Virtual States in the Complex Scaling Method

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1. Cross Sections just above Thresholds

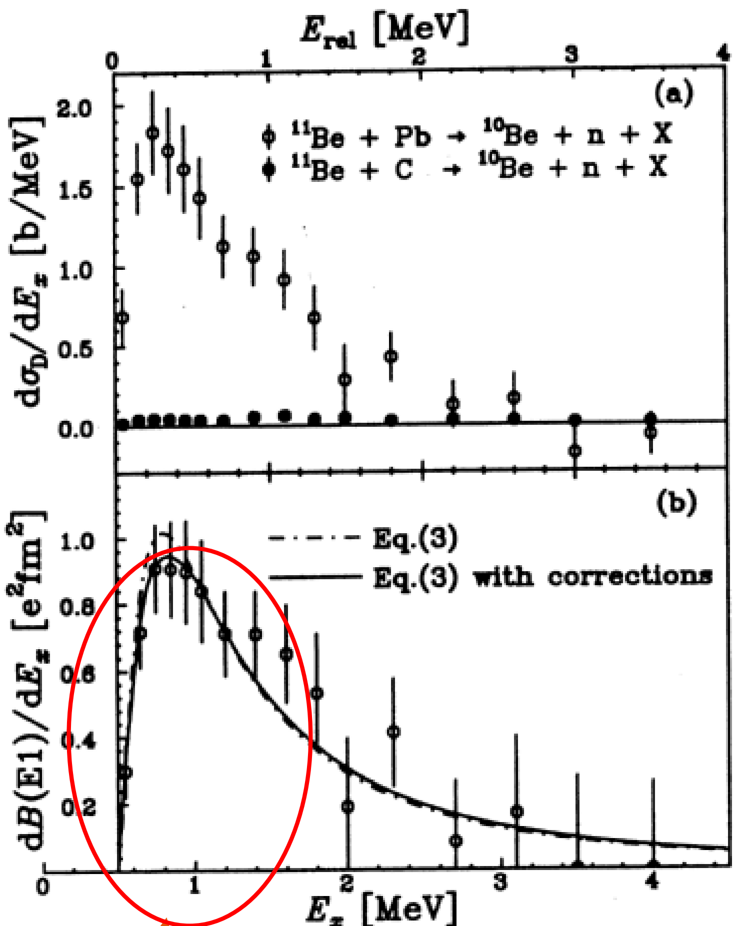
◆Nuclear reaction cross sections observed around threshold energies provide us with interesting information about nuclear cluster structures. ("Ikeda's threshold rule")

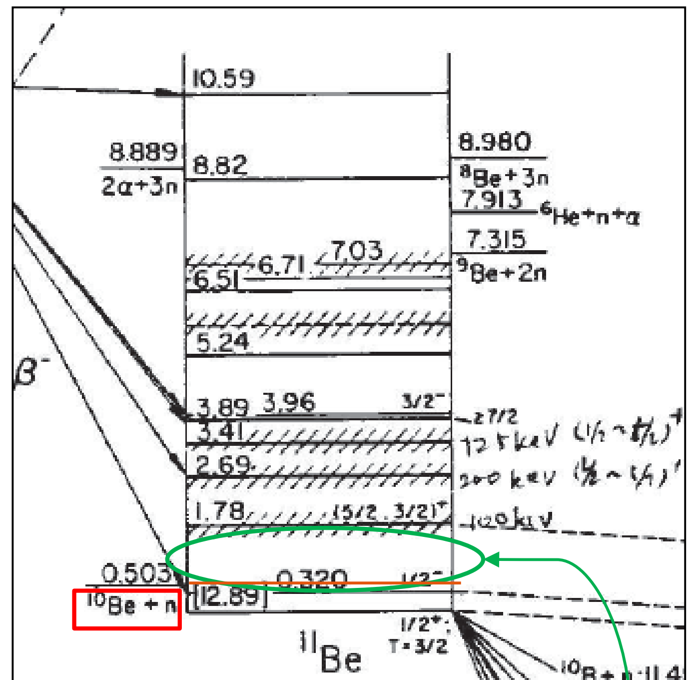
◆Therefore, it is important to investigate its origin of the cross section observed just above threshold carefully.

Outline

- Several photodisintegration cross sections
- The first $1/2^+$ state in ^{9}Be and photodisintegration
- Virtual state in complex scaling method
- Cross sections of virtual and resonant states

