Container evolution and dynamics of cluster formation

Yasuro Funaki

(Kanto Gakuin U.)

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Container picture and THSR ansatz

Hoyle band and other excited states in ^{12}C

Container evolution for ^{16}O

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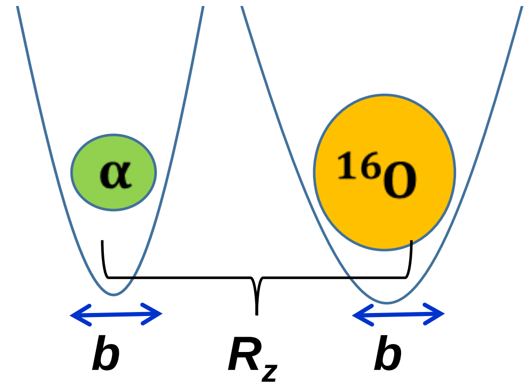
First success of container picture for ordinary cluster state

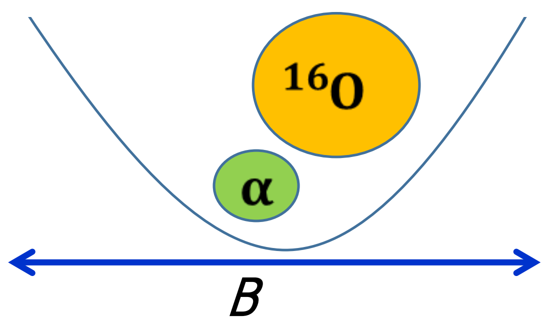
Characterized by rel. distance parameter R_z . Localized clustering.



$$\Psi^{\mathrm{Brink}}_{^{20}\mathrm{Ne}}(R_z, b) = \mathcal{A}\left\{\exp\left(\frac{8(r_z - R_z)^2}{5b^2}\right)\phi_\alpha(b)\phi_{^{16}\mathrm{O}}(b)\right\}$$

VS

Characterized by the size of container *B.* Non-localized clustering.



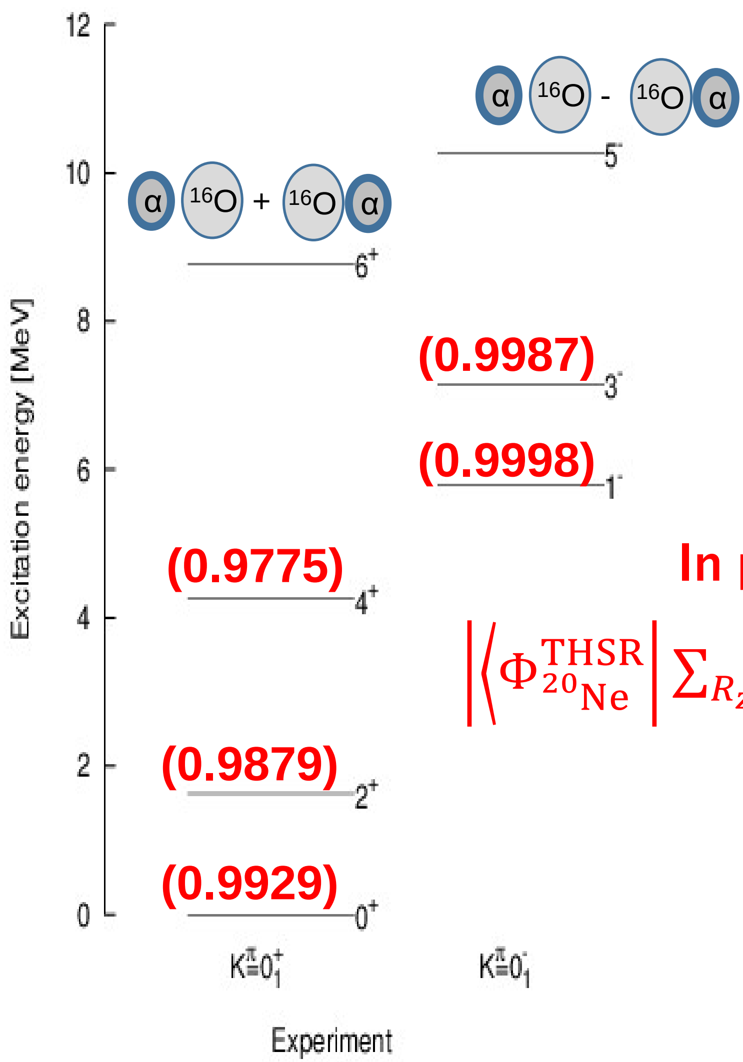
$$\Phi^{\mathrm{THSR}}_{^{20}\mathrm{Ne}}(\beta, b) = \mathcal{A}\left\{\exp\left(\sum_k^{x,y,z}\frac{8r_k^2}{5(b^2 + 2\beta_k^2)}\right)\phi_\alpha(b)\phi_{^{16}\mathrm{O}}(b)\right\}$$

$$B_k^2 = b^2 + 2\beta_k^2 \quad (k = x, y, z)$$

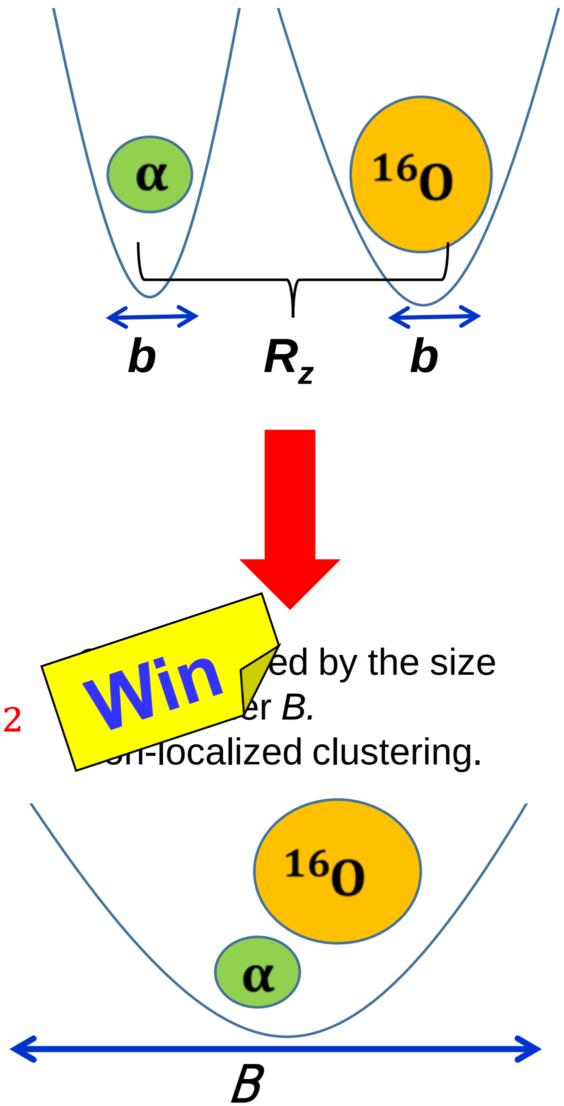
B. Zhou et al., PRC86, 014301 (2012); PRL 110, 262501(2013); PRC 89, 3319 (2014).

First success of container picture for ordinary cluster state

The energy levels of α+^{16}O inversion doublet bands in ^{20}Ne



Characterized by rel. distance parameter R_z . Localized clustering.



In parentheses

$$\left|\left\langle \Phi^{\mathrm{THSR}}_{^{20}\mathrm{Ne}} \middle| \sum_{R_z} f(R_z)\Psi^{\mathrm{Brink}}_{^{20}\mathrm{Ne}}(R_z, b)\right\rangle\right|^2$$

↑

Brink GCM

B. Zhou et al., PRC86, 014301 (2012); PRL 110, 262501(2013); PRC 89, 3319 (2014).

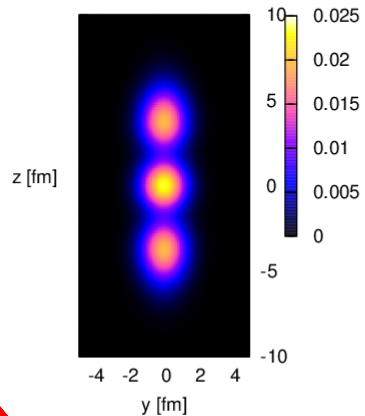
Container picture succeeds in describing other non-gaslike cluster states.

α Linear chain states

T. Suhara et al., PRL112, 062501 (2014)

Overlap between cont. w.f. and full sol.

J=0: 0.9873 $(\beta_x=\beta_y,\beta_z)=(0.1,5.1)$
J=2: 0.9887 $(\beta_x=\beta_y,\beta_z)=(0.1,5.4)$
J=4: 0.9806 $(\beta_x=\beta_y,\beta_z)=(0.1,6.6)$

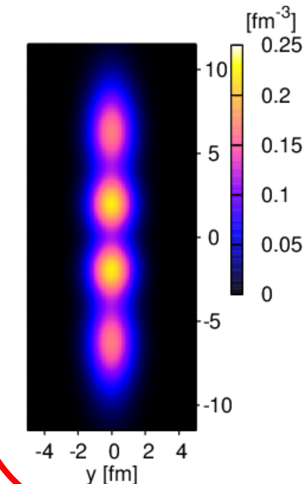


Strongly prolate

The superposition of 100 localized w.fs. coincides with one one-dim. cont. w.f.!

Overlap between cont. w.f. and full sol.

J=0: 0.9440 $(\beta_x=\beta_y,\beta_z)=(0.1, 8.2)$
J=2: 0.9417 $(\beta_x=\beta_y,\beta_z)=(0.1,8.4)$
J=4: 0.9307 $(\beta_x=\beta_y,\beta_z)=(0.1,9.0)$



Strongly prolate

The superposition of 300 localized w.fs. coincides with one one-dim. Cont. w.f.!

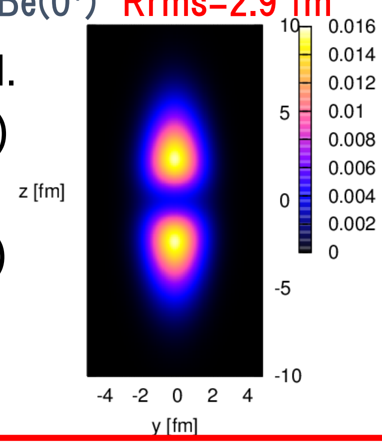
$^{9}_{\Lambda}$Be

Overlap between cont. w.f. and full sol.

J=0: 0.995 $(\beta_x=\beta_y,\beta_z)=(1.6,3.0)$
J=2: 0.994 $(\beta_x=\beta_y,\beta_z)=(0.1,3.0)$
J=4: 0.977 $(\beta_x=\beta_y,\beta_z)=(0.1,2.1)$

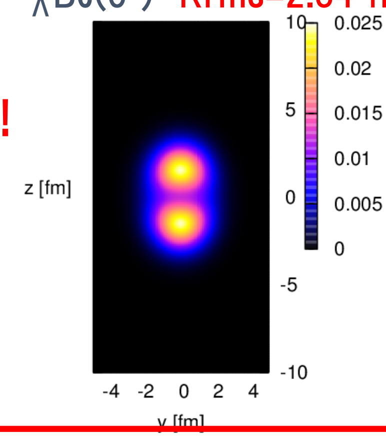
Small size

^{8}Be(0^+) Rrms=2.9 fm



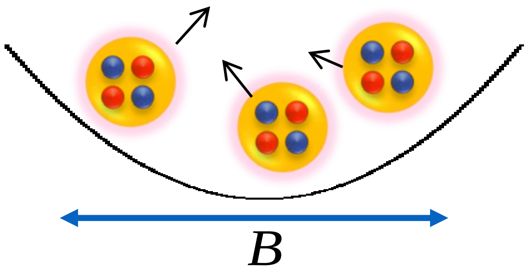
Adding Λ !

