The cluster structure of ^{12}C within the THSR wave function

Bo Zhou

Hokkaido University

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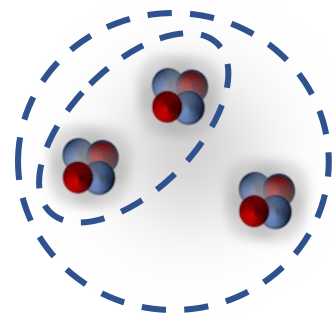
Outline

1. Container picture

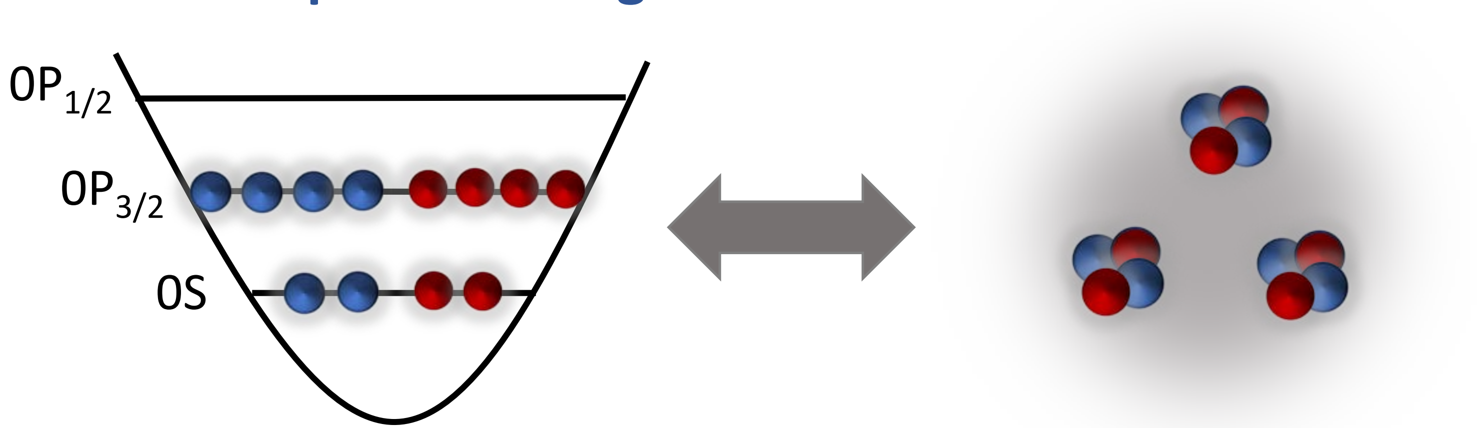


2. The ground and excited 0^+ states of ^{12}C in the container picture

3. The 3^- state of ^{12}C in the container picture (*Preliminary*)

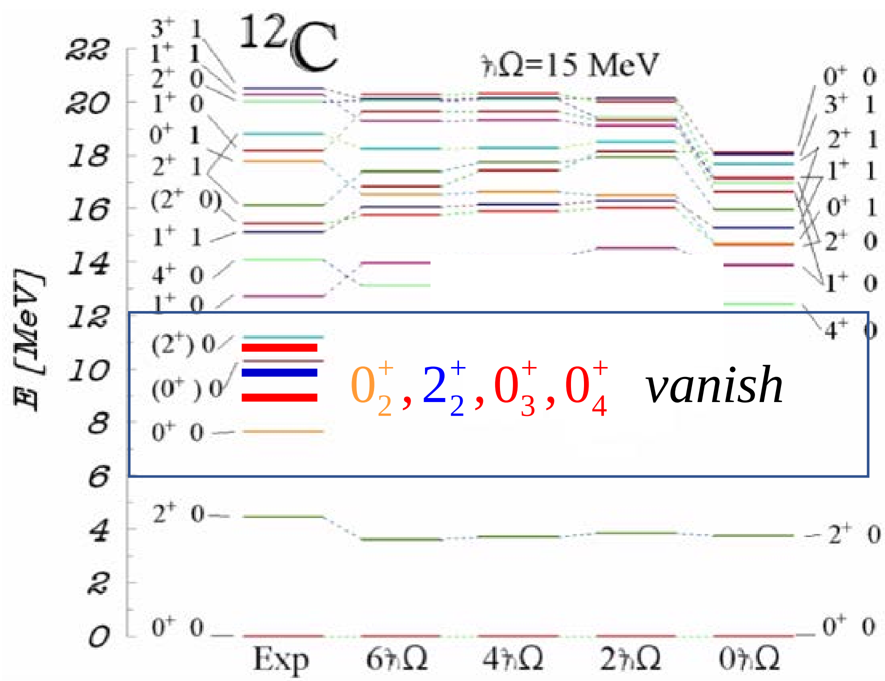
4. Summary and Prospect

Two fundamental pictures in light nuclei.