Coulomb breakup reaction of ^{11}Be



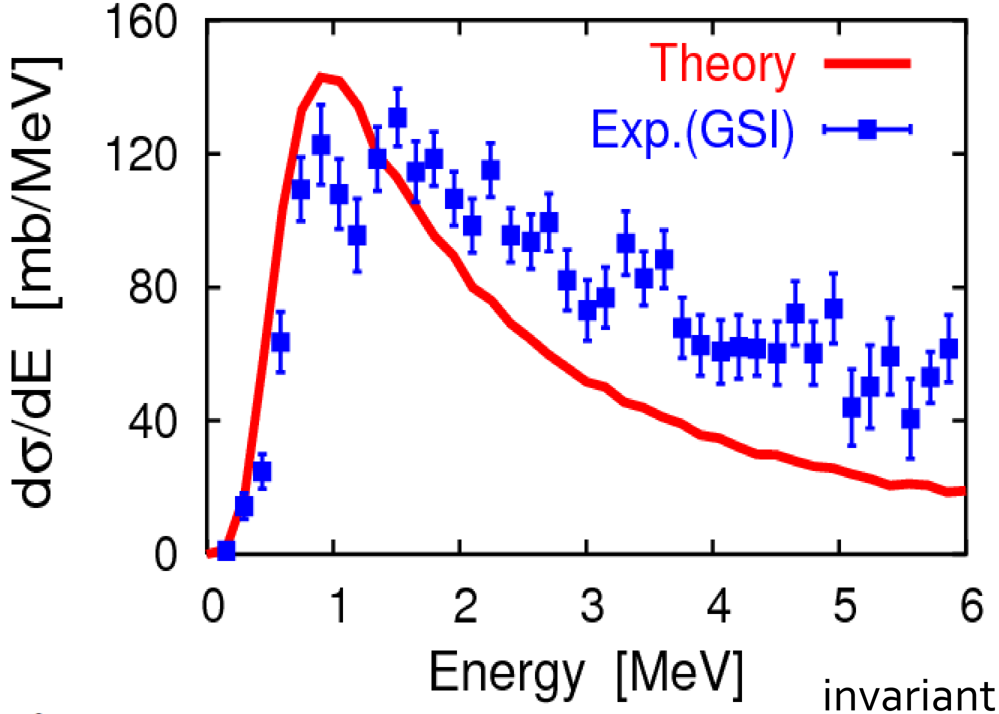


No resonant state

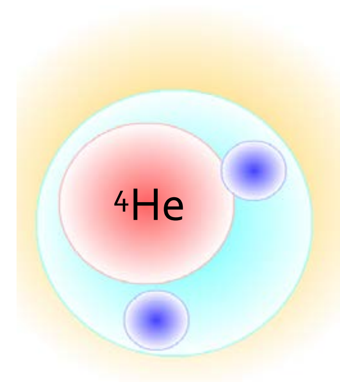
One neutron halo structure in the ground state of ^{11}Be

T. Nakamura et. al. Phys. Lett. **B331**(1994), 296

Coulomb breakup reaction of ^{6}He

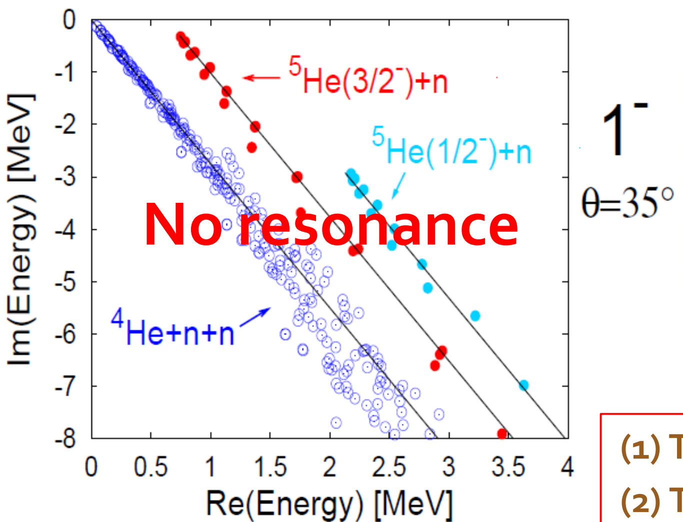


EXP:
T. Aumann et al., PRC**59**(1999)1252
Theory:
T. Myo, et al. PRC**63**(2001)054313.