$^{9}_{\Lambda}$Be(0^+) Rrms=2.34 fm



Y. F. et al., PTEP (2014) 113D01.

This, in general, gives ``container'' picture of nuclear clustering



Nonlocalized clustering

Characterized by a size parameter B of the container, corresponding to nuclear size.

$^{8}Be(0^+)$ *Y. F. et al., PTP108, 297(2002);*

$^{12}C(0_2^+)$ *Y. F. et al., PRC67, 051306(R)(2003);PRC80, 064326(2009).*

^{20}Ne (^{16}O-α) *B. Zhou et al., PRC86, 014301 (2012); PRL 110, 262501(2013); PRC 89, 034319 (2014).*

3α and 4α linear chain states T. Suhara et al., PRL112, 062501 (2014)

$^{9\text{-}10}Be$ M. Lyu et al., PRC91, 014313(2015); 93, 054308 (2016).

^{9}B Q. Zhao et al., arXive: 1801.05964.

$^{9}{}_{\Lambda}Be(0^+)$ Y. F. et al., PTEP 2014, 113D01

$^{12}C(0_1^+)$ *Y. F. et al., PRC80, 064326(2009). B. Zhou et al., PTEP 2014, 101D01.*

Almost equivalent to the w.fs. obtained by the corresponding full cal. (RGM/GCM)

Providing basic understanding of the nuclear clustering

Y. F. H. Horiuchi, A. Tohsaki, PPNP82, 78-132 (2015).

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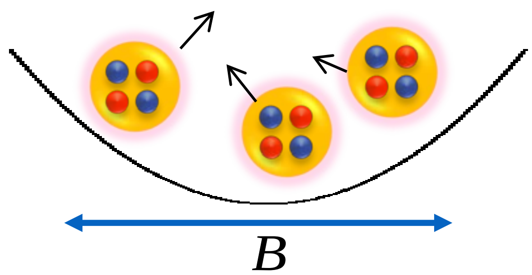

Nonlocalized clustering

Characterized by a size parameter B of the container, corresponding to nuclear size.

New!
$^{12}C(3_1^-)$ by B. Zhou

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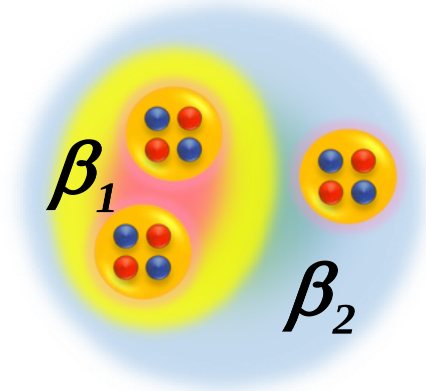
3α extended THSR wave function

$$\Phi_\alpha(\boldsymbol{\beta}, b) = \exp\left(-2\sum_{k}^{x,y,z}\frac{r_k^2}{b^2+2\beta_k^2}\right)\phi_\alpha(b)$$

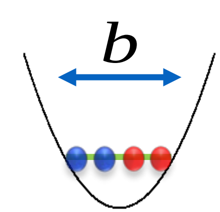
$$\Phi_{^{12}\mathrm{C}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, b) = \Psi_G^{-1}\mathcal{A}\{\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_1, b)\ \Phi_\alpha(\boldsymbol{\beta}_2, b)\}$$

Ψ_G: Total center-of-mass w.f. to be eliminated



Internal w.f. of α particle

b=1.35 fm: fixed

$\phi_\alpha(b) :=$ 

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Ψ_G: Total center-of-mass w.f. to be eliminated

Hill-Wheeler eq. or GCM (generator coordinate method)

$$\sum_{\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2} \left\langle \hat{P}^J_{MK}\Phi_{^{12}\mathrm{C}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, b) \middle| \hat{H} - E \middle| \hat{P}^J_{MK}\Phi_{^{12}\mathrm{C}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2, b)\right\rangle f(\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2) = 0$$

$\hat{P}^J_{MK}$: Angular momentum projection operator

Hamiltonian (NN force: Volkov No.2 force)

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i^{12} \nabla_i^2 - T_G + \sum_{i<j}^{12}(V_{ij}^{(N)} + V_{ij}^{(C)})$$

$\boldsymbol{\beta}_i = (\beta_{ix} = \beta_{iy}, \beta_{iz})$

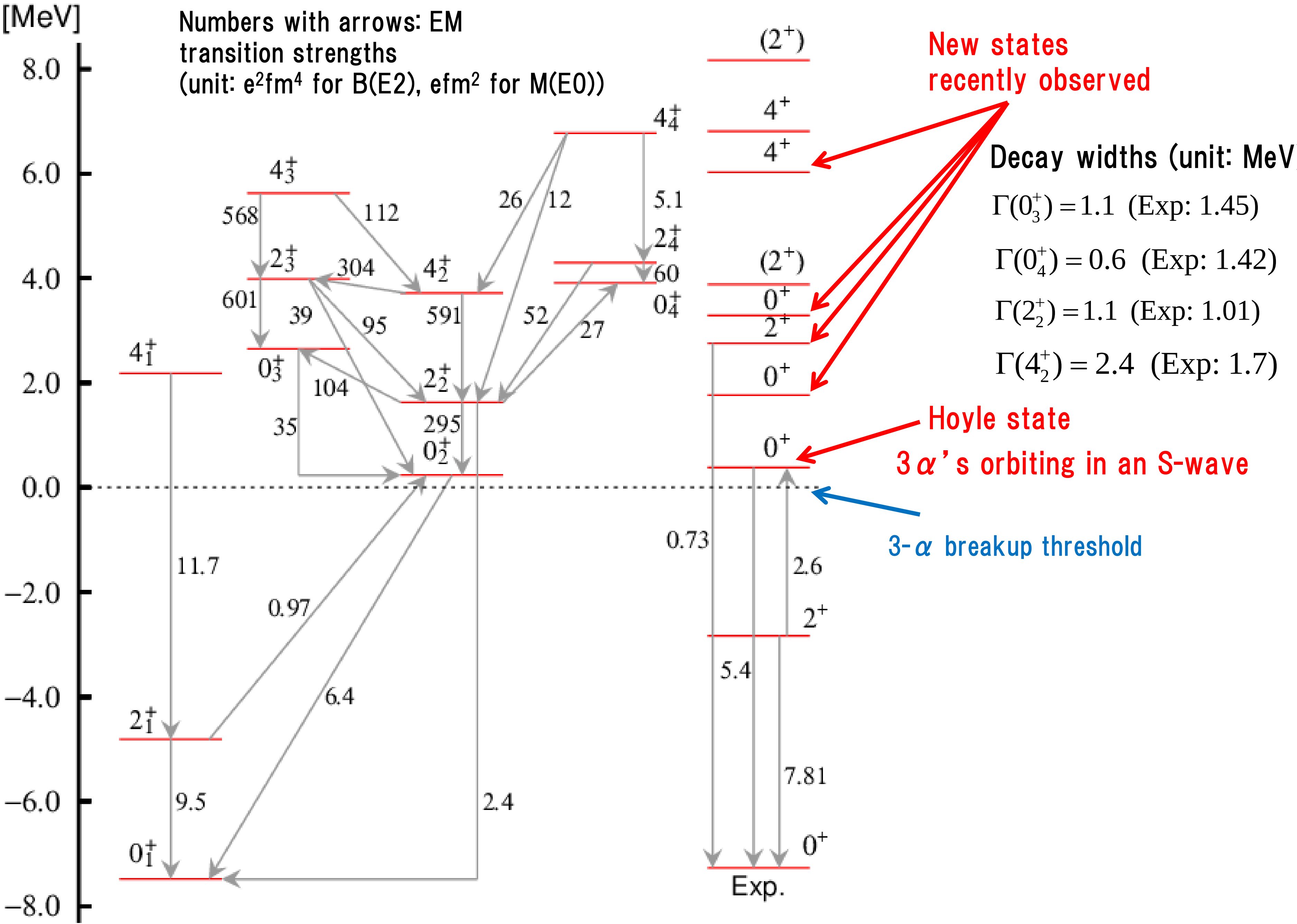
With (axially symmetric) deformation

Spurious continuum components are effectively eliminated by r^2 constraint method.

See Y. F. et al., PTP **115,** 115 (2006).

Y. F. et al., Progress in Particle and Nuclear Physics **82**, 78-132 (2015).

Results (of ^{12}C)



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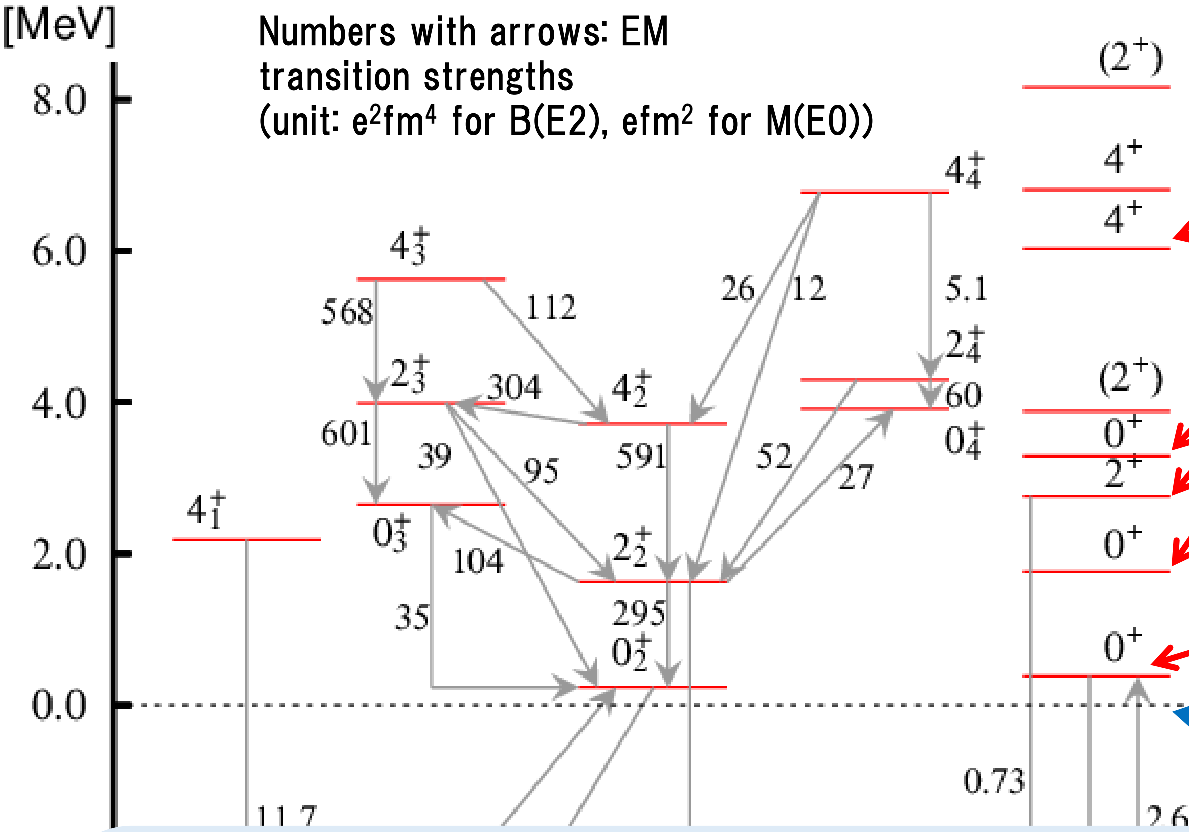


New states recently observed

Decay widths (unit: MeV)

$\Gamma(0_3^+) = 1.1$ (Exp: 1.45)

$\Gamma(0_4^+) = 0.6$ (Exp: 1.42)

$\Gamma(2_2^+) = 1.1$ (Exp: 1.01)

$\Gamma(4_2^+) = 2.4$ (Exp: 1.7)

Hoyle state

3α's orbiting in an S-wave

3-α breakup threshold

New observed states are consistently reproduced.

