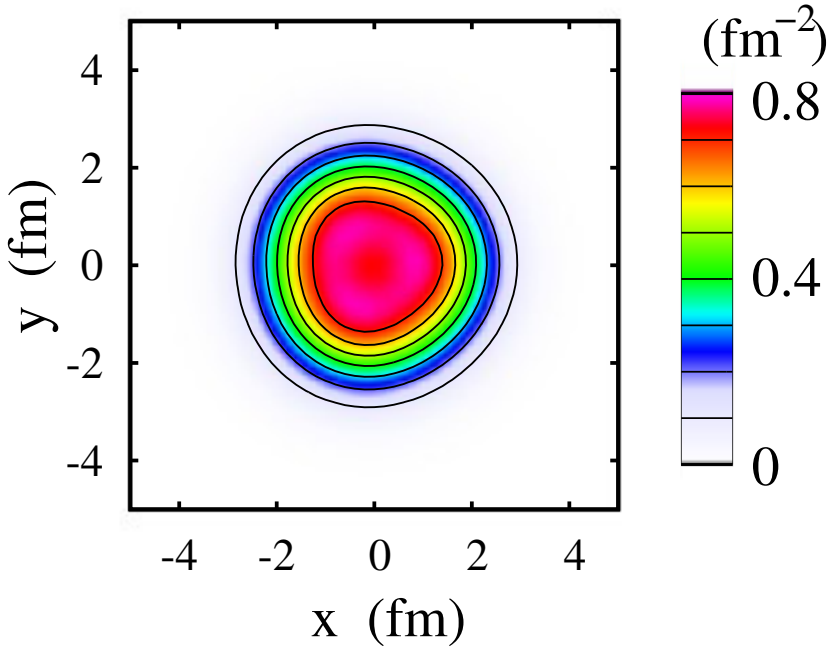
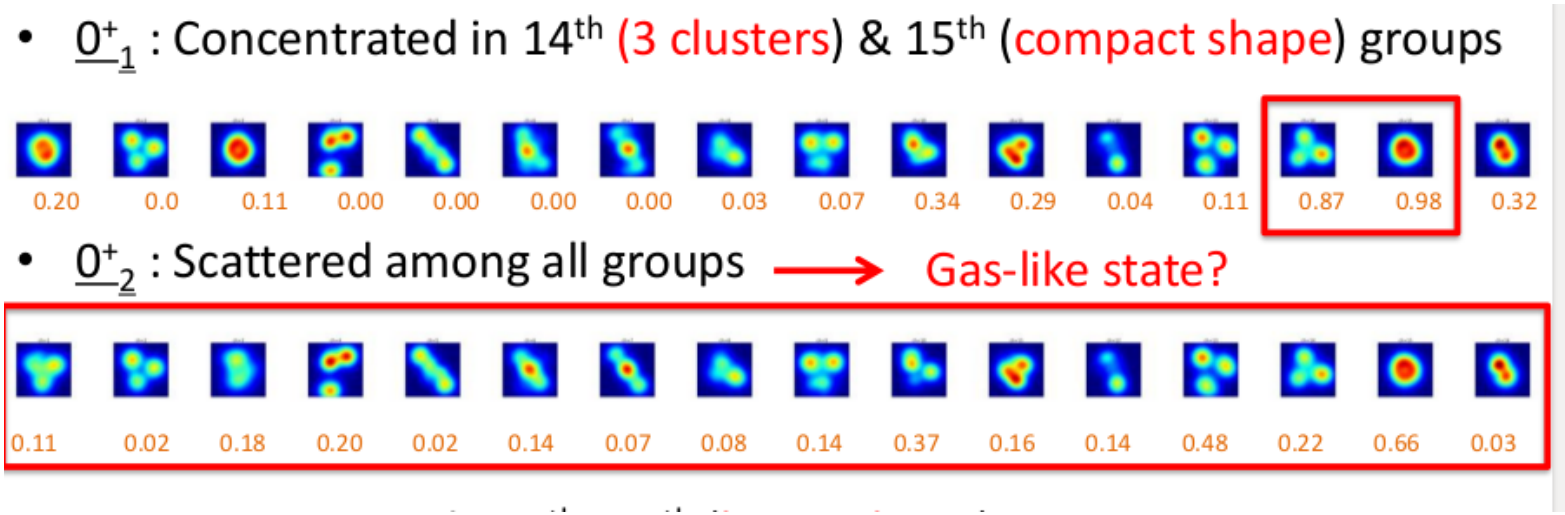



FIG. 4 (color online). Rotational band structure of the ground-state band, the Hoyle band, and the bending vibration in $^{12}$C.