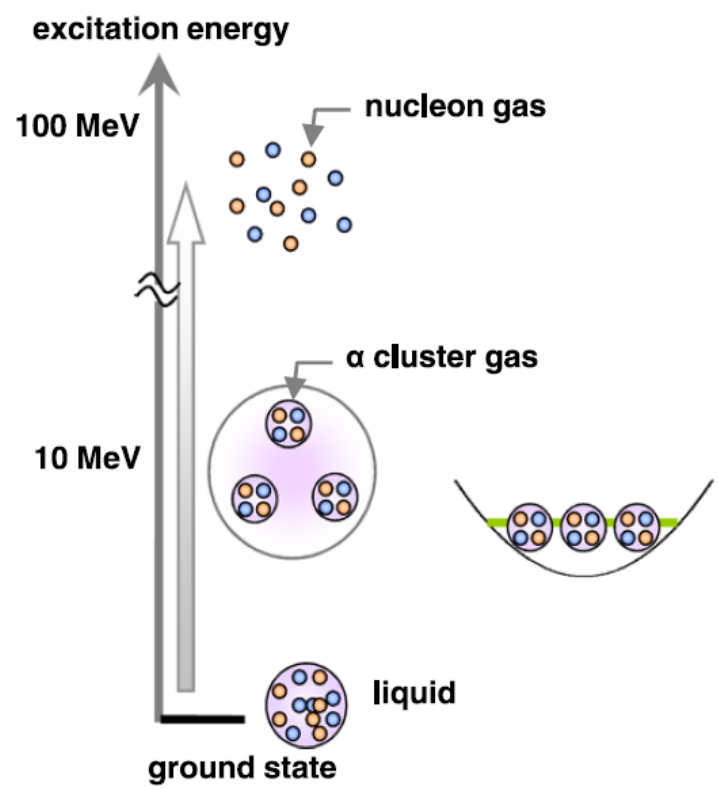


Single-particle motion

Clusters motion

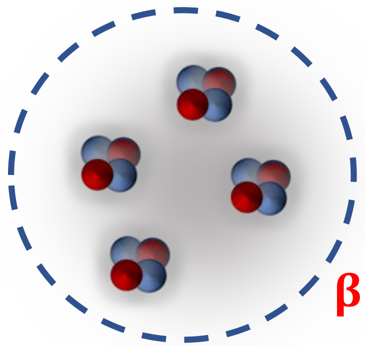


No-core nuclear shell model calculation



P. Navrátil, et al, Phys. Rev. Lett. **84**, 5728 (2000)

Y. Funaki et al., PPNP**82**,78 (2015).



Alpha Cluster Condensation in ^{12}C and ^{16}O

A. Tohsaki,[1] H. Horiuchi,[2] P. Schuck,[3] and G. Röpke[4]
[1]*Department of Fine Materials Engineering, Shinshu University, Ueda 386-8567, Japan*
[2]*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*
[3]*Institut de Physique Nucléaire, F-91406 Orsay Cedex, France*
[4]*FB Physik, Universität Rostock, D-18051 Rostock, Germany*
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A new α-cluster wave function is proposed which is of the α-particle condensate type. Applications to ^{12}C and ^{16}O show that states of low density close to the 3 and 4 α-particle thresholds in both nuclei are possibly of this kind. It is conjectured that all self-conjugate $4n$ nuclei may show similar features.

$$\Phi^{\mathrm{THSR}}(\beta)=\int d^3R_1\ldots d^3R_n\,\mathrm{Exp}\left[-\frac{R_1^2+\ldots+R_n^2}{\beta^2}\right]\Phi^{\mathrm{Brink}}(R_1,\ldots,R_n)$$

$$\propto \phi_G\,\mathcal{A}\left\{\prod_{i=1}^{n}\left[\mathrm{Exp}\left(-\frac{2(X_i-X_G)^2}{B^2}\right)\phi(\alpha_i)\right]\right\}$$

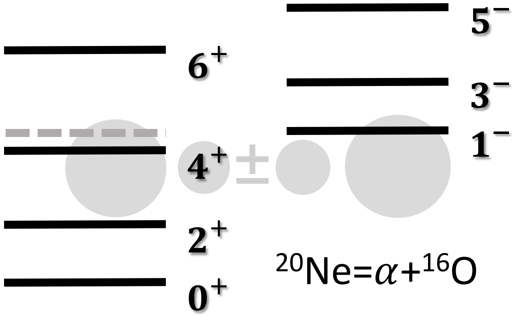
Nonlocalized

$$\phi(\alpha)\propto \exp\left[-\sum_{1\le i<j\le 4}(r_i-r_j)^2/(8b^2)\right]$$

$$B^2=b^2+2\beta^2$$

β can be considered as the size parameter of the nucleus

A. Tohsaki, H. Horiuchi, P. Schuck, and G. Röpke, Rev. Mod. Phys. **89**, 011002 (2017).



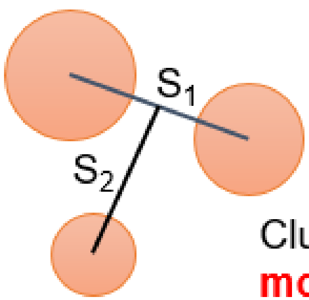
Inversion doublet rotational bands in ^{20}Ne

H.Horiuchi and K.Ikeda,PTP**40**,277(1968)

Container picture for the clusters motion

The clusters make the localized motion confined by the inter-cluster distance parameter *R*.