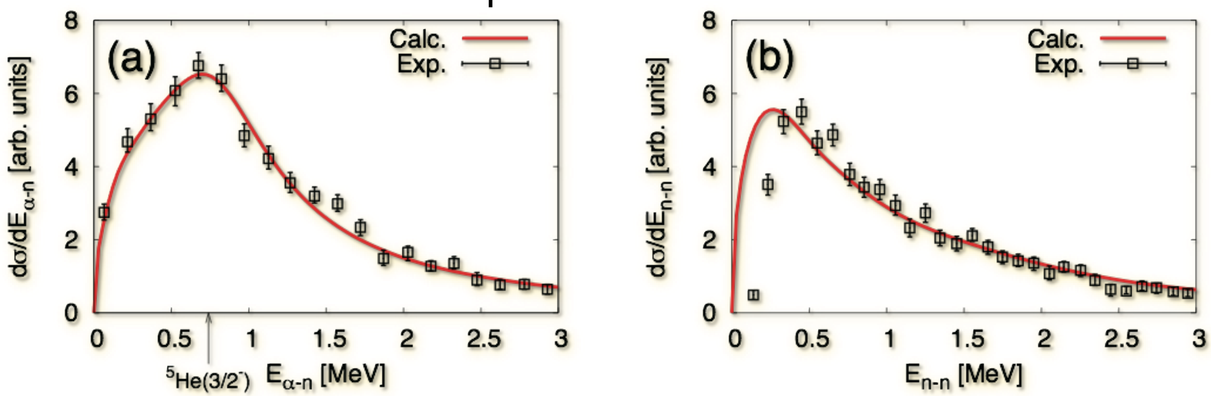


α + n+n model

Y. Kikuchi et al., Prog. Theor. Phys. **122** (2009), 499;
Phys. Rev. C**81** (2010), 044308



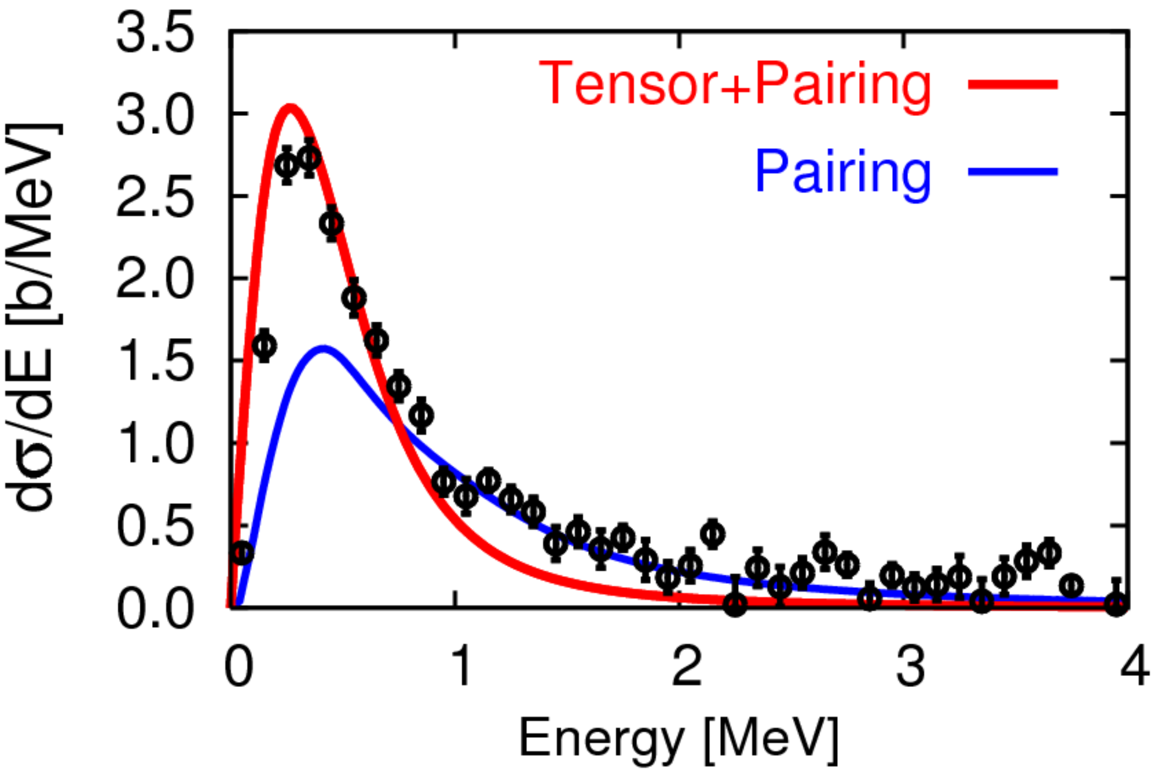
invariant mass spectra of α+n and n+n channels



(1) The result supports the sequential decay: ^{6}He → ^{5}He($3/2^{-}$)+n → ^{4}He+n+n.

(2) The n-n virtual-state correlations in final state has an important effect on the breakup cross section.

Coulomb breakup reaction of ^{11}Li

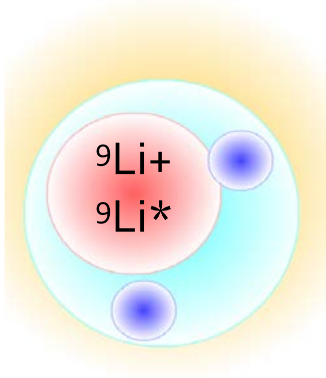


EXP:

T. Nakamura et al., PRL96,252502(2006)

Theory:

T. Myo, et al. PRC76(2007)024305

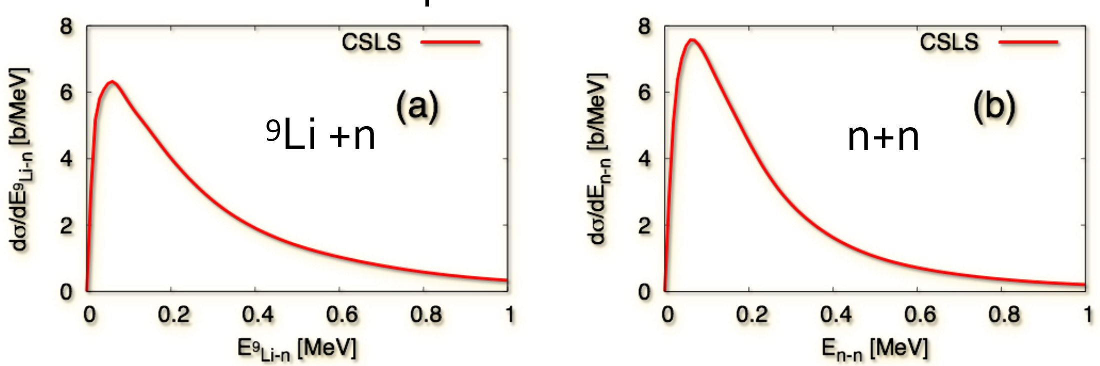


^{9}Li + n+n model

No three-body resonance

- (1) **Both spectra for core-n and n-n subsystems show sharp peaks at a very low-energy region.**
- (2) **These peaks come from the virtual states of ^{10}Li and n-n.**
- (3) **The breakup process of ^{11}Li is dominated by ^{9}Li-n and n-n FSI's which cause the virtual states.**

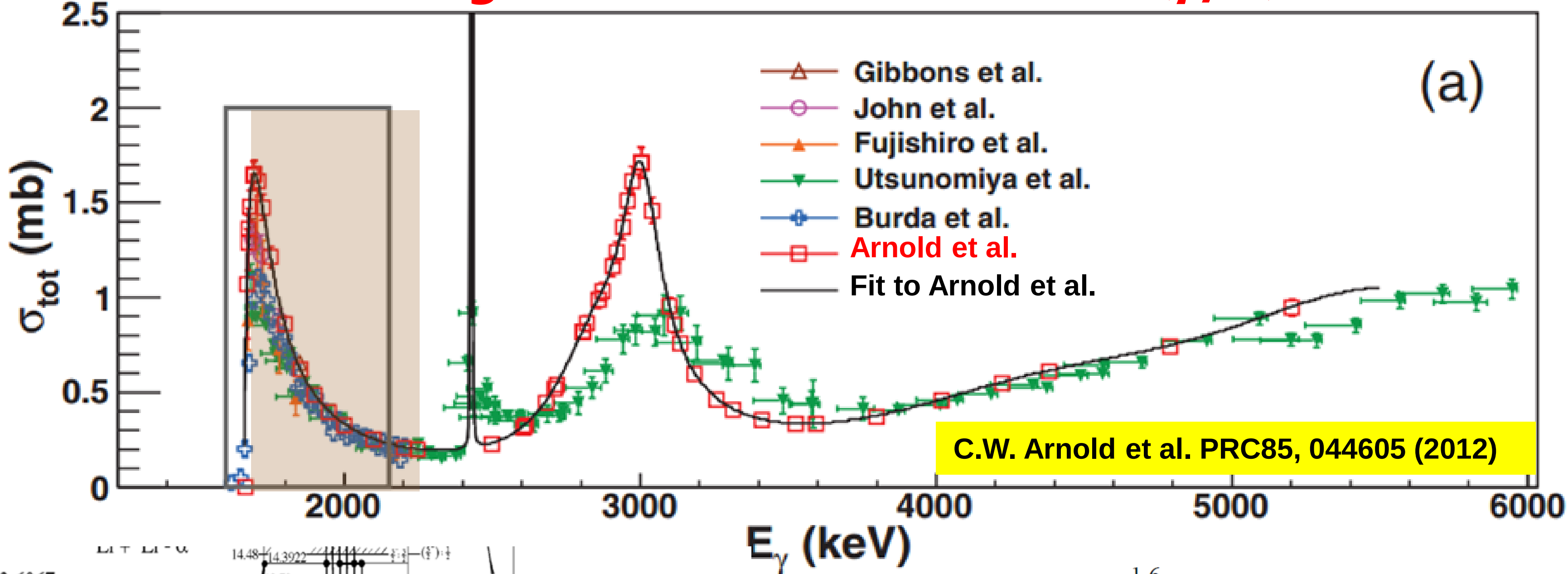
invariant mass spectra of ^{9}Li +n and n+n channels



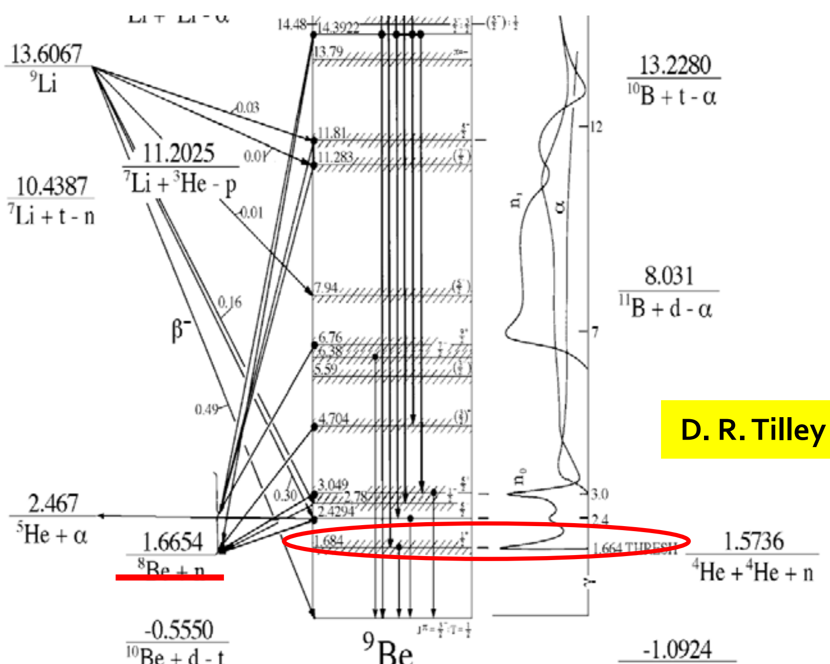
Y. Kikuchi et al., Phys. Rev. C**87** (2013), 034606