AMD, from Kanada-En'yo:



# T. Otsuka et al.: No core SM



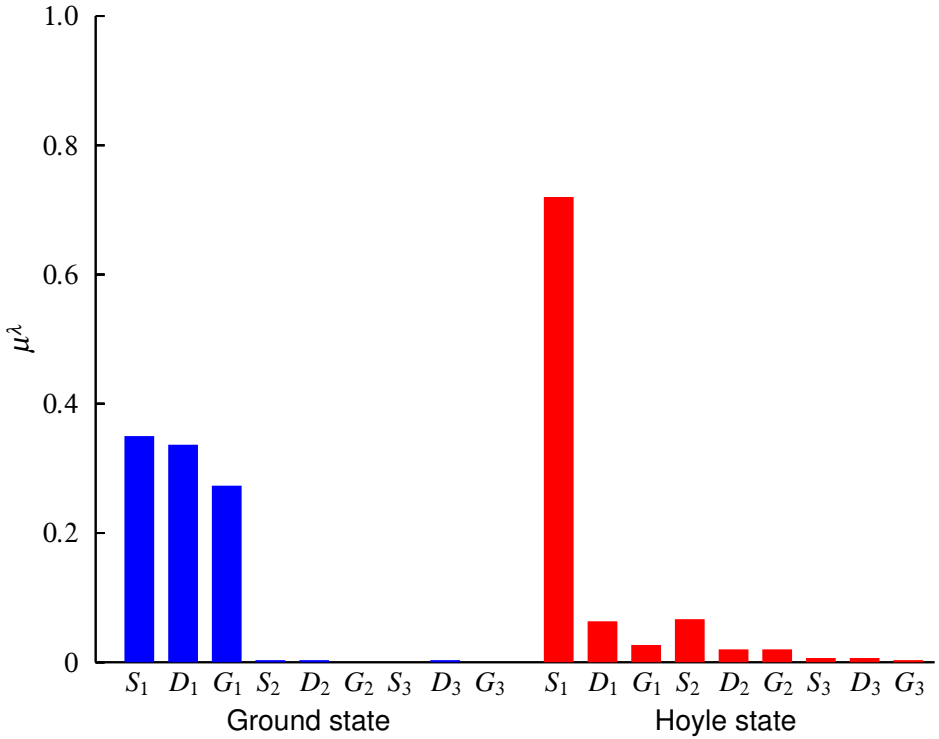

also J. Draayer et al., K. Launey No inelastic form factor so far!

## Radius of second $2^+$ state in 12C: Makoto ITO: to appear in PRC

Thus, the 1 fm enhancement in the nuclear radius in $^{12}C(2_2^+)$ is exotic phenomena, which occurs in much lower excitation energy than the neutron-excess nucleus.

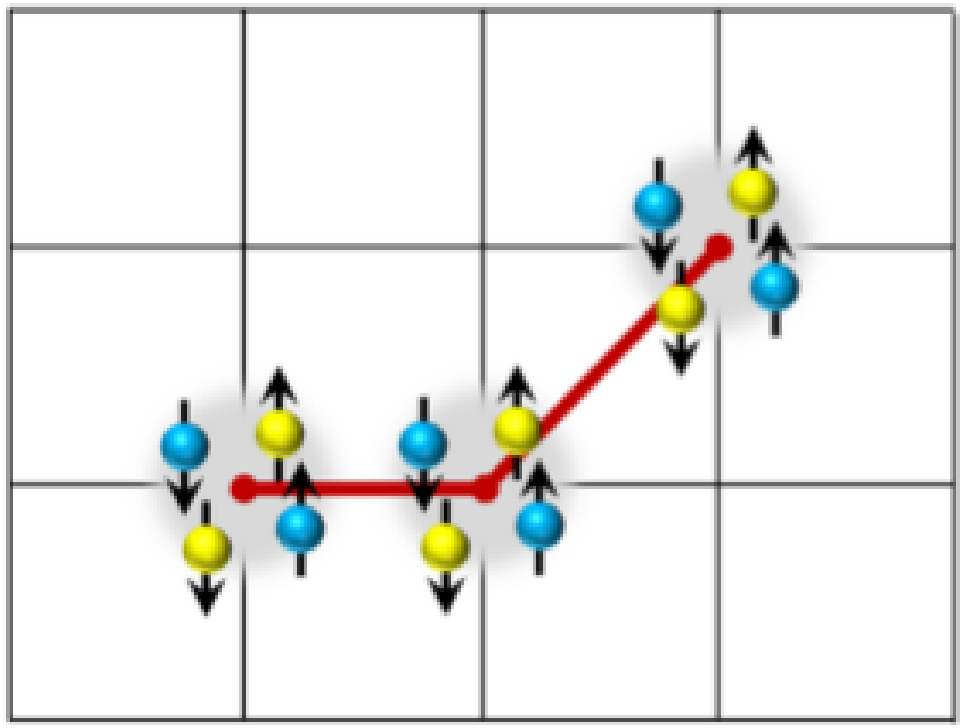
future studies. Since the experimental information of the differential cross section of the $2_2^+$ channel is still insufficient, the measurement of the differential cross section of the excited state at $E_x \sim 10$ MeV and careful MDA to separate the $2_2^+$ component should be extended over a wide angle and energy region.