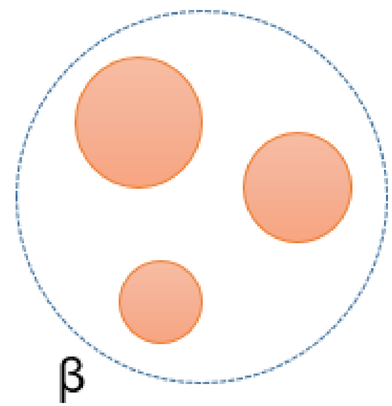
$$\mathcal{A}\{\exp[-\frac{8\,(\boldsymbol{r}-\boldsymbol{R})^2}{5\boldsymbol{b}^2}]\phi(\alpha)\phi(^{16}\mathrm{O})\}$$



S₁ and S₂: inter-cluster distance parameters.

Clusters make the **localised motions** around the fixed positions.

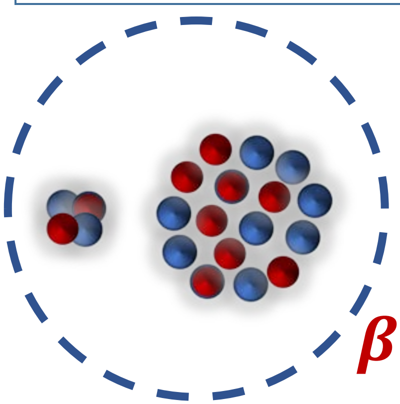
Traditional cluster model



β: the size of the nucleus parameter.

Clusters move around in the **cluster-type mean field**.

Container model



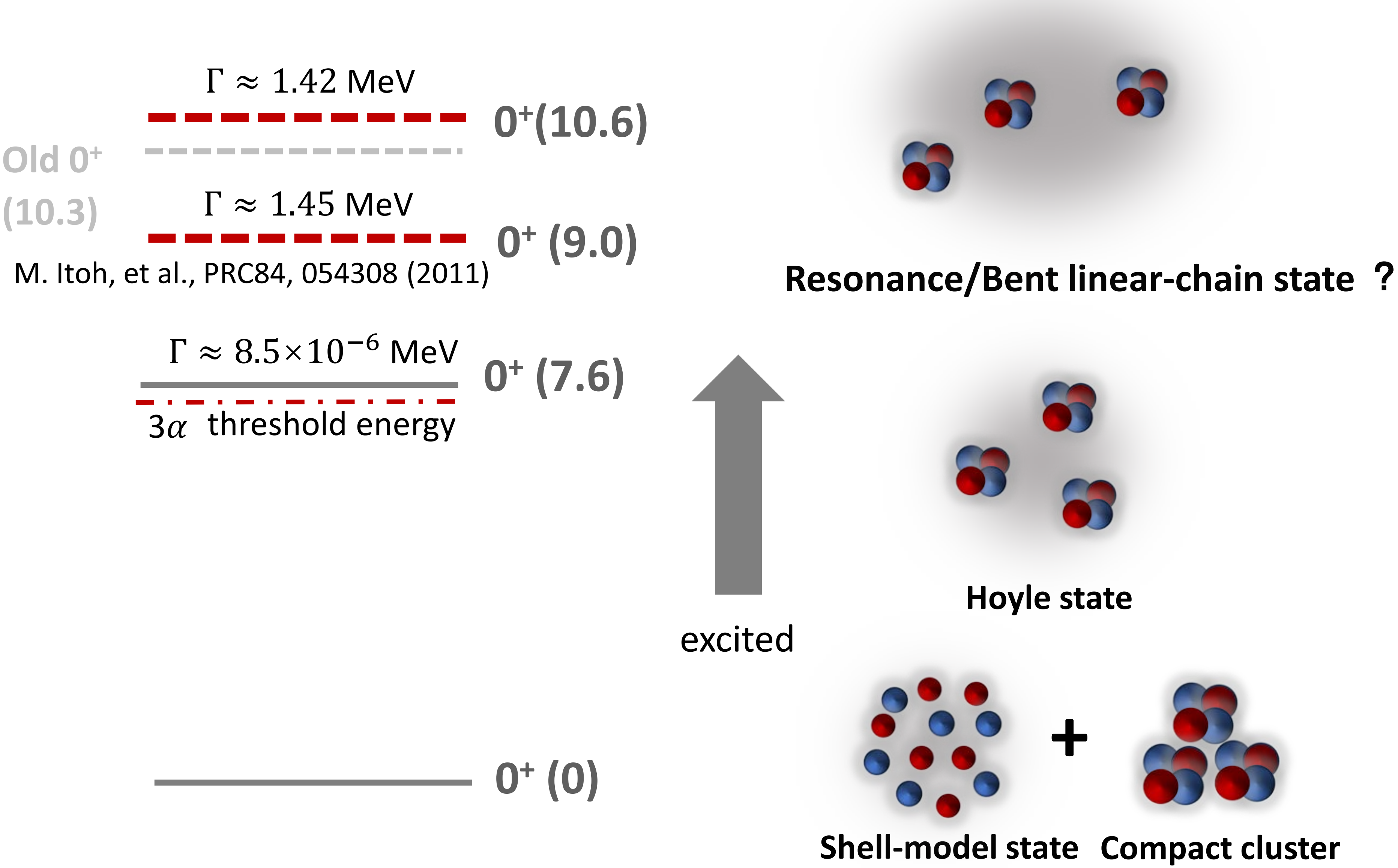
Container picture

The clusters make the nonlocalized motion in a container whose size is described by parameter *β*

$$\mathcal{A}\{\exp[-\frac{8\,r^2}{5(\boldsymbol{b}^2+2\boldsymbol{\beta}^2)}]\phi(\alpha)\phi(^{16}\mathrm{O})\}$$