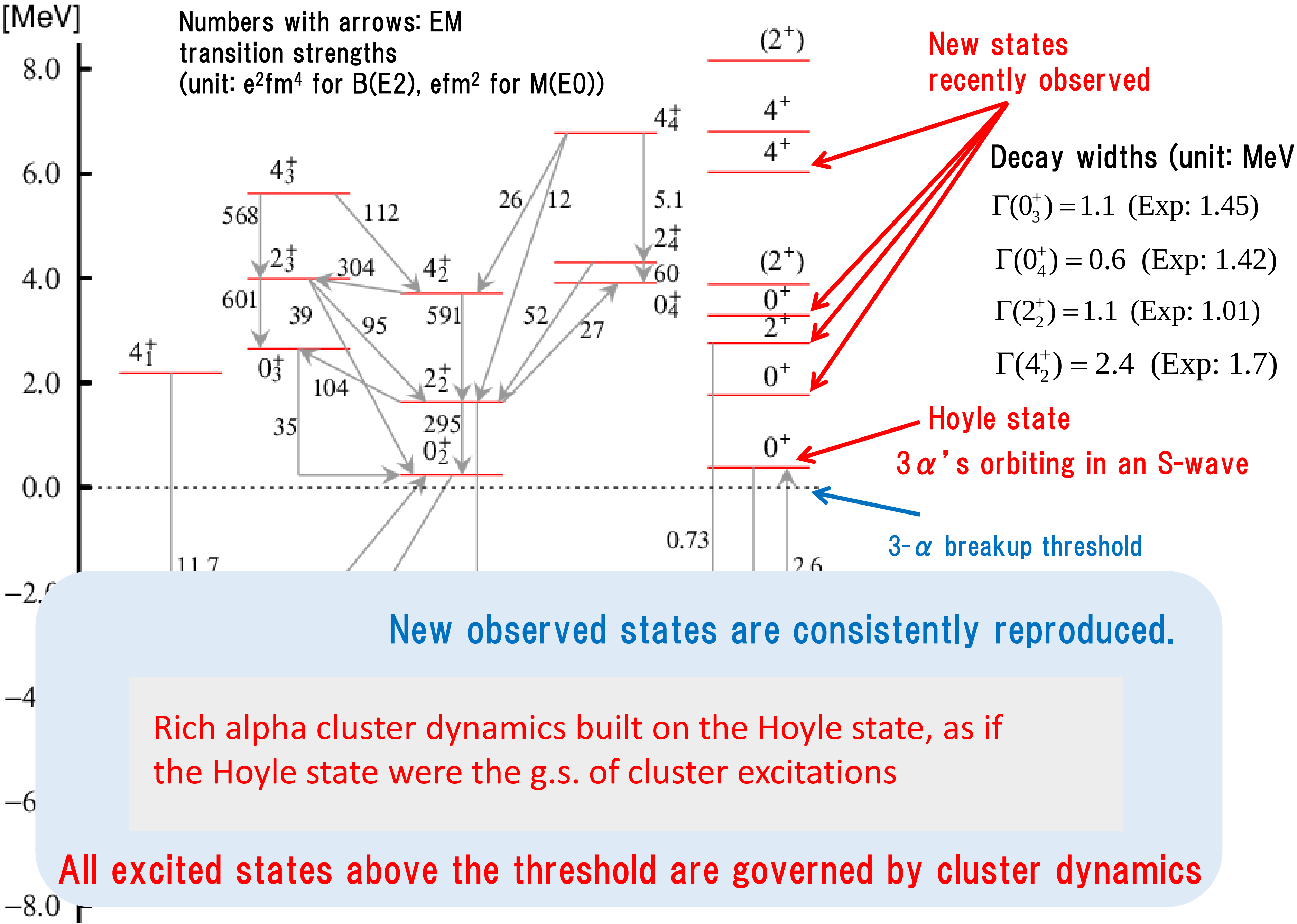
Large spatial size
3.7 fm ～ 4.7 fm

except for the shell-model-like states
($0_1^+, 2_1^+, 4_1^+$: ～2.4 fm)

All excited states above the threshold are governed by cluster dynamics

Results (of ^{12}C)

Y. F. et al., Progress in Particle and Nuclear Physics **82**, 78-132 (2015).



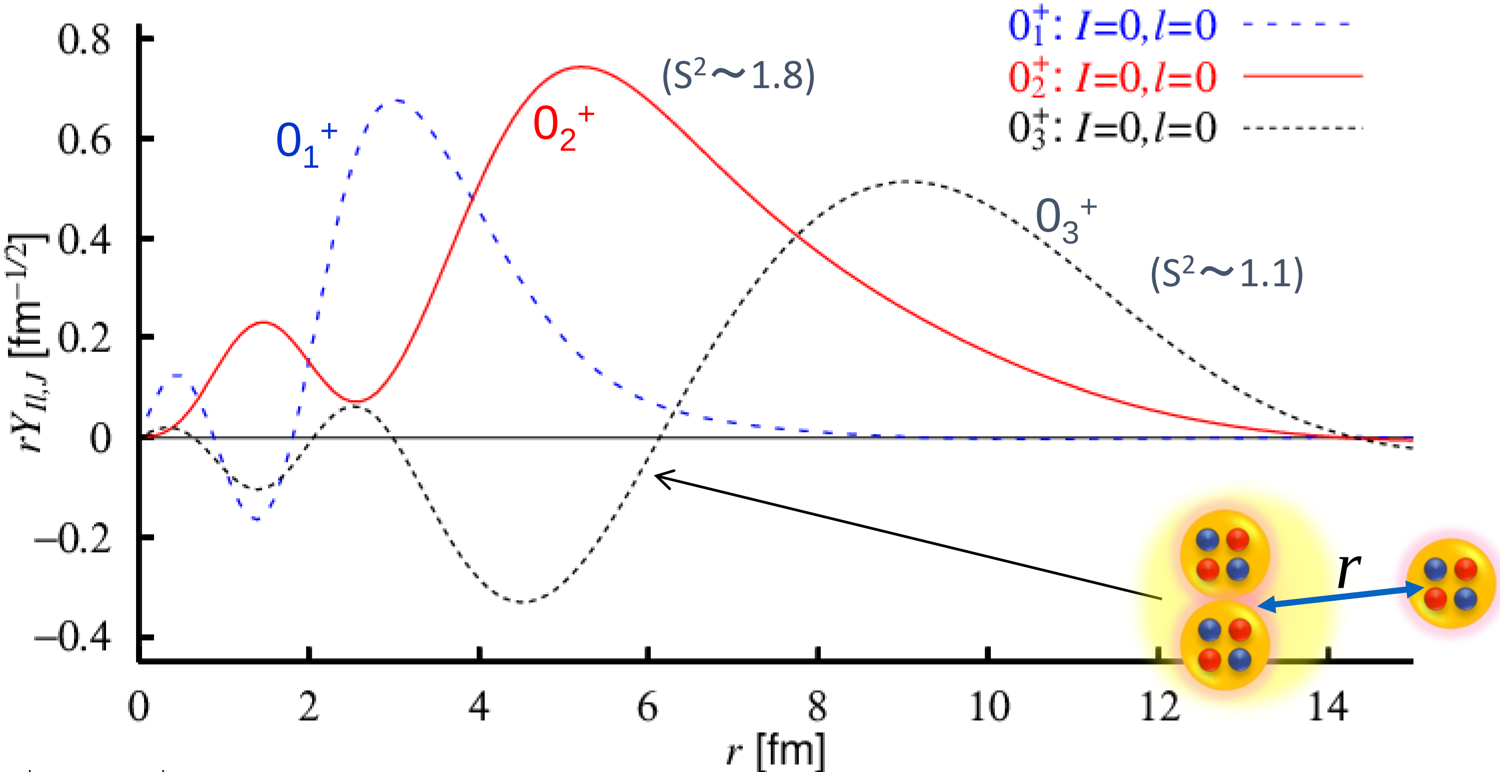
New observed states are consistently reproduced.

Rich alpha cluster dynamics built on the Hoyle state, as if the Hoyle state were the g.s. of cluster excitations

All excited states above the threshold are governed by cluster dynamics

0_3^+ state: higher nodal excitation of the Hoyle state

Overlap functions of the 0_1^+ , 0_2^+ , 0_3^+ states for ^{8}Be(0+)+α(S) channel



$$M(E0;\ 0_3^+ \to 0_2^+) = 34.5$$

Very large monopole transition strength between the 0_2^+ and 0_3^+ states   c.f. $M(E0;\ 0_2^+ \to 0_1^+) = 6.4$

0_1^+ state: 2 nodes

0_2^+ state: 3 nodal oscillation (nodes disappear due to the dissolution of ^{8}Be core)

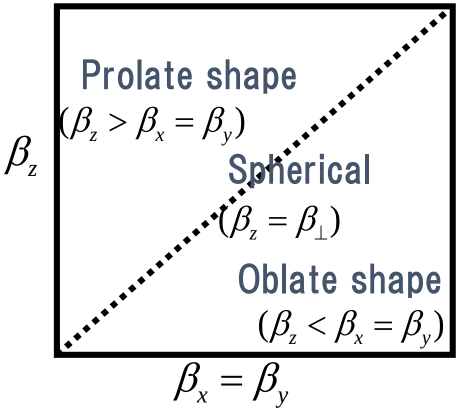
0_3^+ state: 4 nodes (higher nodal structure)

Squared overlap with single THSR config. for the 0_4^+ state of ^{12}C

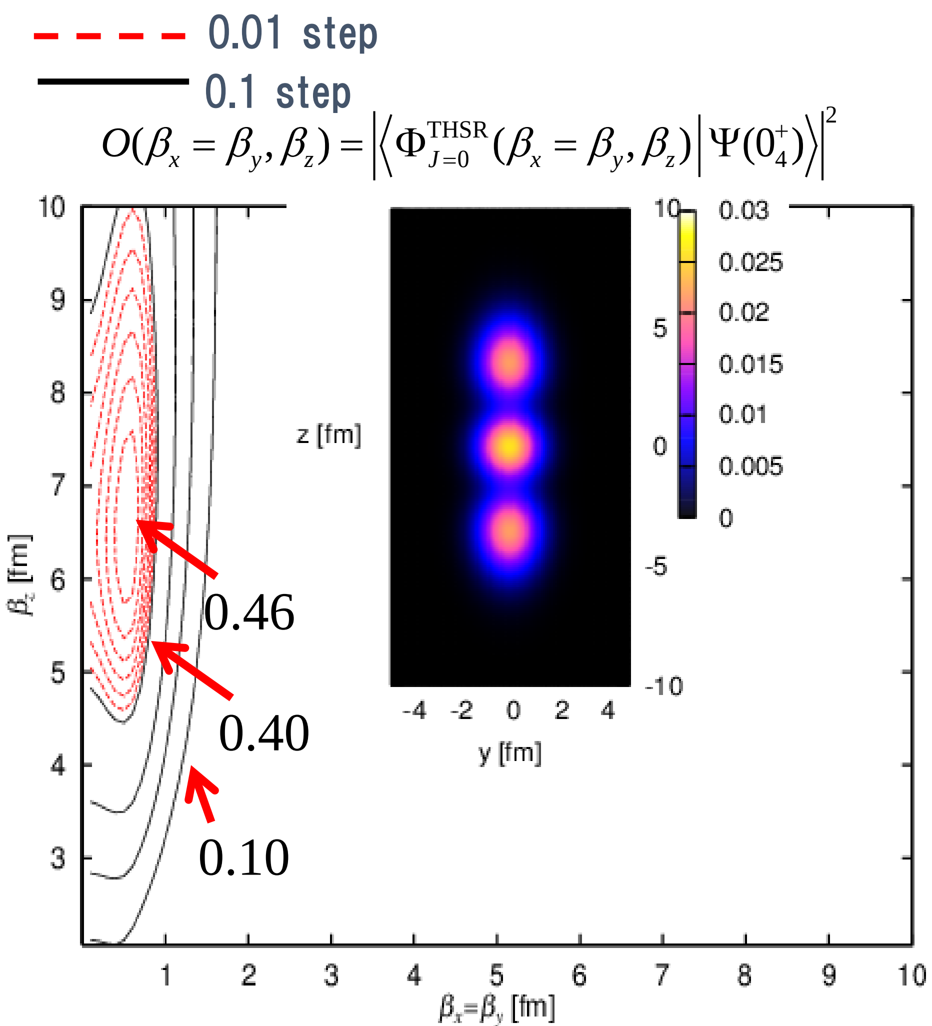
Y. F., PRC **94**, 024344 (2016) .