## Alpha condensation, 3-alpha decay out of Hoyle: **S. Ishikawa**

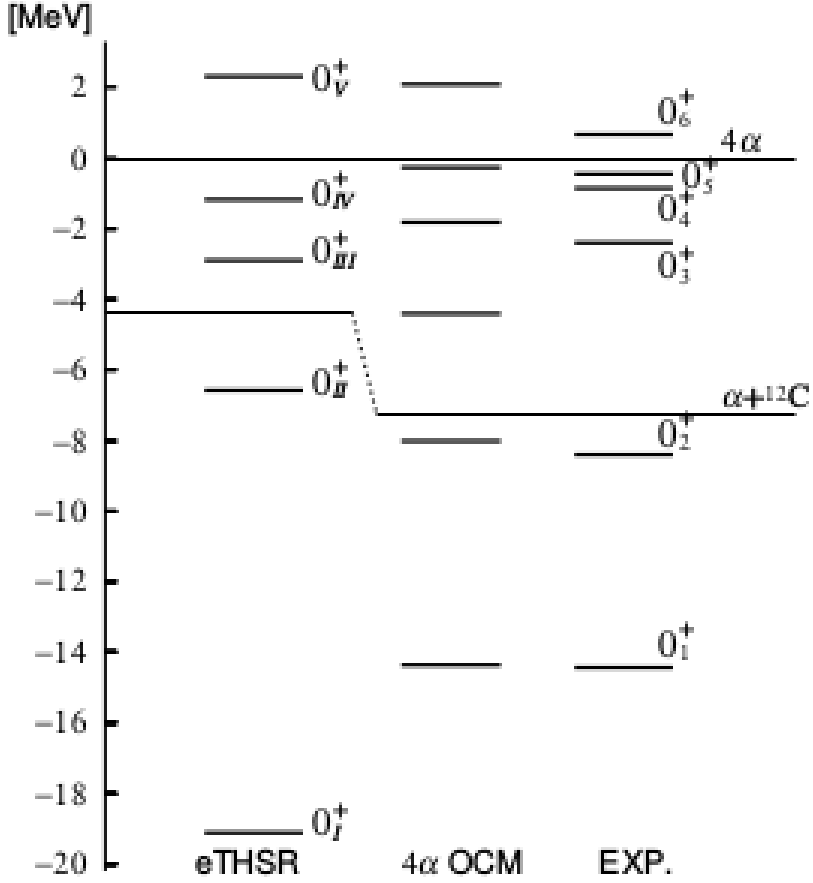



S-wave dominance: 80 percent

Meissner et al., PRL 109, 252501 Lattice QMC; lattice spacing: 2fm



Yasuro FUNAKI: extended THSR for 4 alpha's in 16O, PRC 97, 021304.
experimental problem with 6-th $^{+}0$ state around 15.1 MeV



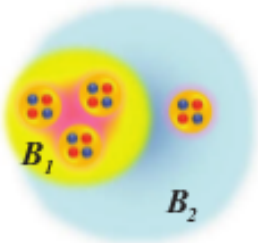
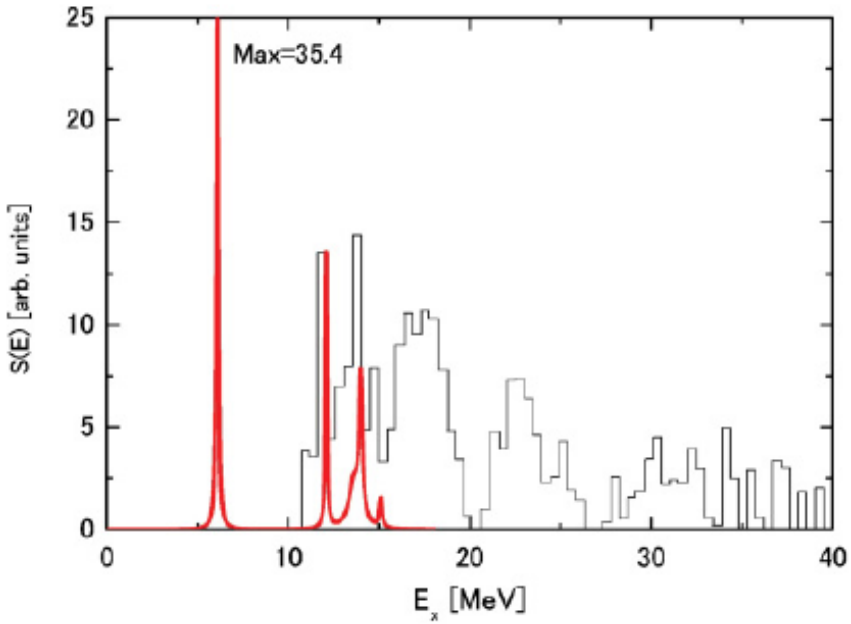