B. Z, Y. Funaki, et al., PRL**110**(2013),PRC**89** (2014).

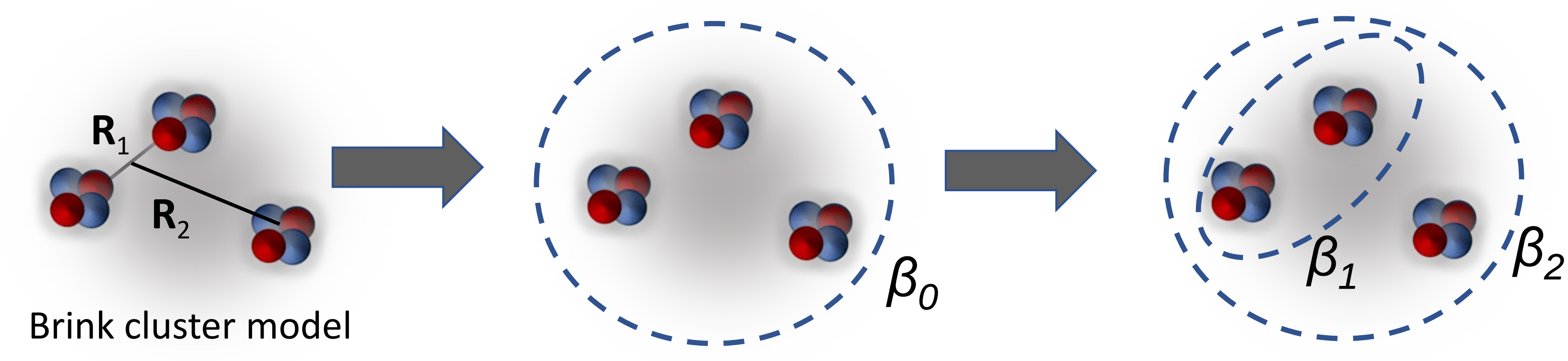
Rich cluster structures of 0^+ states in ^{12}C



OCM_K : C. Kurokawa and K. Kato, PRC 71, 021301(2005); NPA 792, 87 (2007).

OCM_o : S. Ohtsubo, Y. Fukushima, M. Kamimura, and E. Hiyama,PTEP, 2013, 073D02.

Extended 2α+α THSR Wave Function



B.Z, et al.,PTEP.2014,101D01.

$$\Phi^B(\boldsymbol{R}_1, \boldsymbol{R}_2) \propto \phi_G \mathcal{A}\{\exp(-\frac{(\boldsymbol{r}_1 - \boldsymbol{R}_1)^2}{b^2} - \frac{(\boldsymbol{r}_2 - \boldsymbol{R}_2)^2}{\frac{3}{4}b^2})\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\}$$

$$\Phi(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2) = \int d^3R_1 d^3R_2 \exp[-\sum_{i=1}^{2}(\frac{R_{ix}^2}{\beta_{ix}^2} + \frac{R_{iy}^2}{\beta_{iy}^2} + \frac{R_{iz}^2}{\beta_{iz}^2})]\Phi^B(\boldsymbol{R}_1, \boldsymbol{R}_2)$$

$$\propto \phi_G \mathcal{A}\{\exp[-\sum_{i=1}^{2}(\frac{r_{ix}^2}{B_{ix}^2} + \frac{r_{iy}^2}{B_{iy}^2} + \frac{r_{iz}^2}{B_{iz}^2})]\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\}$$

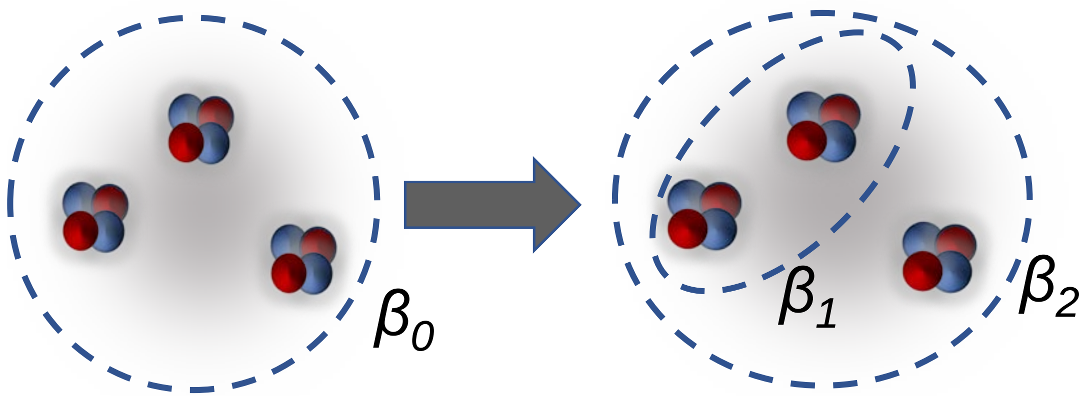
$$B_{1k}^2 = b^2 + \beta_{1k}^2, \; B_{2k}^2 = \frac{3}{4}b^2 + \beta_{2k}^2$$