For the 0_4^+ state

Clear linear-chain structure

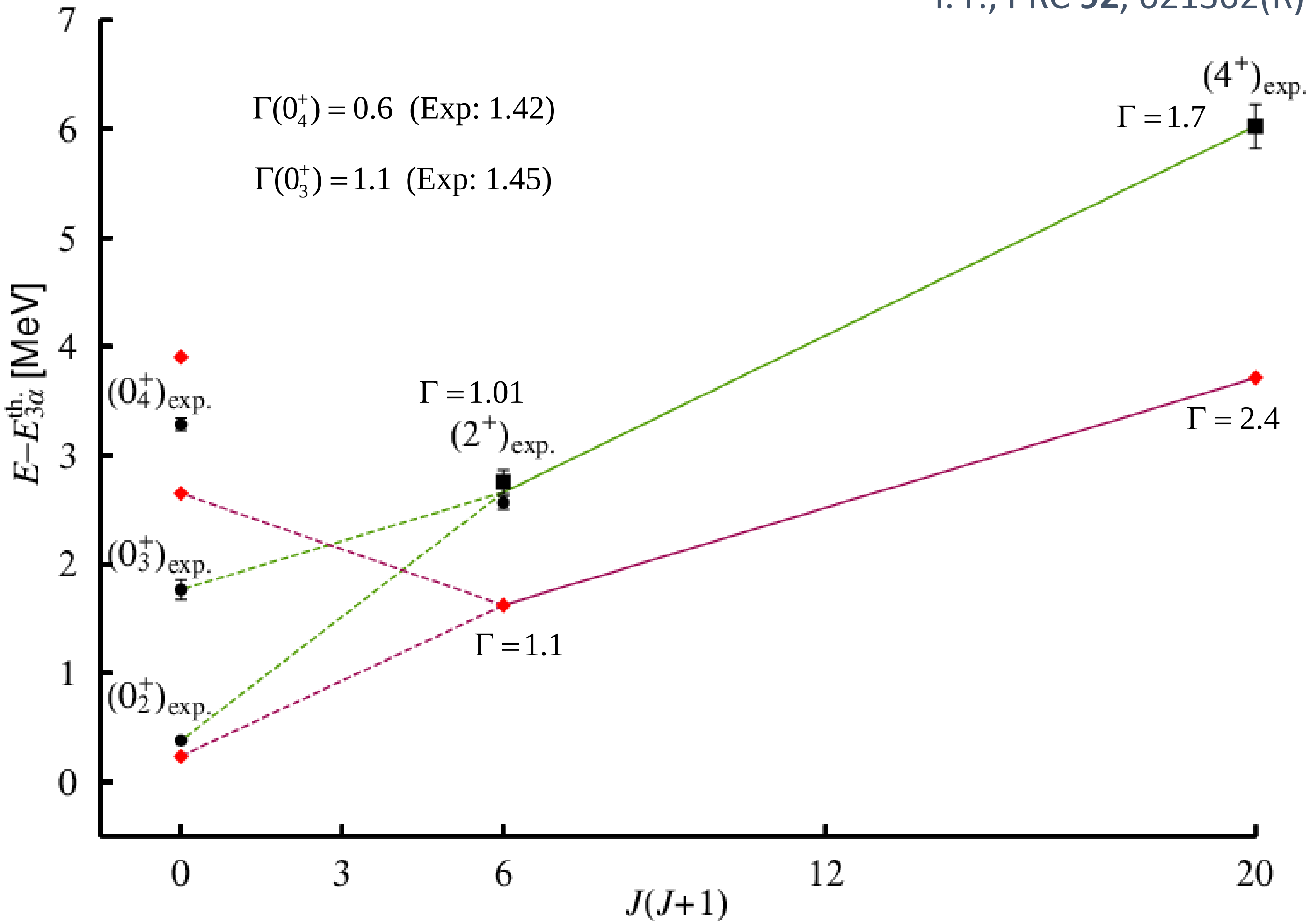


$$O(\beta_x = \beta_y, \beta_z) = \left| \left\langle \Phi^{\mathrm{THSR}}_{J=0}(\beta_x = \beta_y, \beta_z) \middle| \Psi(0_4^+) \right\rangle \right|^2$$



Rotational structure of the Hoyle band

Y. F., PRC **92**, 021302(R) (2015)



$B(E2;\ 4_2^+ \to 2_2^+) = 591$

$B(E2;\ 2_2^+ \to 0_2^+) = 295$

$B(E2;\ 2_2^+ \to 0_3^+) = 104$

$B(E2;\ 2_2^+ \to 0_4^+) = 27$

Fragmented into the Hoyle state and 0_3^+ state

The Hoyle state is not a simple bandhead of α+^{8}Be rotation

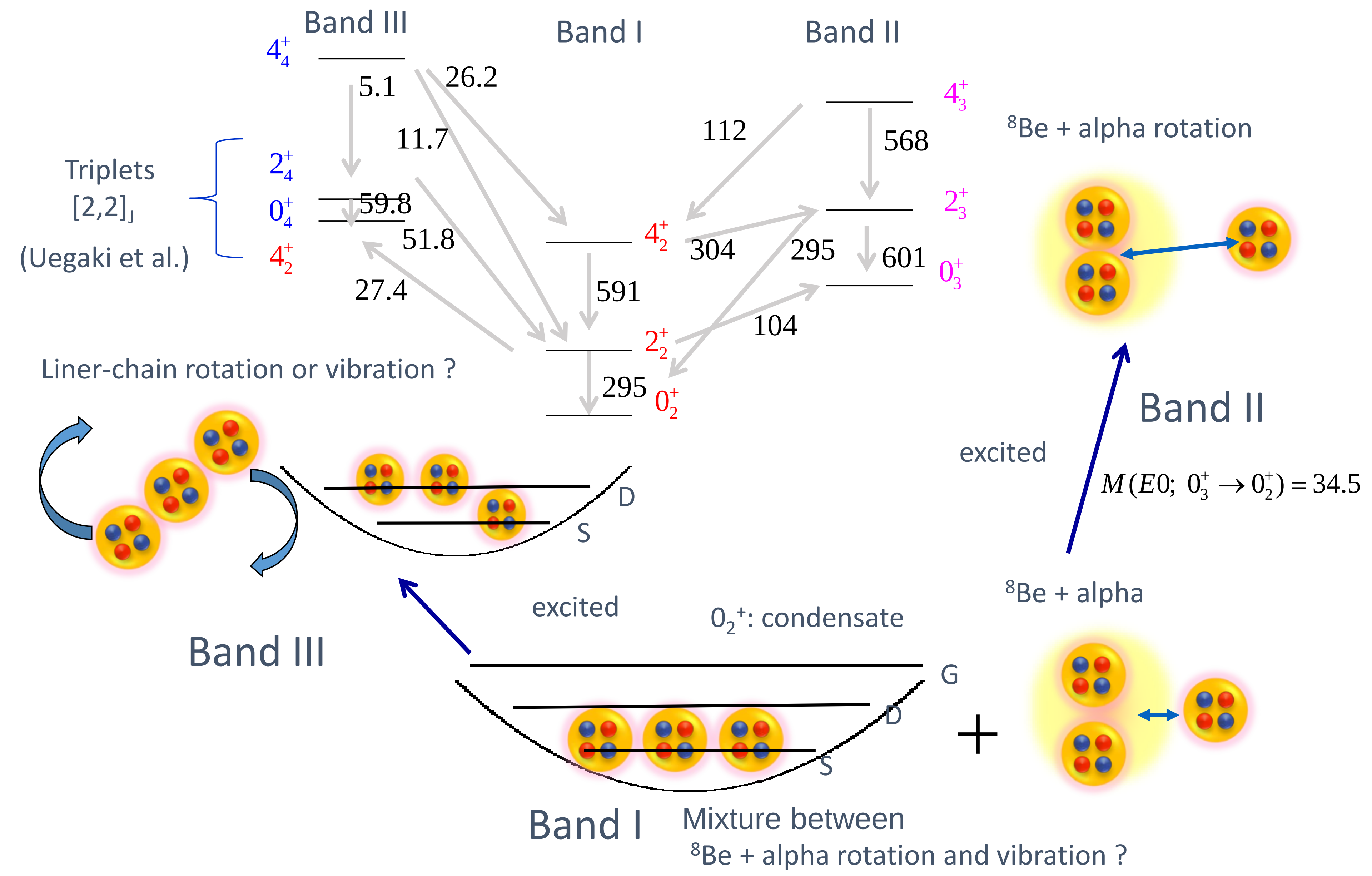
Specificity of the Hoyle state as the 3-alpha condensate

α rotates outside the core -> α rotates inside the core (α cond.)

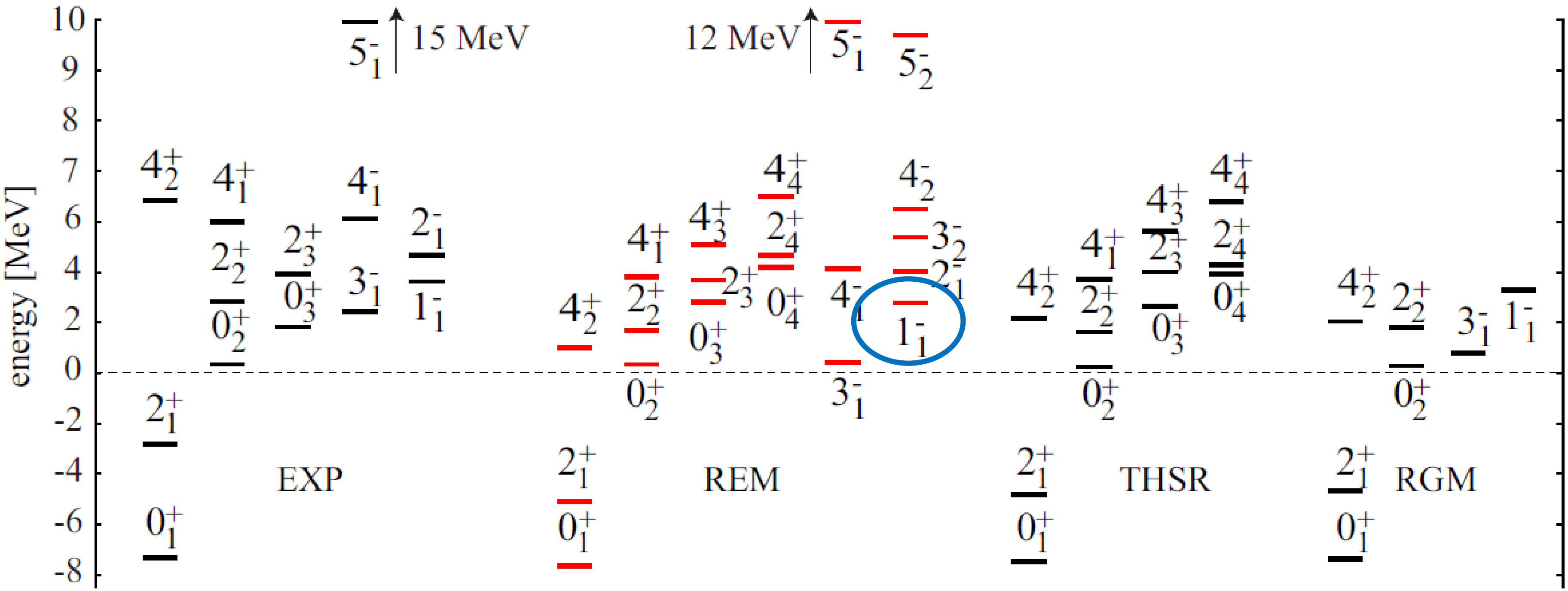
Possible interpretations of these bands

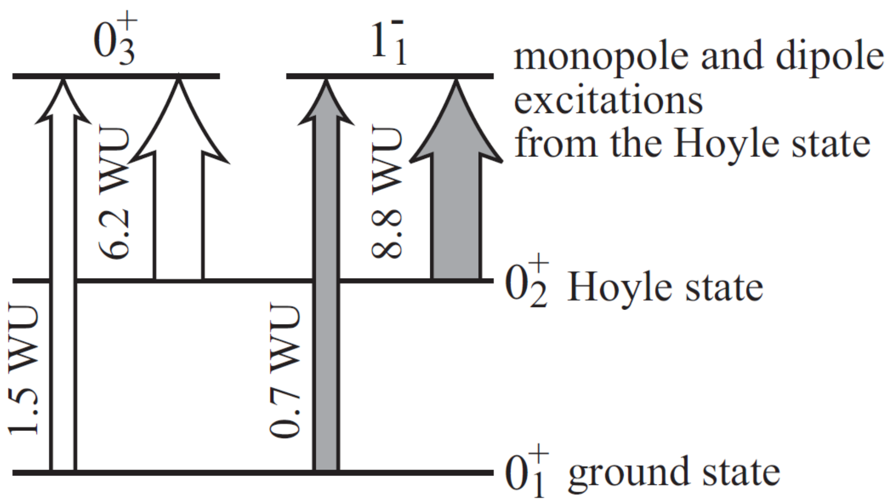
Y. F. PRC **92**, 021302(R) (2015).