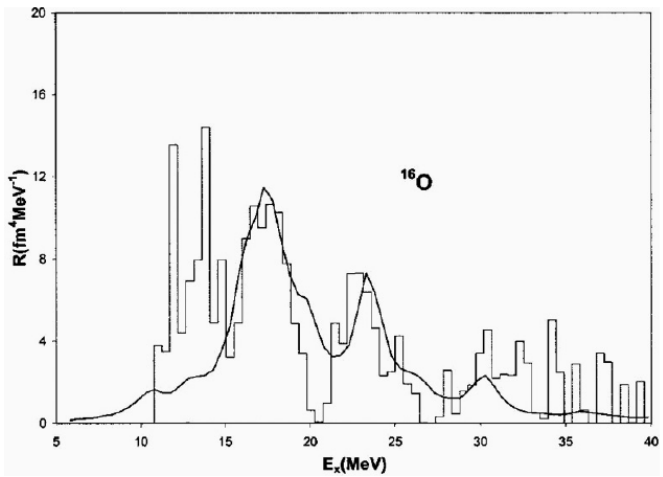

FIG. 2. Energy spectra of the low-lying $J^\pi = 0^+$ states calculated with the extended THSR ansatzes. The corresponding observed spectrum (Exp.) [36] and result by the $4\alpha$ OCM [9] are also shown. The $0^+_4$ state in experiment is taken from Ref. [37].

$$|\mathrm{THSR} - 16O\rangle = \alpha^+\alpha^+\alpha^+\alpha^+|vacuum\rangle$$

Taiichi YAMADA: Monopole transitions in 16O:

TAIICHI YAMADA *et al.*



## THSR extension to alpha-clusters plus valence neutrons: M. Lyu et al.

**New cluster approach on properties of $^{8-11}$Be isotopes with isospin-dependent spin-orbit potential**

Neutron separation of 8-11Be

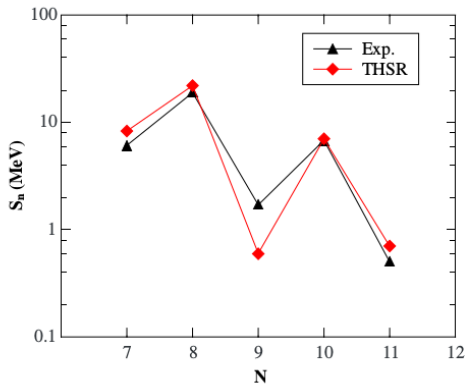



FIG. 3. The one-neutron separation energies $S_n$ of $^{6-11}$Be isotopes. "THSR" denotes calculated results with the THSR wave function and "Exp" denotes the experimental values from Ref. [38].

PRC 93, 054308 (2017)

M. Kimura: 14, 16C, chain states

# Alpha condensation in infinite matter, Sogo et al., PRC 81, 064310

We also need the in-medium four-body Schrödinger equation for the order parameter, in analogy with the pairing case shown in Eq. (4) in the previous section. It is given by

$$\varepsilon_{1234}\langle c_4 c_3 c_2 c_1\rangle + \sum_{1'2'3'4'} V_{1234;1'2'3'4'}\langle c_{4'} c_{3'} c_{2'} c_{1'}\rangle = 0, \qquad (12)$$

where

$$V_{1234;1'2'3'4'} = (1-\rho_1-\rho_2)\tfrac{1}{2}\bar{v}_{12;1'2'}\delta_{33'}\delta_{44'} + (1-\rho_1-\rho_3) \times \tfrac{1}{2}\bar{v}_{13;1'3'}\delta_{22'}\delta_{44'} + \text{permutations}. \qquad (13)$$



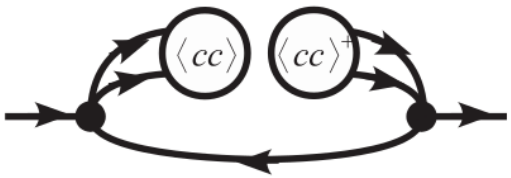



FIG. 1. Graphic representation of the BCS mass operator in Eq. (2).