Effective nucleon-nucleon interaction: $$V_N = \sum_{i>j}^{20}\{(1-M) - MP_\sigma P_\tau\}_{ij} \sum_{n=1}^{2} v_n e^{-\frac{r_{ij}^2}{a_n^2}}.$$

Radius-Constraint Method for removing the continuum states.

Y. Funaki, et al., Prog.Theor.Phys.115,115(2006).

The ground state of ^{12}C in the two-β THSR wave function



[1] Y.Fukushima and M.Kamimura in Proceedings of the International Conference on Nuclear Structure (1977). M.Kamimura, Nucl.Phys.A 351,456(1981)**(RGM)**

[2] E.Uegaki, S.Okabe, Y.Abe and H.Tanaka, PTP 57,1262(1977); 59,1031(1978); 62,1621(1979) **(GCM)**

[3] Y. Funaki et al., PRC 67, 051306(2003) **(THSR)**

Pot	Emin (β_0)[3]	Emin (β_1,β_2)	Full solution (GCM/RGM)	GCM (β_0)[3]	GCM (β_1 ,β_2)	Squared overlap
F1	-86.09	-87.28	-87.92 [2]	-87.81	-87.98	0.975
F2	-87.68	-89.05	-89.4 [1]	-89.52	-89.65	0.978

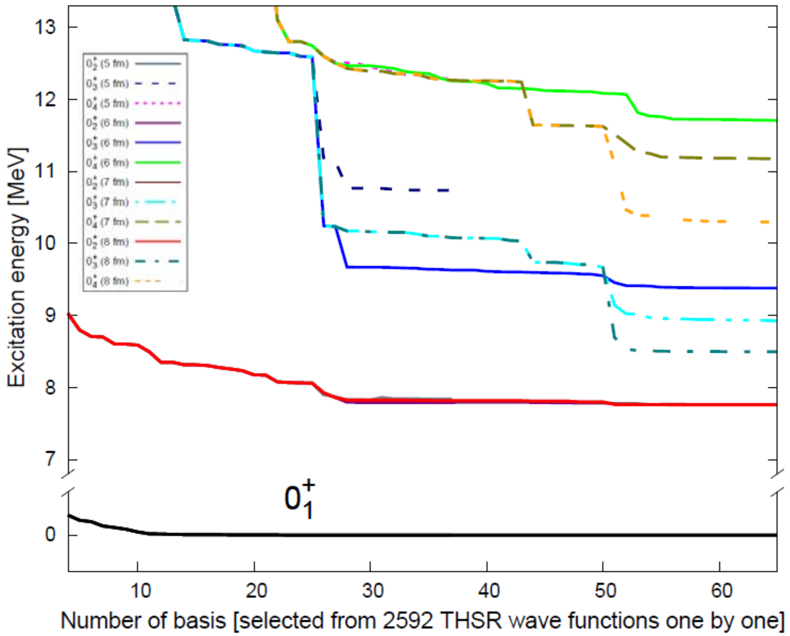
Squared overlap: $|< \Phi_{min}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2)|\Phi_{GCM}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2) > |^2$

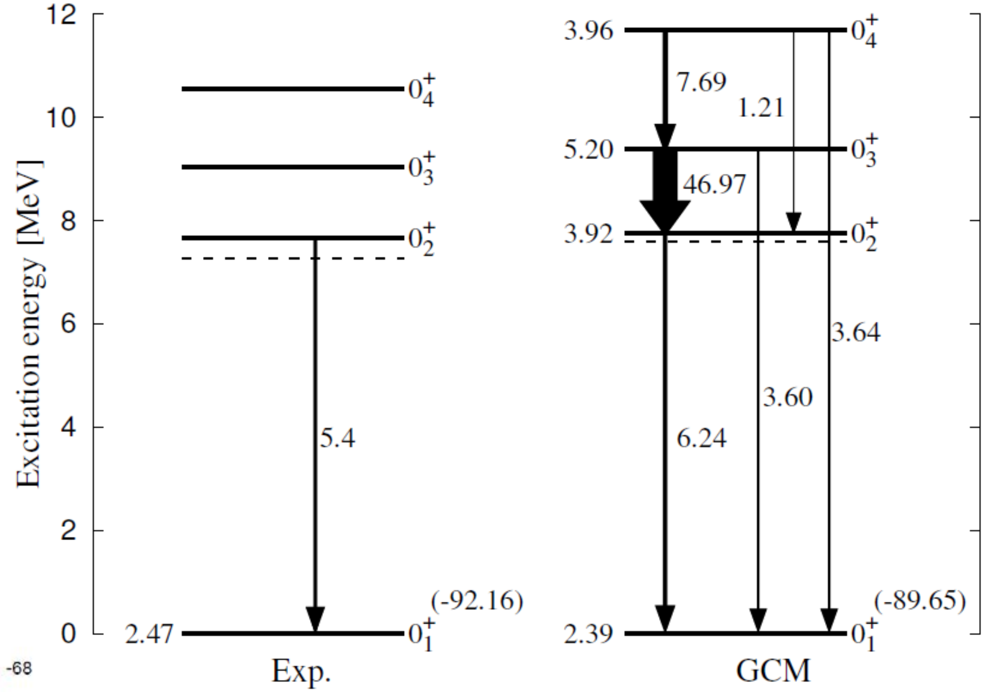
The single THSR wave function is almost equivalent to the RGM/GCM wave function.