2. Photodisintegration of ^{9}Be

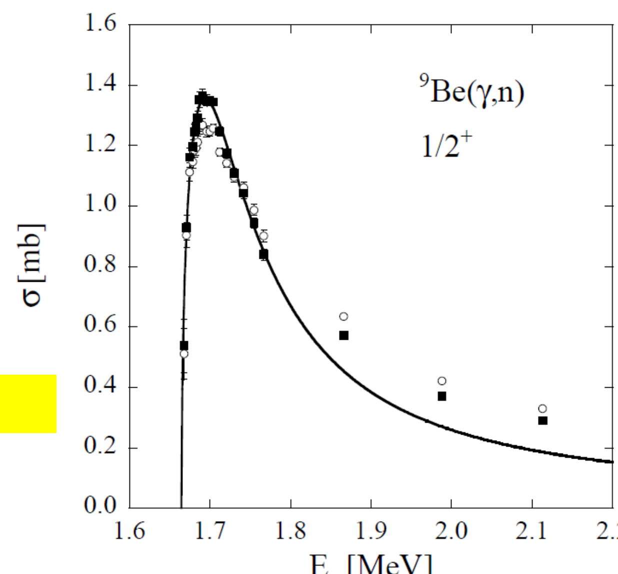
Photodisintegration cross section of $^{9}Be(\gamma, n)^{8}Be$.



C.W. Arnold et al. PRC85, 044605 (2012)



D. R. Tilley et al. NPA 745, 155 (2004)

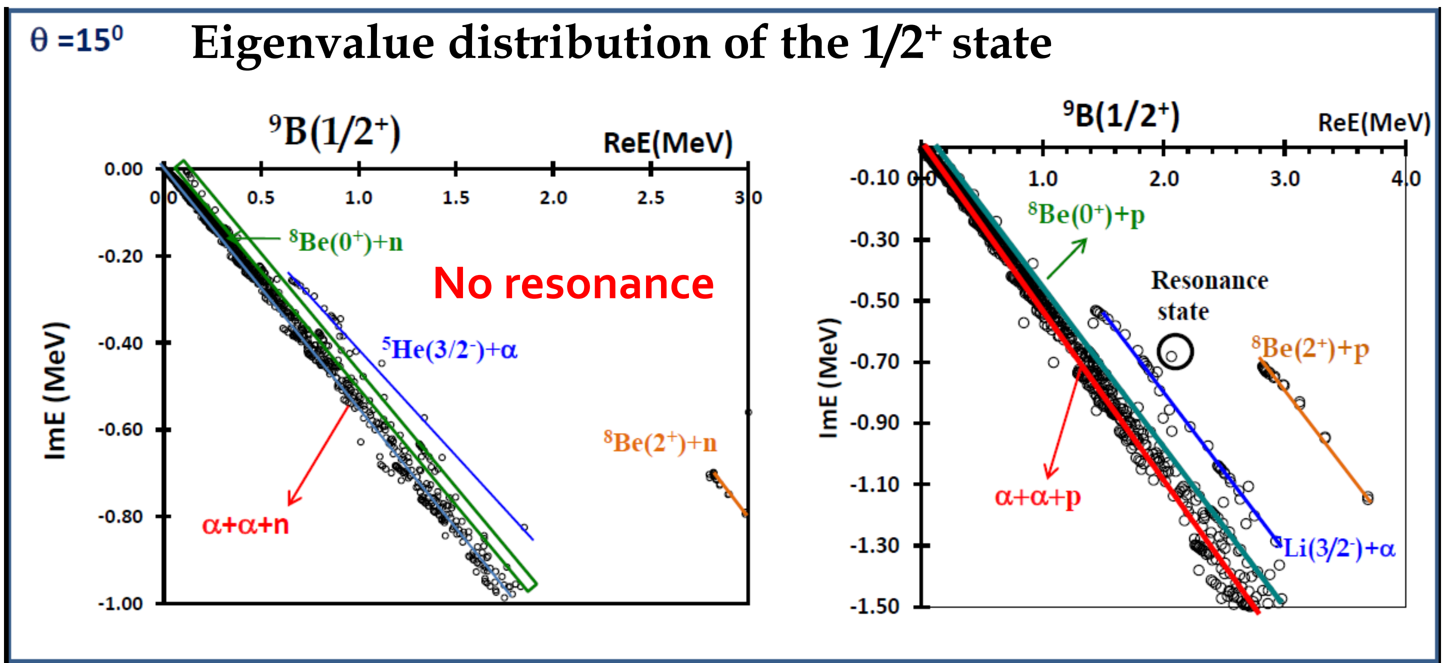


H. Utsunomiya et al. PRC92, 064323 (2015)

- From this observed cross section peak, we cannot immediately conclude the existence of resonances (ex. Coulomb breakup cross sections in neutron halo nuclei)
- We investigated whether or not this peak is caused by the ½$^+$ resonant state in ^{9}Be applying the complex scaling method to an α+α+n model.