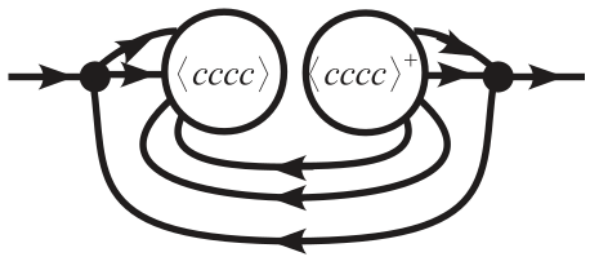
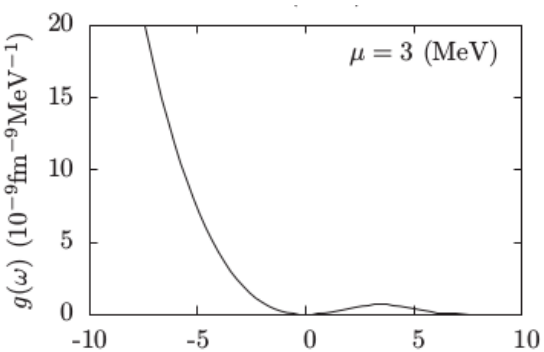
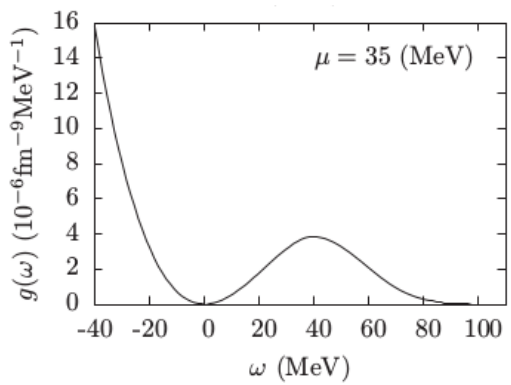


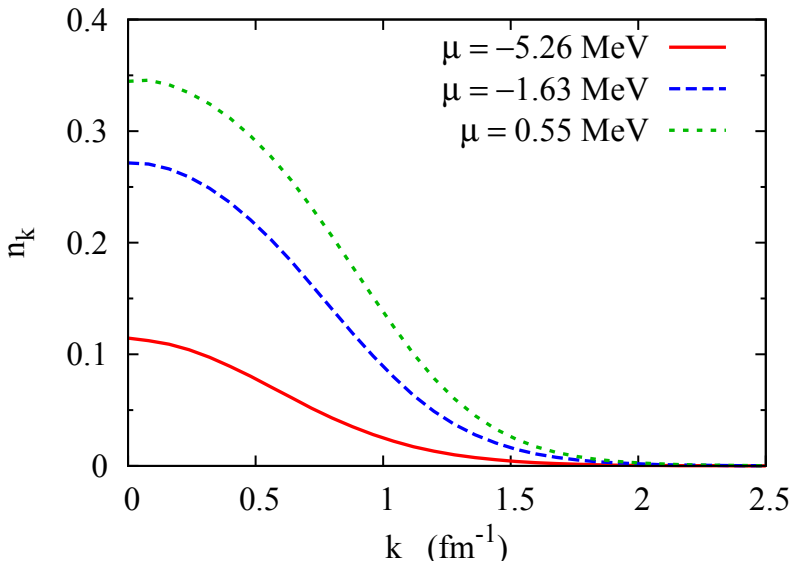

FIG. 4. Graphical representation of the approximate $\alpha$-BEC mass operator $M^{\text{quartet}}$ of Eq. (17).