In the container picture, 2α correlation is very important for the ground state of ^{12}C.

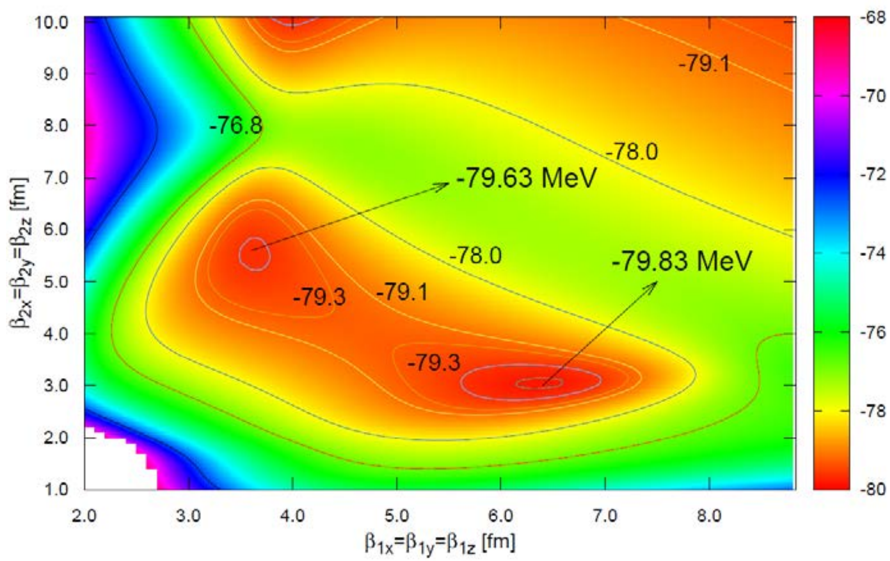
B.Z, A.Tohsaki, et al. Prog. Theor. Exp. Phys. **2014**, 101D01

The 0_3^+ and 0_4^+ states of ^{12}C





$$|\langle \hat{\Phi}^{0_3^+}_{\rm min1}(\beta_1 = 6.4, \beta_2 = 3.0)|\hat{\Phi}^{0_3^+}_{\rm gcm}\rangle|^2$$ = **0.903**



$$P_2^{\perp} = 1 - n_1|\hat{\Phi}^{0_1^+}\rangle\langle\hat{\Phi}^{0_1^+}| - n_2|\hat{\Phi}^{0_2^+}\rangle\langle\hat{\Phi}^{0_2^+}|,$$

The existence of the 0_3^+ state was confirmed in two ways.

Very large radius, strong monopole transitions between the 0_3^+ state and the Hoyle sate.

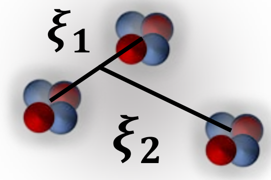
B. Z, A. Tohsaki, H. Horiuchi, and Z. Ren, Phys. Rev. C **94**, 044319 (2016).

Relative wave functions between $2\alpha-\alpha$ and $\alpha-\alpha$ in ^{12}C

$$O_B=\sum_{i=1}^{12}(\boldsymbol{r}_i-\boldsymbol{r}_{\rm cm})^2 \;\Rightarrow\; O_B=\sum_{k=1}^{3}\sum_{i\in\alpha_k}(\boldsymbol{r}_i-\boldsymbol{X}_k)^2+2\xi_1^2+\frac{8}{3}\xi_2^2,$$

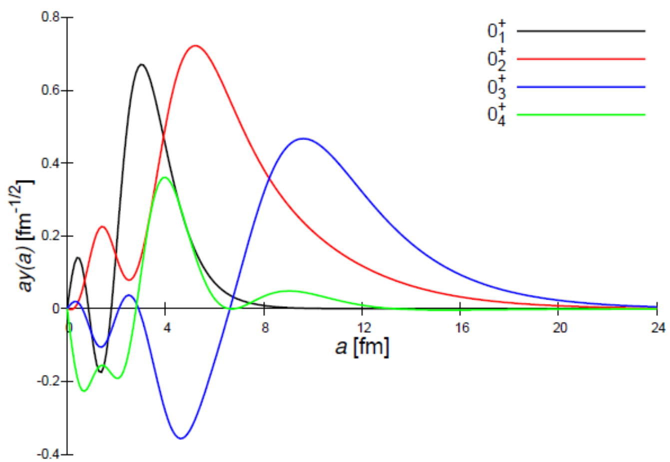
The reduced width amplitude (RWA) of ^{12}C can be written as,

$$\mathcal{Y}(a)=\sqrt{\frac{12!}{4!8!}}\langle[\hat{\Phi}_{2\alpha}^{0^+},Y_{00}(\hat{\boldsymbol{\xi}}_2)]_{00}\frac{\delta(\xi_2-a)}{\xi_2^2}\phi(\alpha)|\hat{\Phi}_{\rm gcm}^{0_k^+}\rangle.$$