α+α+N calculations

M. Odsuren et al. PRC92, 014322 (2015)



Complex scaling Method

J.Aguilar and J. M. Combes; Commun. Math. Phys. 22 (1971), 269.
E. Balslev and J.M. Combes; Commun. Math. Phys. 22(1971), 280

0. Coordinates

$$U(\theta): \quad r \to re^{i\theta}, \qquad k \to ke^{-i\theta}$$

1. Wave function

$$U(\theta)\psi(r) = e^{3\theta i/2}\psi(re^{i\theta})$$

→ The resonance wave function has a non-singular asymptotic behavior.

2. Hamiltonian (Operator)

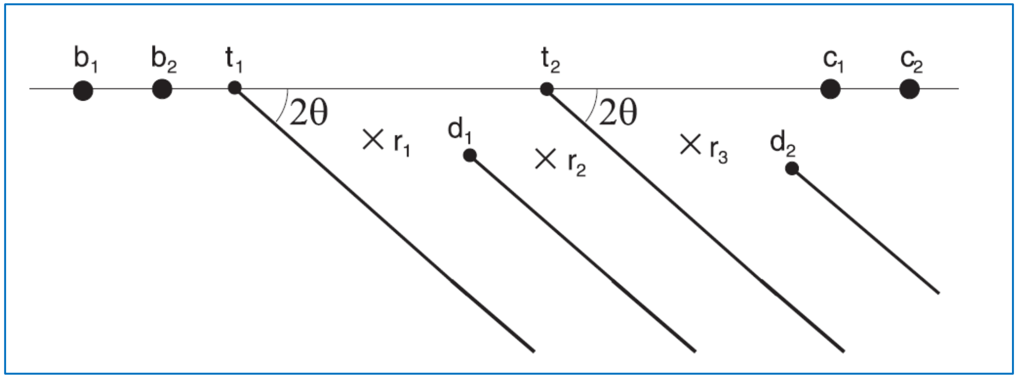
$$\hat{H}(\theta) = U(\theta)\hat{H}U^{-1}(\theta), \qquad \left(\hat{O}(\theta) = U(\theta)\hat{O}U^{-1}(\theta)\right)$$

3. Eigenvalue Problem of H(θ)

$$H(\theta)\Psi_\theta = E_\theta\Psi_\theta$$

Very useful in investigation of many-body resonances and continuum states

Eigenvalue distribution of a many-body system



S. Aoyama, T. Myo, K. Kato, and K. Ikeda, Prog. Theor. Phys.116, (2006) 1.
T. Myo, Y. Kikuchi, H. Masui, and K. Kato, Prog. in Part. and Nucl. Phys. 79 (2014) 1.