# Quantum Phase Transition in infinite matter and 16O

Lazauskas, Sogo et al., PRC 79, 051301





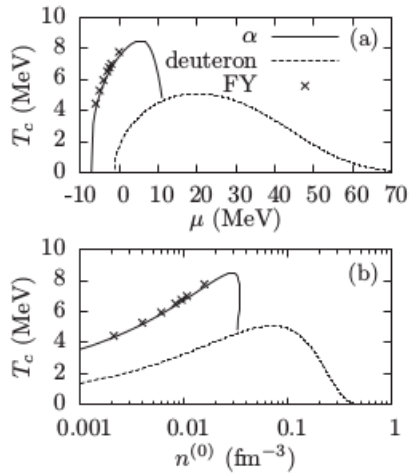
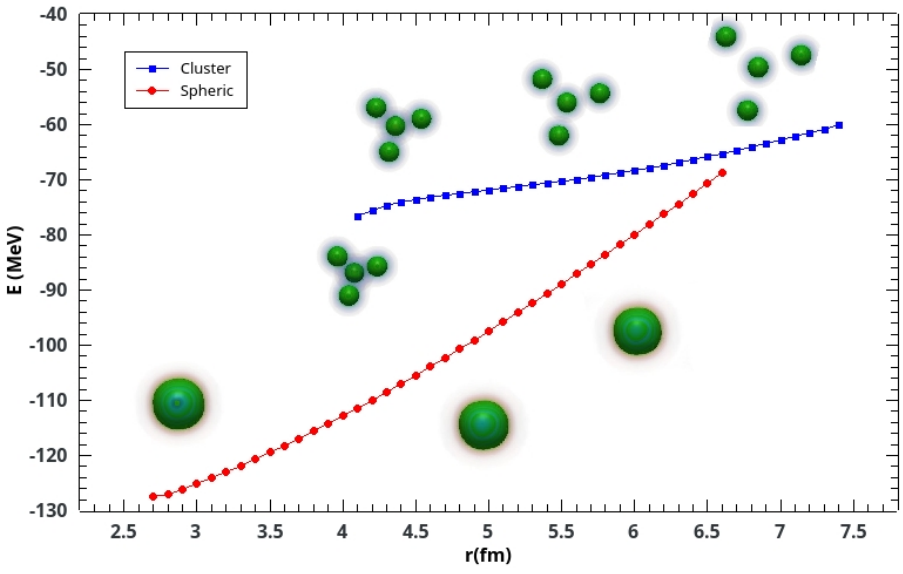



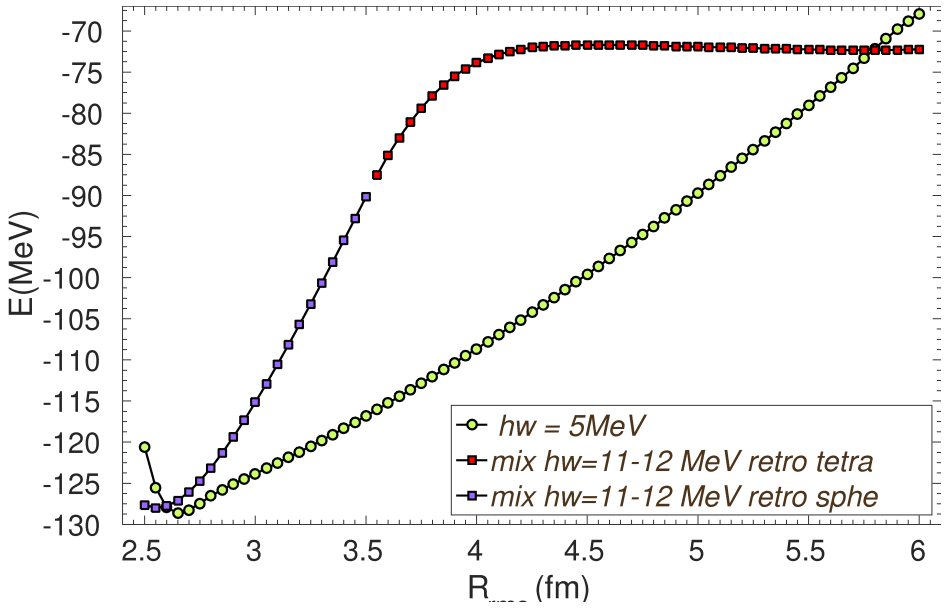

FIG. 2. Critical temperature of $\alpha$ and deuteron condensations as functions of (a) chemical potential and (b) density of free nucleon, derived from Eq. (4) for the $\alpha$ particle and Eq. (11) for the deuteron. Crosses ($\times$) correspond to calculations of Eq. (1) with the Malfliet-Tjon interaction (MT I-III) using the FY method.

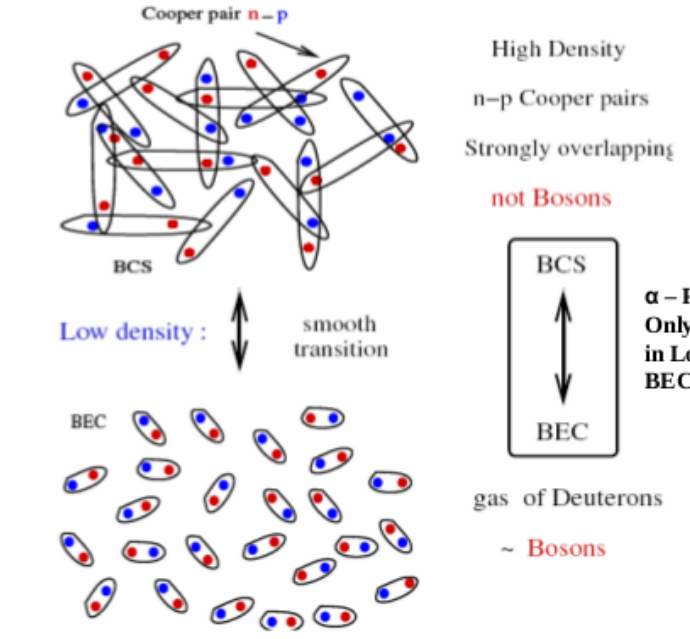
## QPT in finite nuclei

Gogny force D1S, M. Girod;