We define a new 2α relative wave function of ^{12}C as follows,

$$\chi(a)=N_0\sqrt{\frac{12!}{4!4!4!}}\langle[e^{-\frac{\xi_{2x}^2}{B_{2x}^2}-\frac{\xi_{2y}^2}{B_{2y}^2}-\frac{\xi_{2z}^2}{B_{2z}^2}}\phi^3(\alpha)]^{0^+}\frac{\delta(\xi_1-a)}{\xi_1^2}Y_{00}(\hat{\boldsymbol{\xi}}_1)|\hat{\Phi}_{\rm gcm}^{0_k^+}\rangle$$



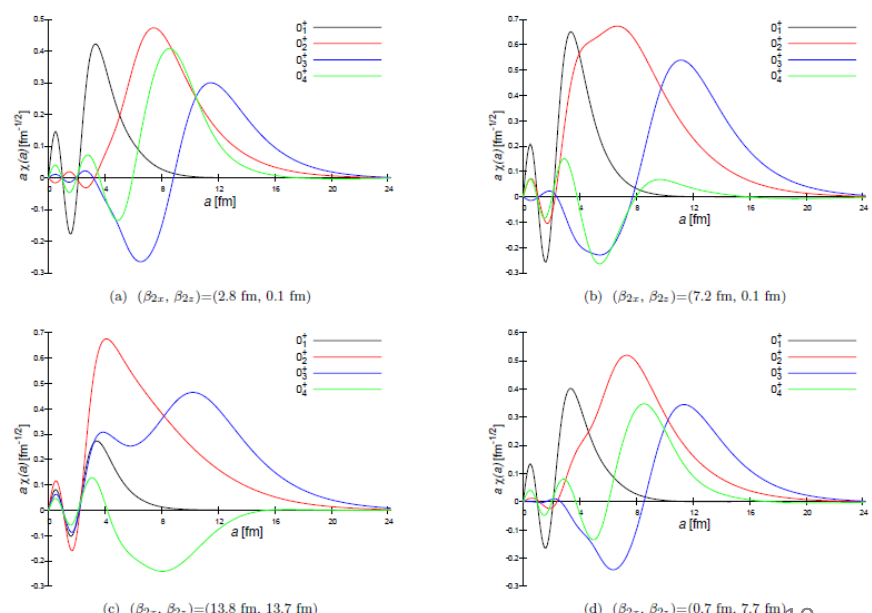
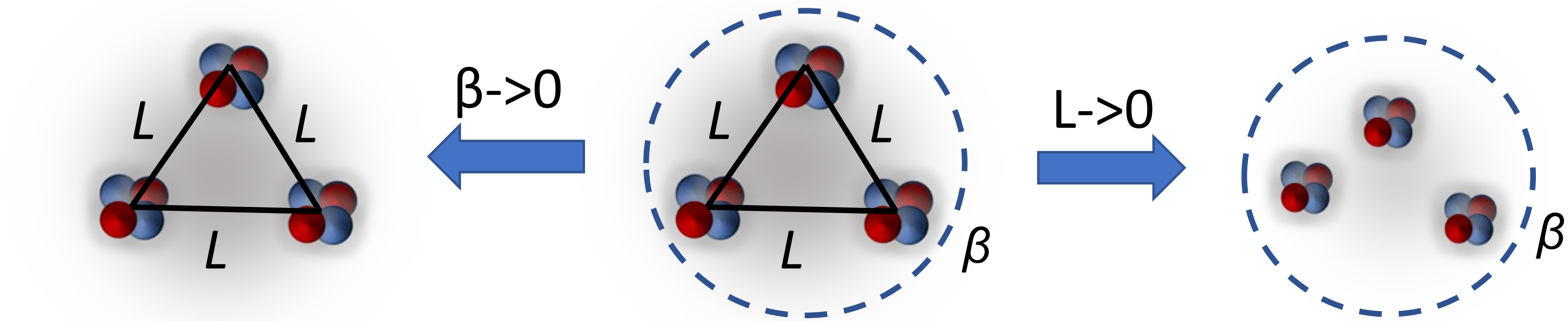


FIG. 5: The calculated 2α correlation wave functions of the $0_1^+,0_2^+,0_3^+$, and 0_4^+ states using four sets of β_2 parameters in ^{12}C.