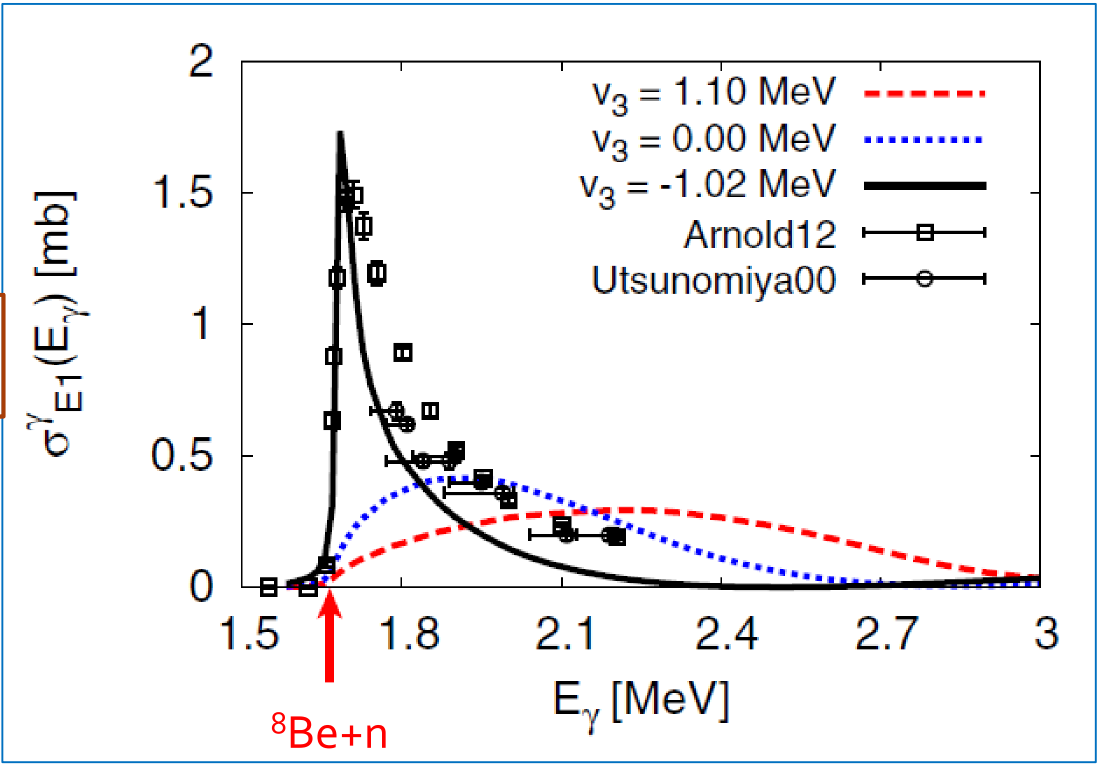
α+α+n three-body model

Hamiltonian

$$\hat{H} = \sum_{i=1}^{3} \hat{t}_i - \hat{T}_{\rm cm} + \sum_{i=1}^{2} \hat{V}_{\alpha\text{-}n}(\boldsymbol{\xi}_i) + \hat{V}_{\alpha\text{-}\alpha} + \hat{V}_3 + V_F$$

Inter-3 cluster potential

$$V_3 = v_3 \exp(-\mu\rho^2), \quad \rho^2 = 2r^2 + \frac{8}{9}R^2$$



M. Odsuren et al. PRC92, 014322 (2015)

The peak observed above the threshold is understood to be originated from a virtual state of n+^{8}Be.

How to confirm it.

In the CSM, the virtual state cannot be solved as an isolate eigenstate.

Virtual state in Complex Scaling Method

M. Odsuren et al., PRC95, 064305 (2017)

To simulate relative motion of the ^{8}Be+n system

A schematic two-body model

Hamiltonian

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r),$$

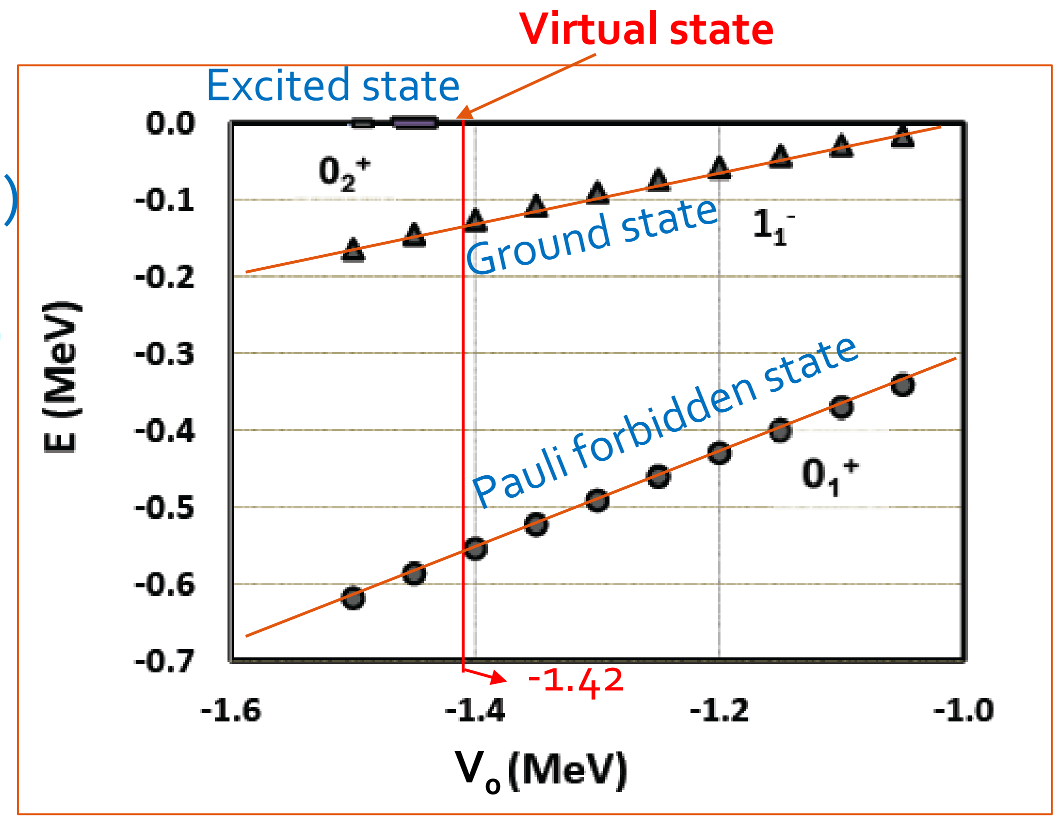
$V(r)=V_o\exp(-ar^2)$;   a =0.16 fm^{-2}

Complex scaling method (CSM)