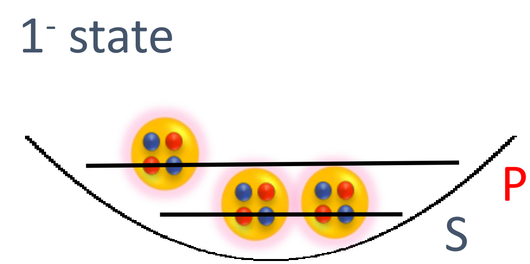
Rich alpha cluster dynamics built on the Hoyle state, as if the Hoyle state were the g.s. of cluster excitations



R. Imai et al., arXiv: 1802.03523.







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4α extended THSR wave function

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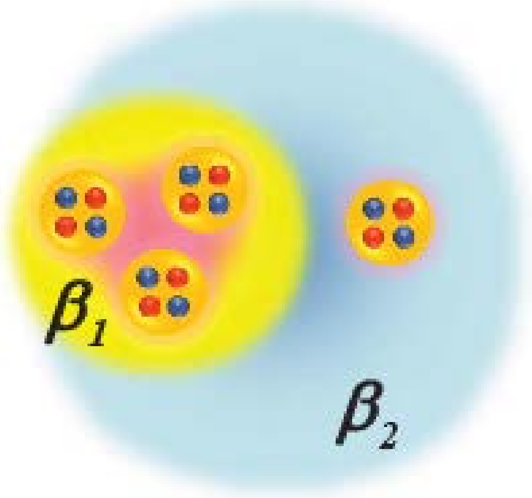
$$\Phi_{^{16}\mathrm{O}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, b) = \Psi_G^{-1}\mathcal{A}\{\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_2, b)\}$$

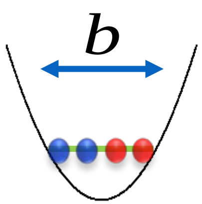
Ψ_G: Total center-of-mass w.f. to be eliminated



Internal w.f. of α particle

b=1.44 fm: fixed

$\phi_\alpha(b) :=$



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$\hat{P}_{MK}^J$: Angular momentum projection operator

Hamiltonian (NN force: F1 force)

A. Tohsaki, PRC **49**, 1814 (1994).

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{i}^{16} \nabla_i^2 - T_G + \sum_{i<j}^{16}(V_{ij}^{(N)} + V_{ij}^{(C)}) + \sum_{i<j<k}^{16} V_{ijk}^{(N)}$$

$\boldsymbol{\beta}_i = \left(\beta_{ix} = \beta_{iy}, \beta_{iz}\right)$

With (axially symmetric) deformation

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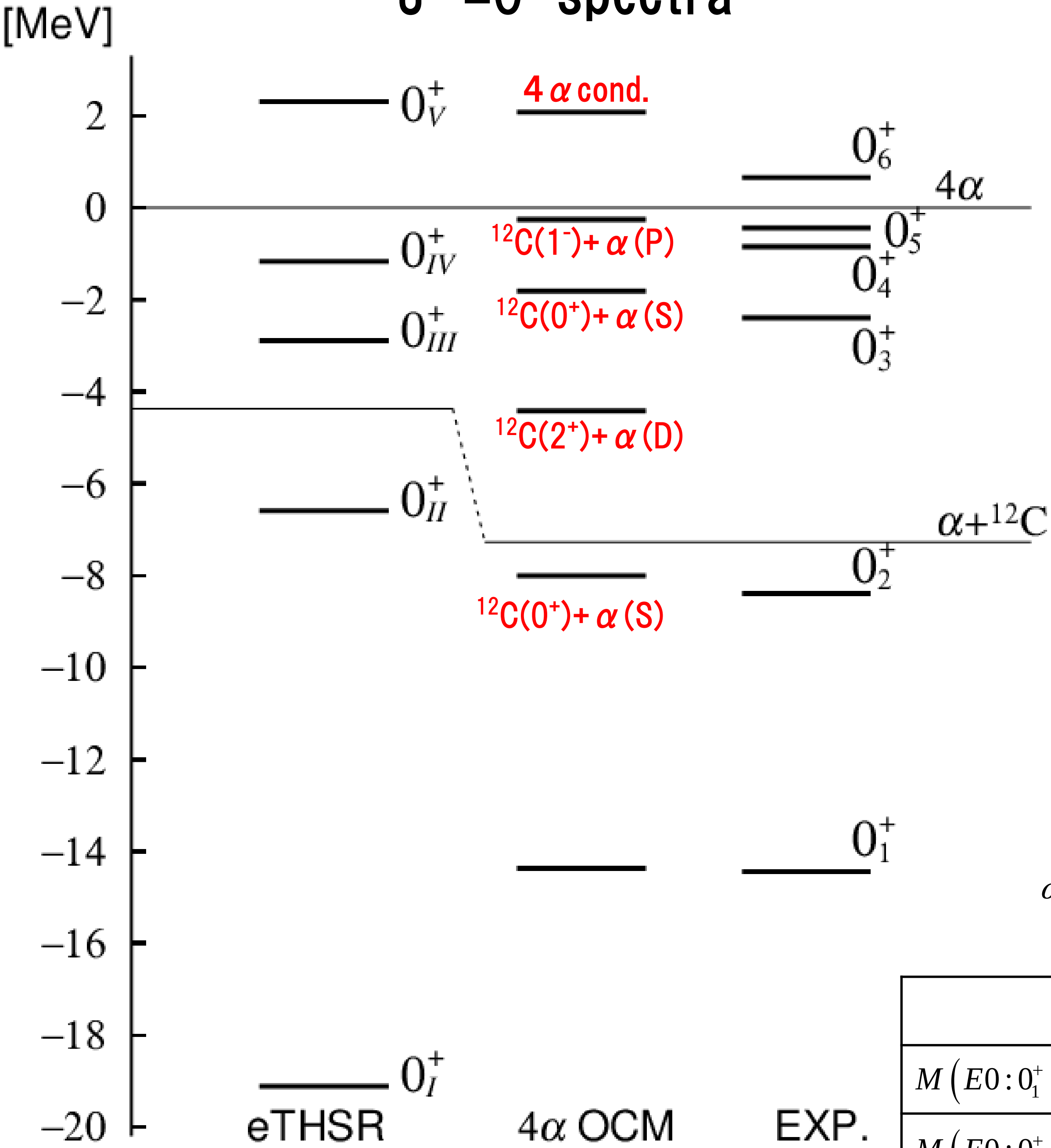
See Y. F. et al., PTP **115,** 115 (2006).

$J^{\pi}=0^{+}$ spectra

Y. F, PRC 97, 021304(R)(2018)



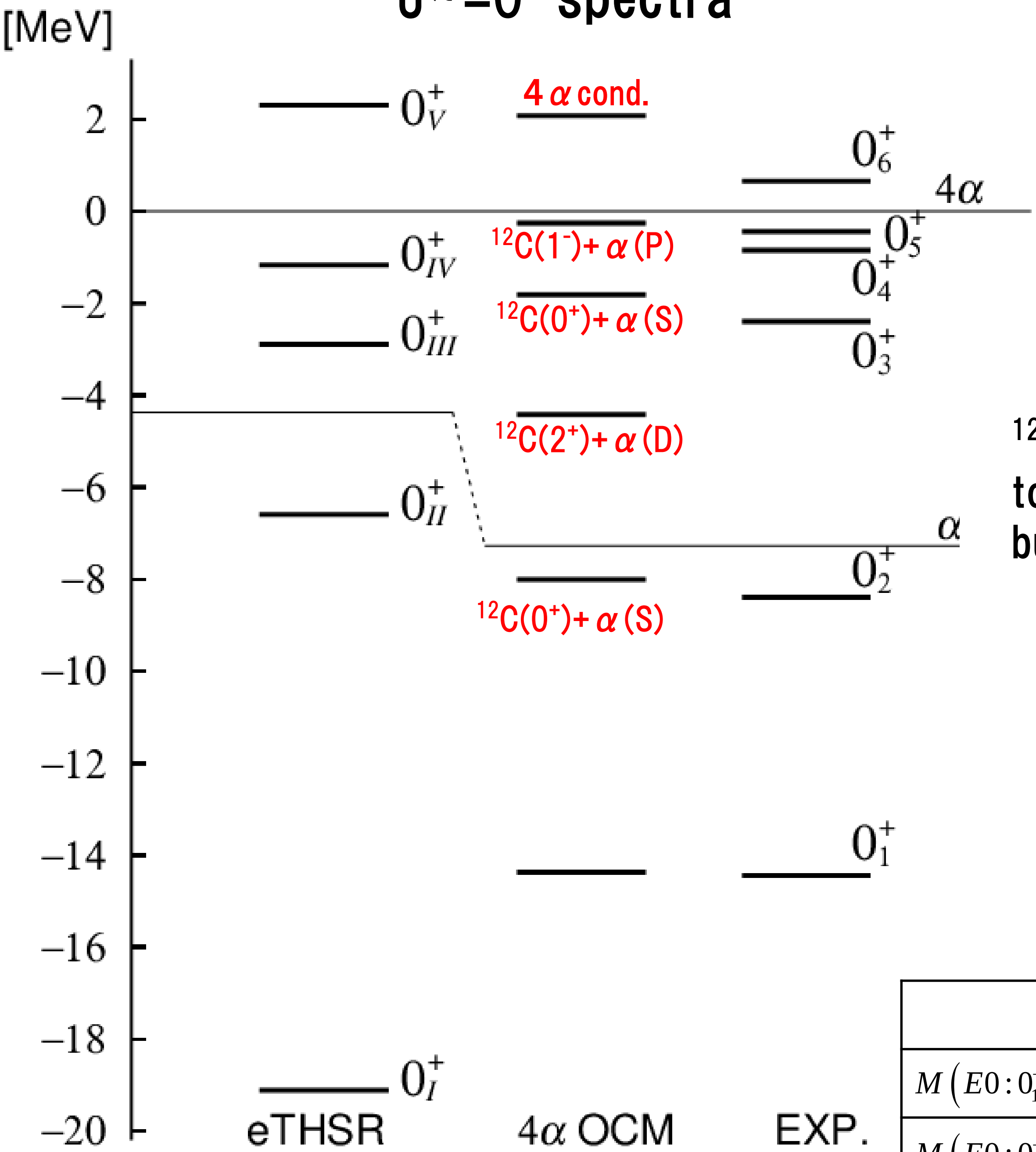
$\alpha+^{12}$C OCM

Y. Suzuki, PTP **55**, 1751 (1976); **56**, 111 (1976).

	eTHSR	$\alpha+^{12}$C OCM	EXP.
$M\left(E0:0_1^+\rightarrow 0_2^+\right)$	5.9	3.88	3.66 ± 0.55
$M\left(E0:0_1^+\rightarrow 0_3^+\right)$	5.7	3.50	4.40 ± 0.44

$J^{\pi}=0^{+}$ spectra

Y. F, PRC 97, 021304(R)(2018)



$^{12}C(1^-)+\alpha$ (P) structure is difficult to describe by the present eTHSR but extension is possible.