B. Zhou, A. Tohsaki, H. Horiuchi, and Z. Ren, Phys. Rev. C **94**, 044319 (2016).

Negative-parity states in the container picture

Preliminary

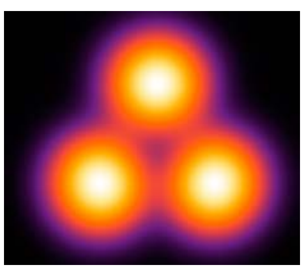


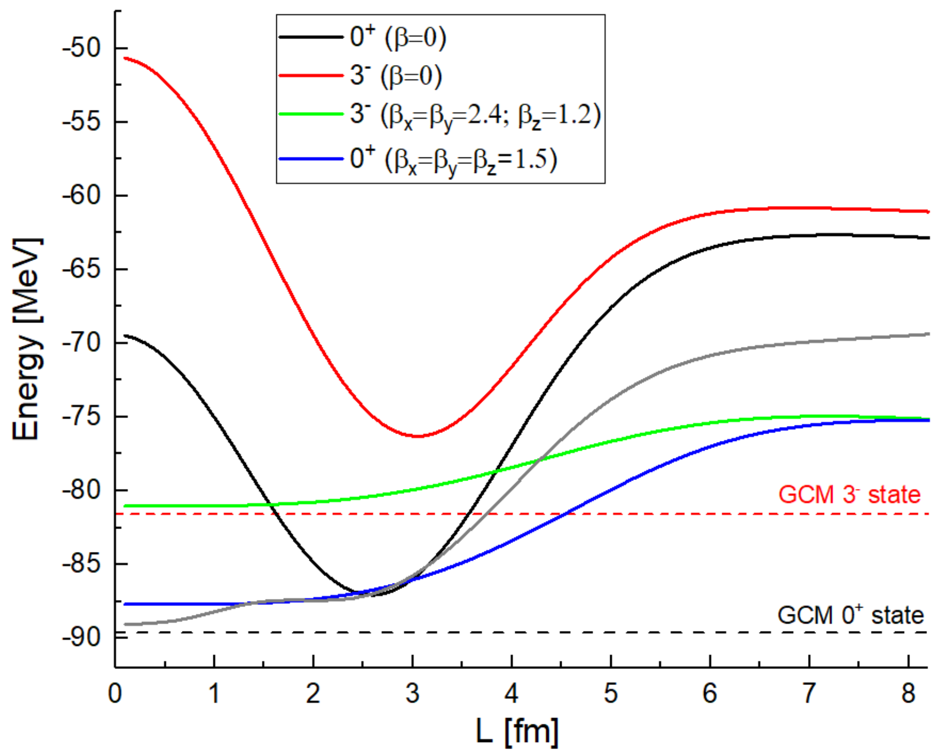
The Brink cluster model with equi-triangle shape

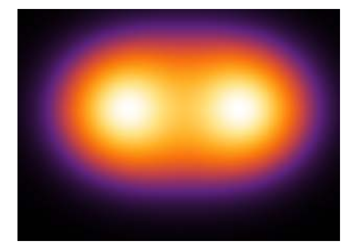
Hybrid-THSR-Brink wave function

The THSR picture with the size parameter

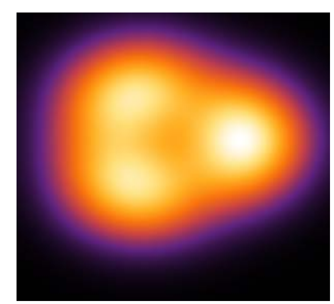
$$\propto \mathcal{A}\{\exp[-\frac{(\boldsymbol{\xi}_1-\boldsymbol{R}_1)^2}{b^2+2\beta^2}-\frac{(\boldsymbol{\xi}_2-\boldsymbol{R}_2)^2}{3/4(b^2+2\beta^2)}\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)]\}$$







L=0.5



L=2

Summary and Prospect

1. The 2α+α wave function was constructed and it is found that the 2α correlation in the ground state of 12C is important.

2. The existence of the 0_3^+ and 0_4^+ states around 10 MeV excitation energy in ^{12}C is confirmed by using the THSR-GCM+Radius-Constraint Method and the constructed single THSR wave function. This 0_3^+ state is considered to be a breathing-like excited state of the Hoyle state.

Future:

The negative-parity state of ^{12}C will be studied in the Hybrid-THSR-Brink wave function

$$\propto \mathcal{A}\{\exp[-\frac{(\boldsymbol{\xi}_1-\boldsymbol{R}_1)^2}{b^2+2\beta^2}-\frac{(\boldsymbol{\xi}_2-\boldsymbol{R}_2)^2}{3/4(b^2+2\beta^2)}\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)]\}$$

to discuss the **intrinsic shape** of the ^{12}C

Thanks for my collaborators and your attention!

Hisashi Horiuchi	(RCNP)
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Chang Xu	(Nanjing Univ.)
Mengjiao Lyu	(Nanjing Univ.)
Qing Zhao	(Nanjing Univ.)
Gerd Röpke	(Rostock Univ.)
Peter Schuck	(Paris-Sud Univ.)
Yasuro Funaki	(Kanto Gakuin Univ.)
Taiichi Yamada	(Kanto Gakuin Univ.)