'RMF', JP. Ebran






High Density

n–p Cooper pairs

Strongly overlapping

not Bosons

BCS

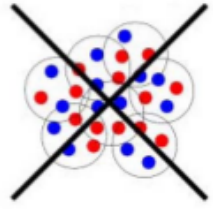
BEC

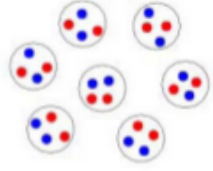
**α – Particles Only Exist in Low Density BEC Phase**

gas of Deuterons

~ Bosons

## Quartetting



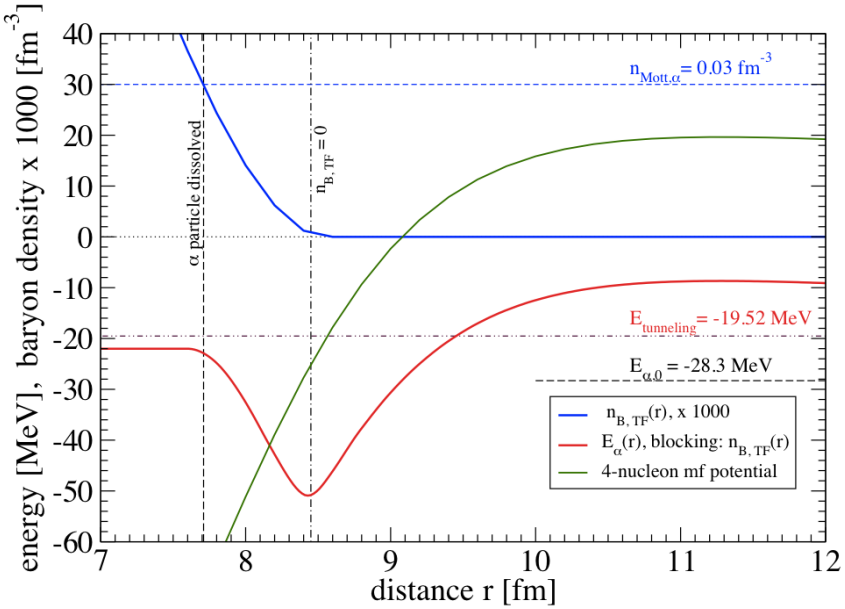


**No BCS phase (dense phase) of α-particles possible!**

**Bose-Einstein-Condensation of α-particles (dilute)**

## Alpha pocket at the surface of 208Pb

Chang Xu, G. Roepke et al., PRC 95;  Double folding: Adamian et al.,PRC 94



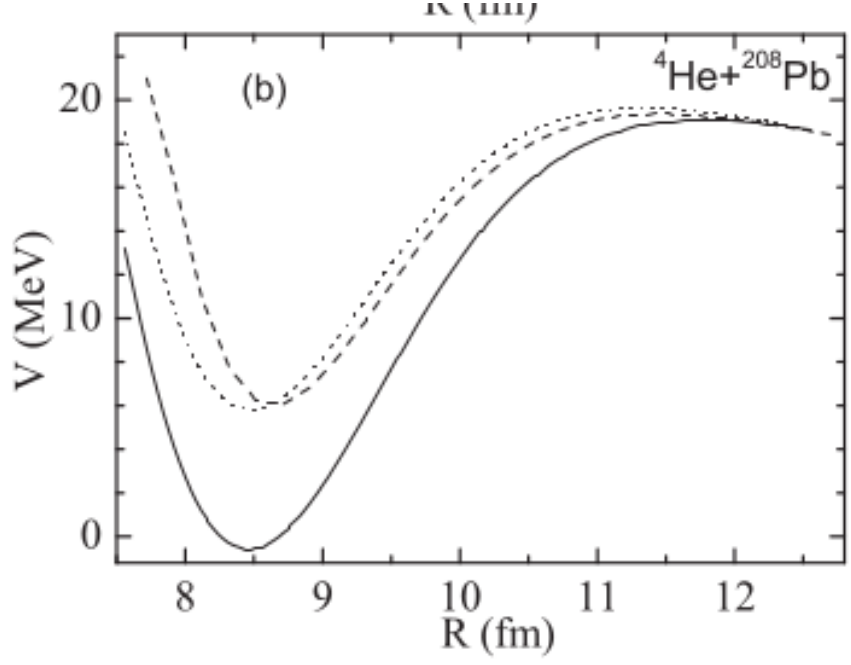




FIG. 13. The same as in Fig. 12, but for the asymmetric reactions indicated. The dotted line is obtained with the same parameters as the solid line, but with $a = 0.39$ fm for $^4$He.