Talk by B. Zhou in this morning.

$\alpha+^{12}C$ OCM

Y. Suzuki, PTP **55**, 1751 (1976); **56**, 111 (1976).

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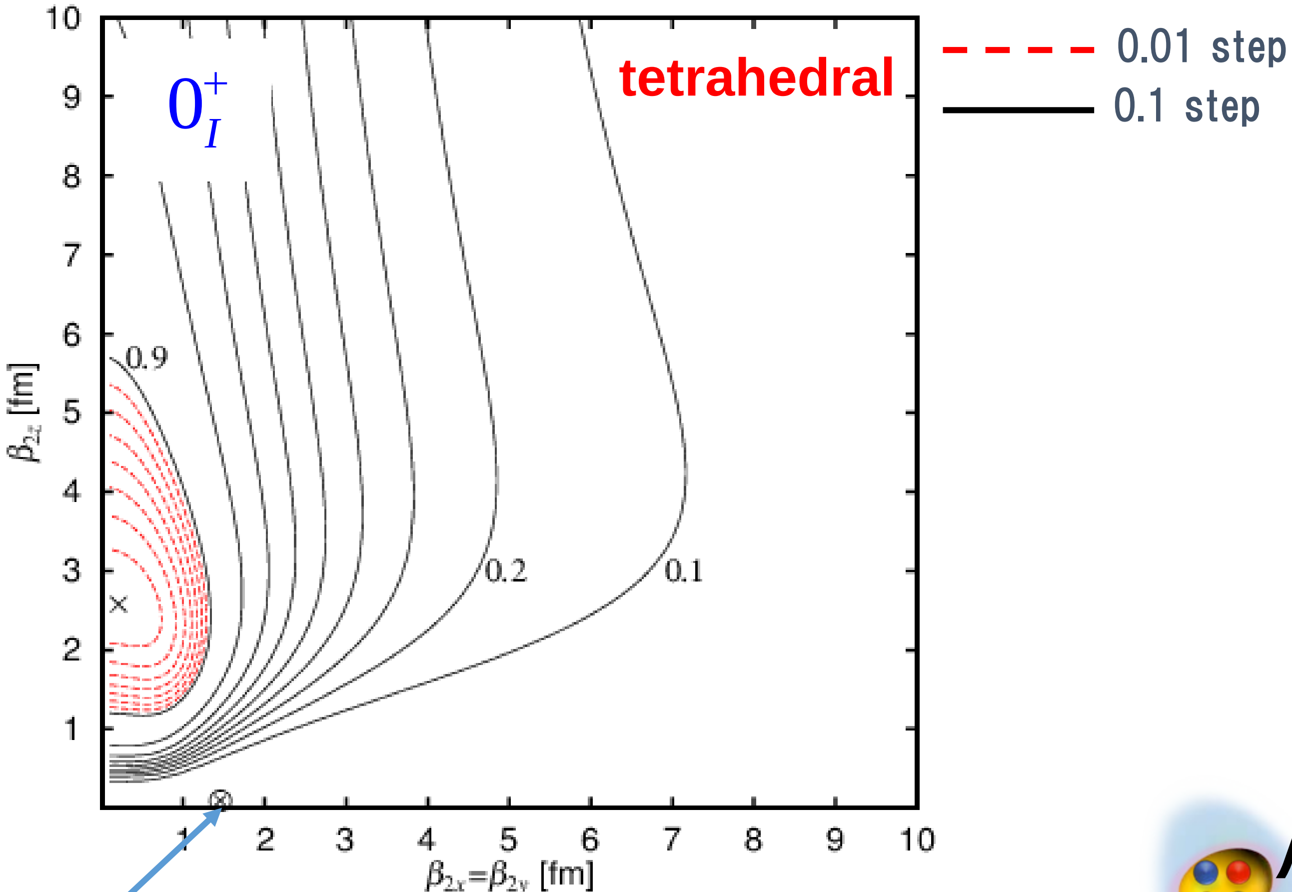
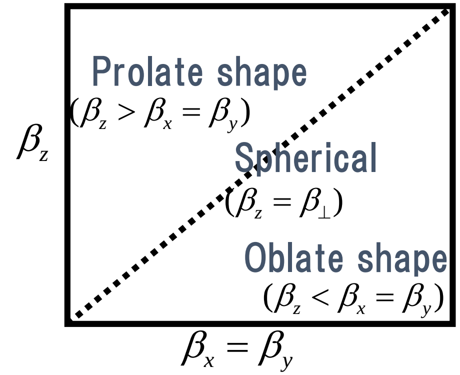
$M\left(E0:0^+_1\rightarrow 0^+_3\right)$	5.7	3.50	4.40 ± 0.44

Squared overlap surface with single config. of eTHSR



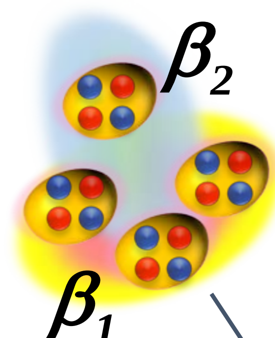


$(\beta_{1x} = \beta_{1y}, \beta_{1z})$: fixed at ⊗

Container for 3 α

× : maximum

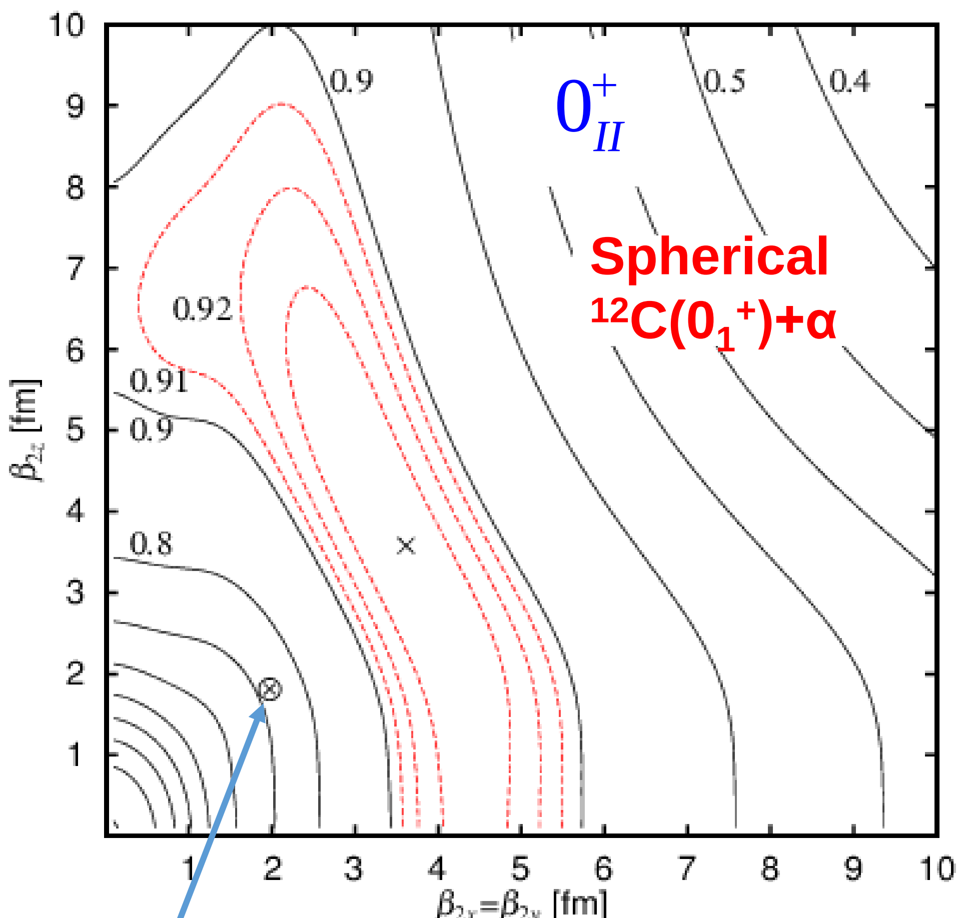
For the fourth α



THSR+GCM

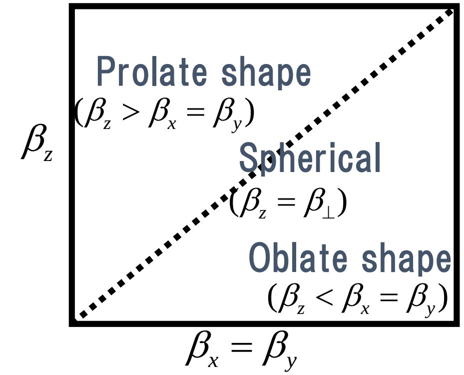
$$\left|\langle \Phi(\beta_1, \beta_2) | 0_\lambda^+ \rangle\right|^2$$

Squared overlap surface with single config. of eTHSR



0.01 step

0.1 step



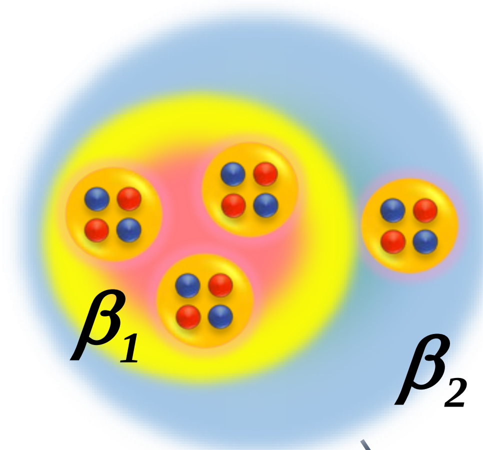
	Sq. overlap	$(\beta_{1x} = \beta_{1y}, \beta_{1z})$
$^{12}C(0_1^+)$	0.93	(1.9,1.8fm)
$^{12}C(2_1^+)$	0.90	(1.9, 0.5fm)
$^{12}C(0_2^+)$	0.99	(5.6,1.4fm)

$(\beta_{1x} = \beta_{1y}, \beta_{1z})$: fixed at ⊗

Container for 3α

× : maximum

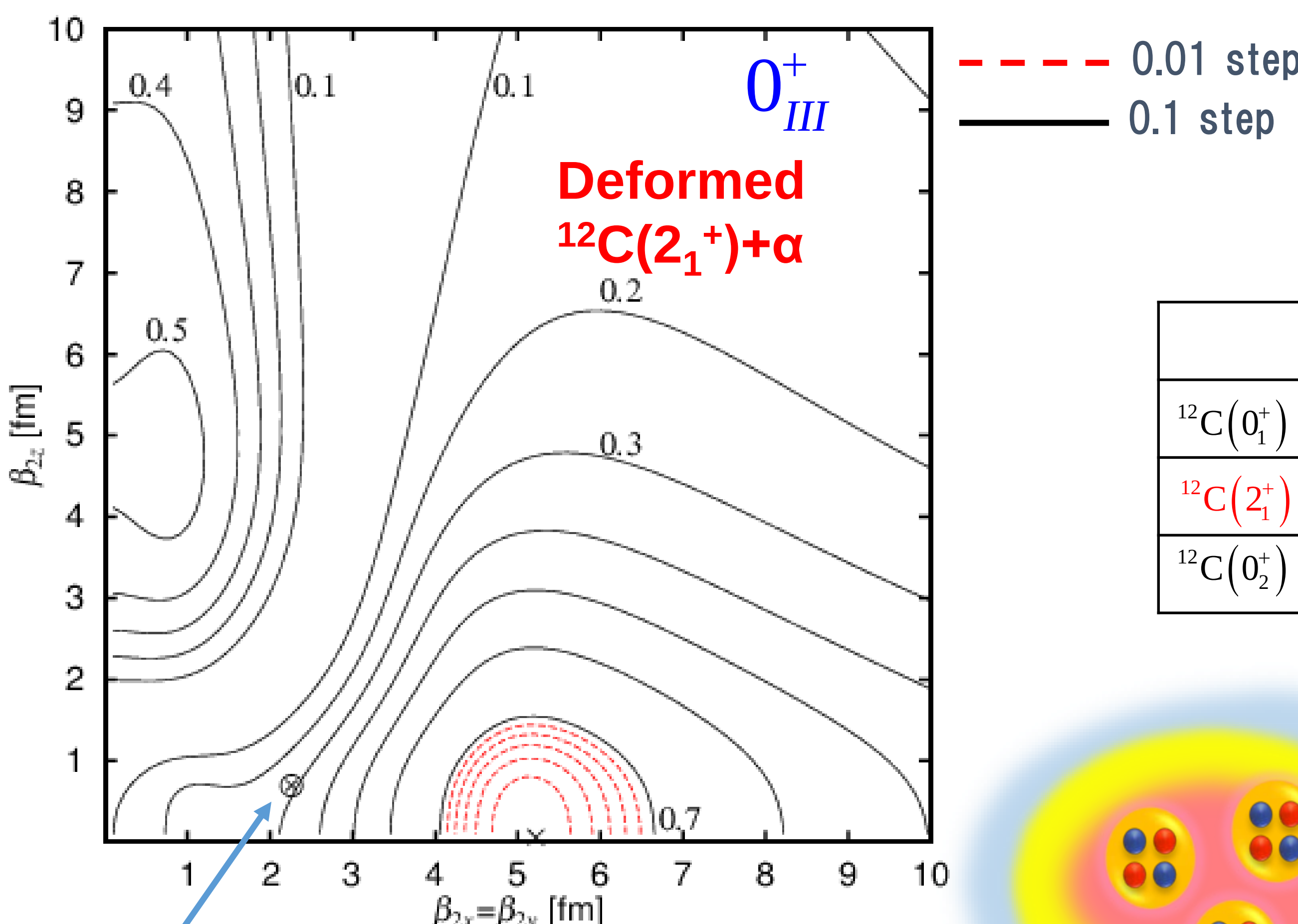
For the fourth α

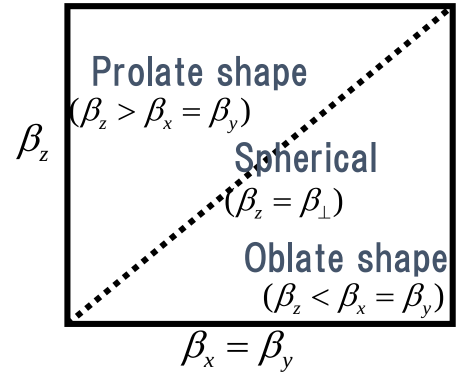


THSR+GCM

$$\left|\langle \Phi(\beta_1, \beta_2) | 0_\lambda^+ \rangle\right|^2$$

Squared overlap surface with single config. of eTHSR





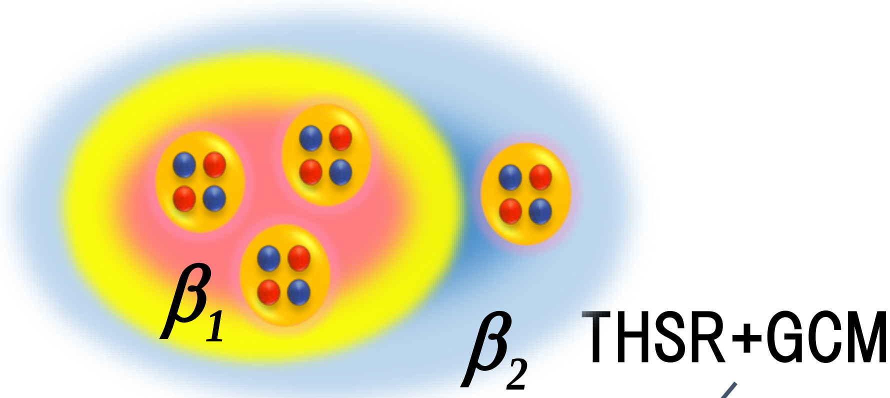
	Sq. overlap	$(\beta_{1x}=\beta_{1y},\beta_{1z})$
$^{12}C(0_1^+)$	0.93	(1.9,1.8fm)
$^{12}C(2_1^+)$	0.90	(1.9, 0.5fm)
$^{12}C(0_2^+)$	0.99	(5.6,1.4fm)

$(\beta_{1x}=\beta_{1y},\beta_{1z})$: fixed at ⊗

Container for 3 α

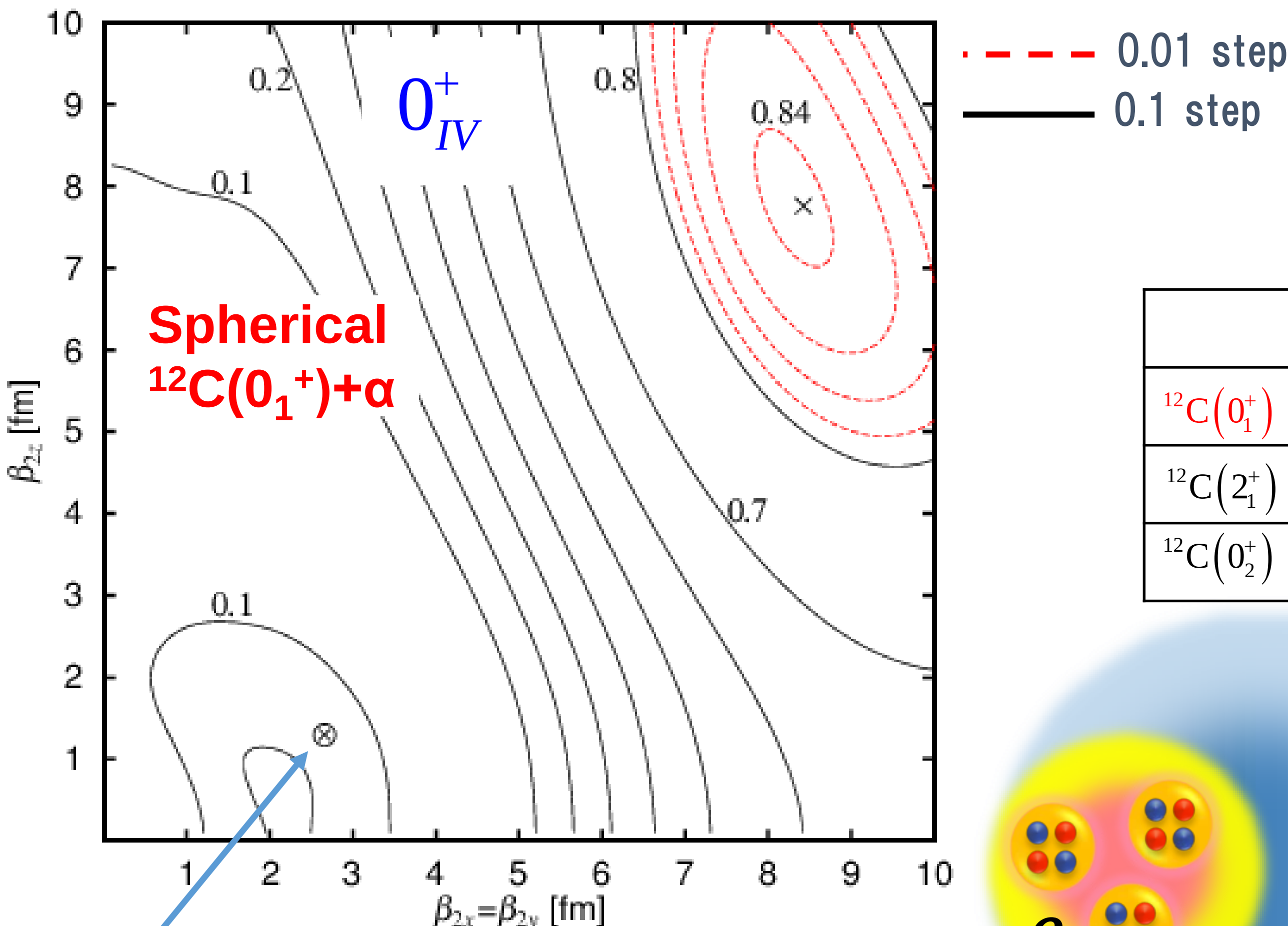
× : maximum

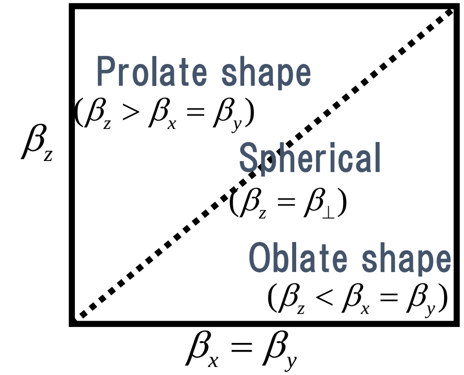
For the fourth α



$$|\langle \Phi(\beta_1,\beta_2)|0_\lambda^+\rangle|^2$$

Squared overlap surface with single config. of eTHSR





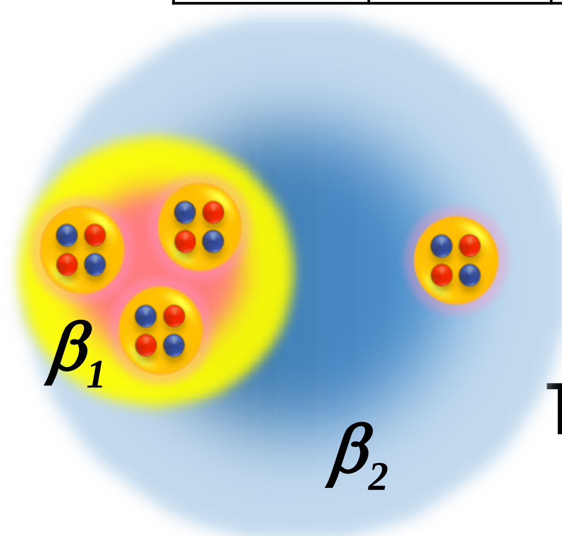
	Sq. overlap	$(\beta_{1x}=\beta_{1y},\beta_{1z})$
$^{12}C(0_1^+)$	0.93	(1.9,1.8fm)
$^{12}C(2_1^+)$	0.90	(1.9, 0.5fm)
$^{12}C(0_2^+)$	0.99	(5.6,1.4fm)