**Future challenge: alpha decay of deformed 226Ra**

Alpha-ring: M. Girod et al., J.Phys. G 37, 064012 (2010)
7-alpha ring measured?? **Nature-article**: Natowitz et al.

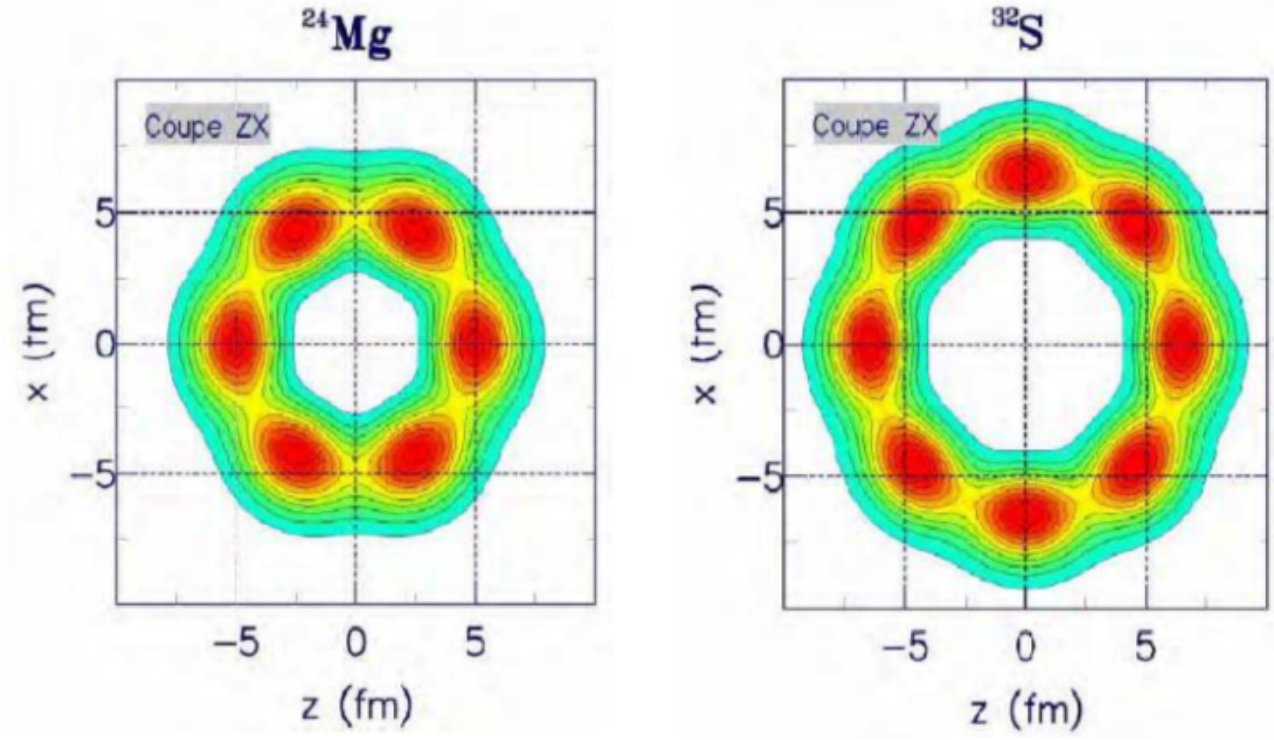



**Figure 1.** Left (right) panel shows the $6\alpha$ ($8\alpha$) ring structure in $^{24}$Mg ($^{32}$S) with constrained HFB calculations [28].

**Conclusions**

Immense activity in nuclear cluster physics

Mostly for self-conjugate nuclei but also with valence neutrons (protons)

Progress with Hoyle family of states in 12C

Some progress with alpha gas states in 16O but experimental ambiguities with sixth zero plus state at about 15.1 MeV

Alpha-condensation is a Quantum-Phase-transition

Alpha-decay 212Po

Alpha rings

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**Alpha Cluster Condensation in $^{12}$C and $^{16}$O**

A. Tohsaki,[1] H. Horiuchi,[2] P. Schuck,[3] and G. Röpke[4]

All started with this....

**THANK YOU!**