$(\beta_{1x}=\beta_{1y},\beta_{1z})$: fixed at ⊗

Container for 3 α

× : maximum

For the fourth α

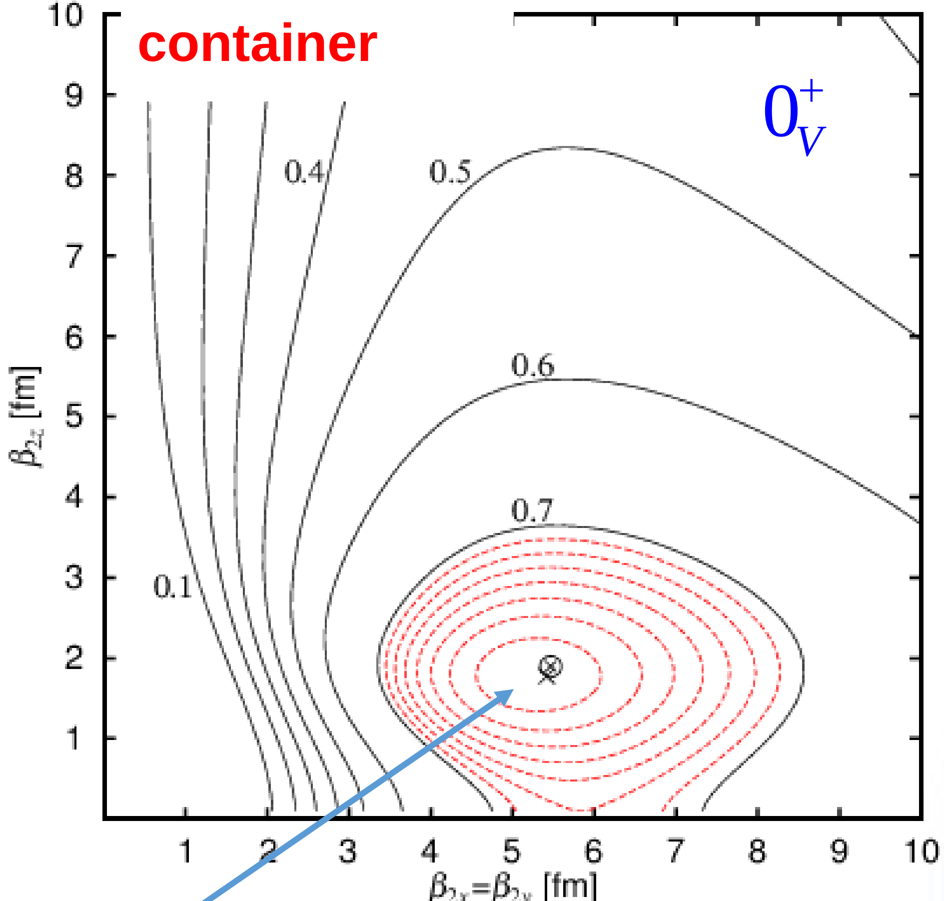


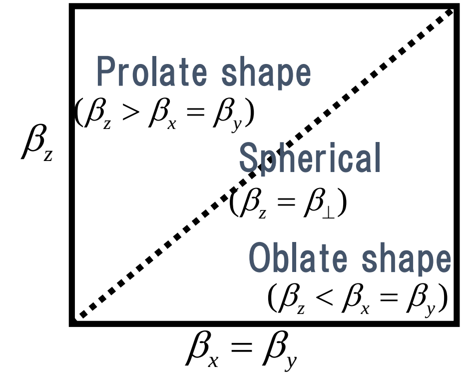
THSR+GCM

$$\left|\langle\Phi(\beta_1,\beta_2)|0_\lambda^+\rangle\right|^2$$

Squared overlap surface with single config. of eTHSR

4α in a common container





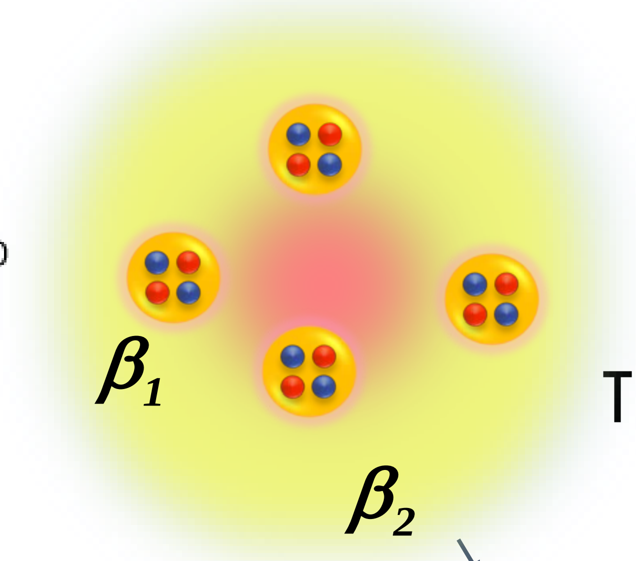
	Sq. overlap	$(\beta_{1x}=\beta_{1y},\beta_{1z})$
$^{12}C(0_1^+)$	0.93	(1.9,1.8fm)
$^{12}C(2_1^+)$	0.90	(1.9, 0.5fm)
$^{12}C(0_2^+)$	0.99	(5.6,1.4fm)

$(\beta_{1x}=\beta_{1y},\beta_{1z})$: fixed at ⊗

Container for 3 α

× : maximum

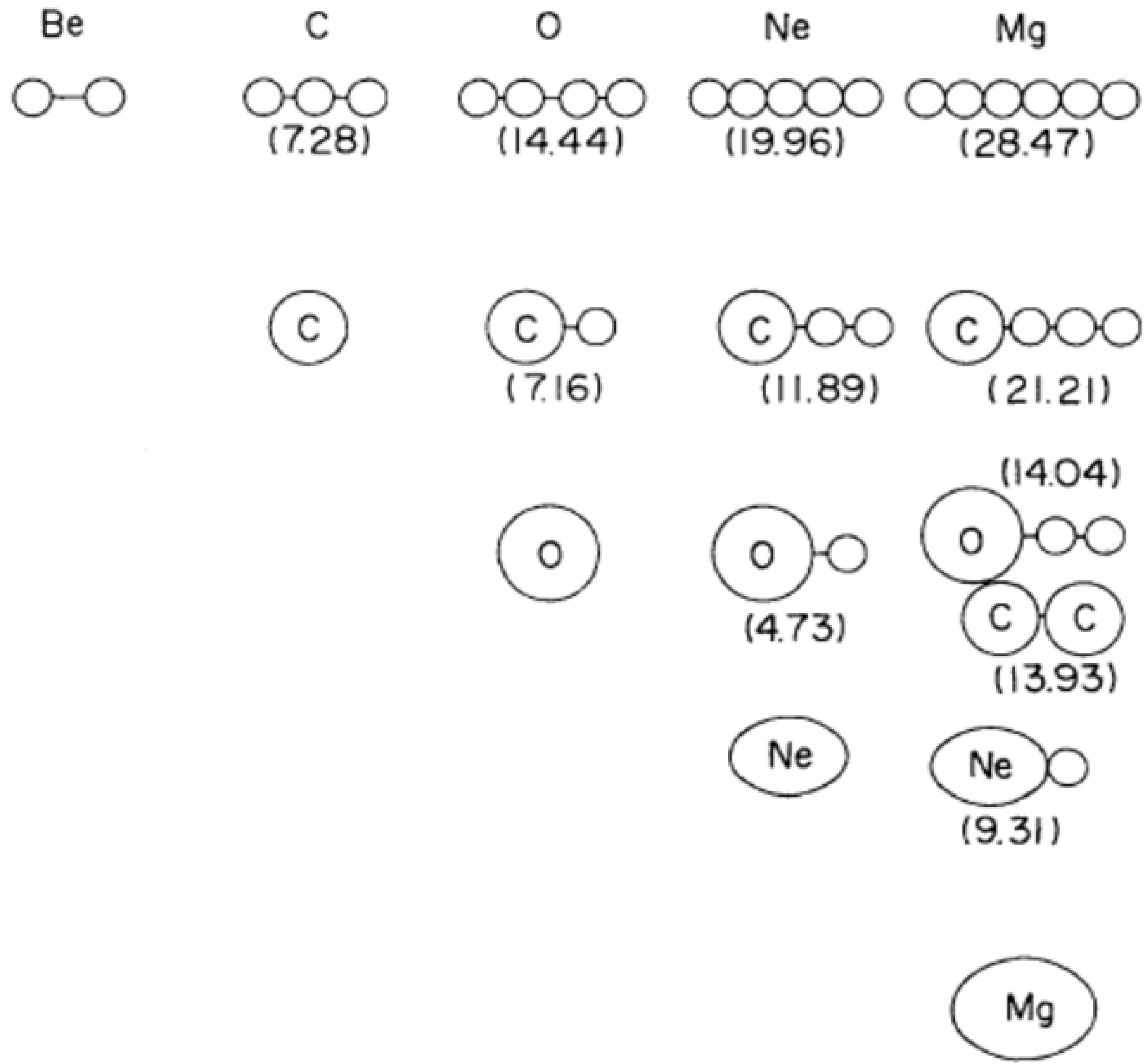
For the fourth α

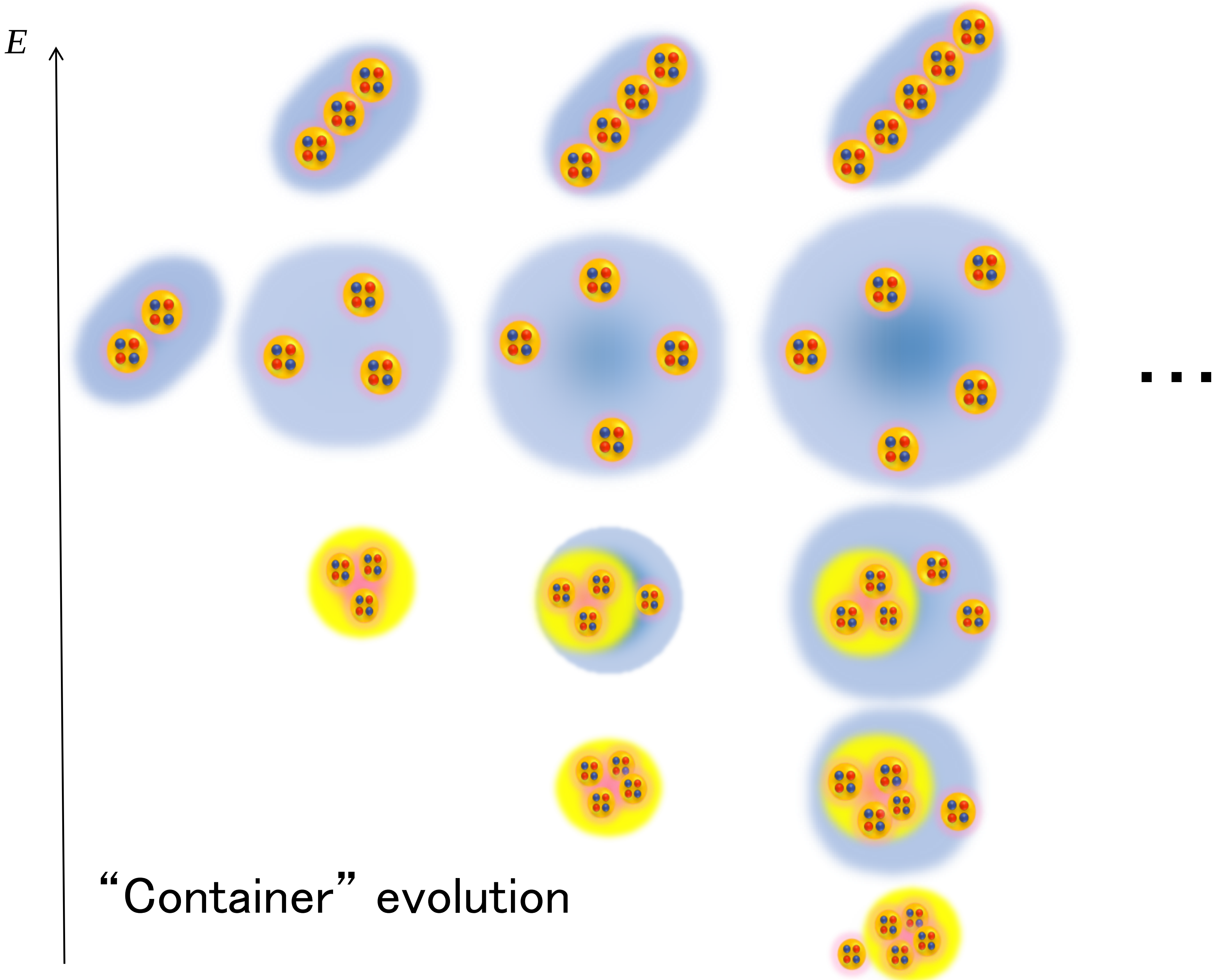


THSR+GCM

$$\left|\langle\Phi(\beta_1,\beta_2)|0_\lambda^+\rangle\right|^2$$

50th anniversary of Ikeda diagram (1968)




E
...
“Container” evolution

Thanks

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