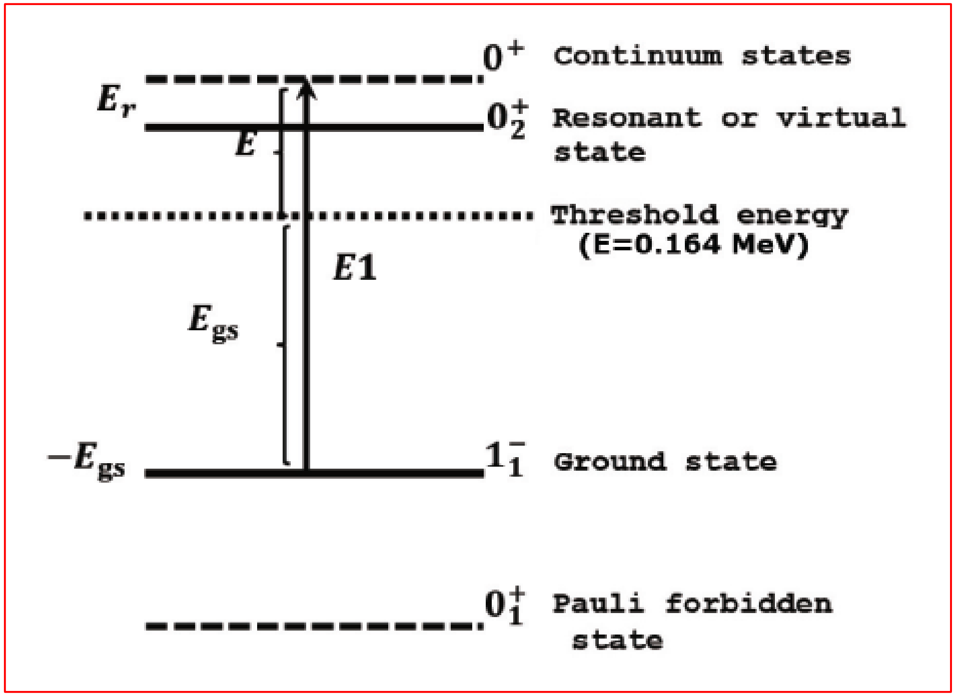
$$\vec{r} \to \vec{r}e^{i\theta}, \qquad \vec{k} \to \vec{k}e^{-i\theta},$$

$$H^\theta \Psi^\nu_{J^\pi}(\theta) = E^\theta_\nu \Psi^\nu_{J^\pi}(\theta).$$

$$\Psi^\nu_{J^\pi}(\theta) = \sum_{n=1}^{N} c_n^{J^\pi\nu}(\theta)\phi_n(r),$$



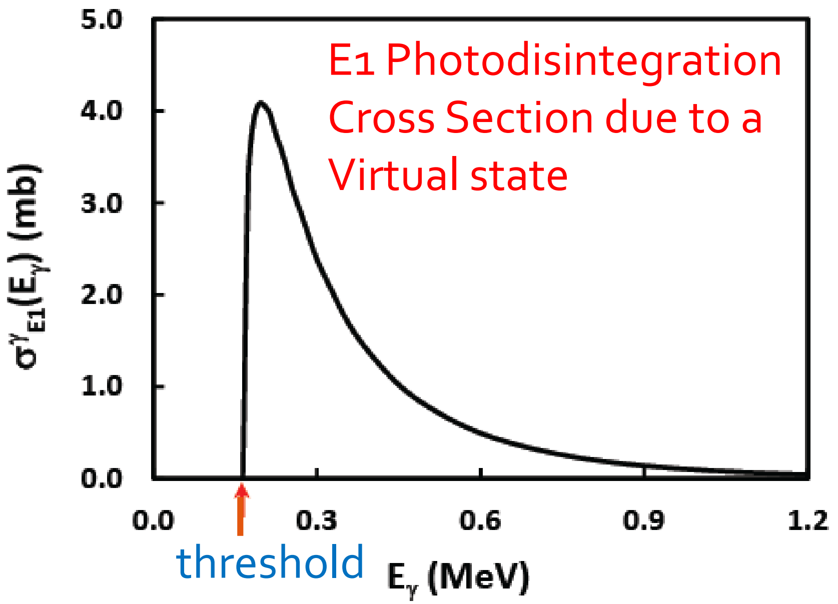
Energy level of the schematic two-body model



E1 transition from 1^-_1 to 0^+_2 states

$$\sigma^{\gamma}_{E1}(E_\gamma) = \frac{16\pi^3}{9} \cdot \left(\frac{E_\gamma}{\hbar c}\right) \cdot \frac{dB(E1, E_\gamma)}{dE_\gamma},$$

$$\frac{dB(E1, E_\gamma)}{dE_\gamma} = -\frac{1}{\pi} \cdot \frac{1}{2J_{gs}+1} \times \mathrm{Im}\left[\sum_\nu \left\langle \tilde{\Psi}^{gs}_{J^\pi} \| (\hat{O}^\theta)^\dagger (E1) \| \Psi^\nu_{J^\pi}(\theta) \right\rangle \times \frac{1}{E - E^\theta_\nu} \left\langle \tilde{\Psi}^\nu_{J^\pi}(\theta) \| \hat{O}^\theta (E1) \| \Psi^{gs}_{J^\pi} \right\rangle \right],$$



Virtual state in Complex Scaling Method

Scattering length

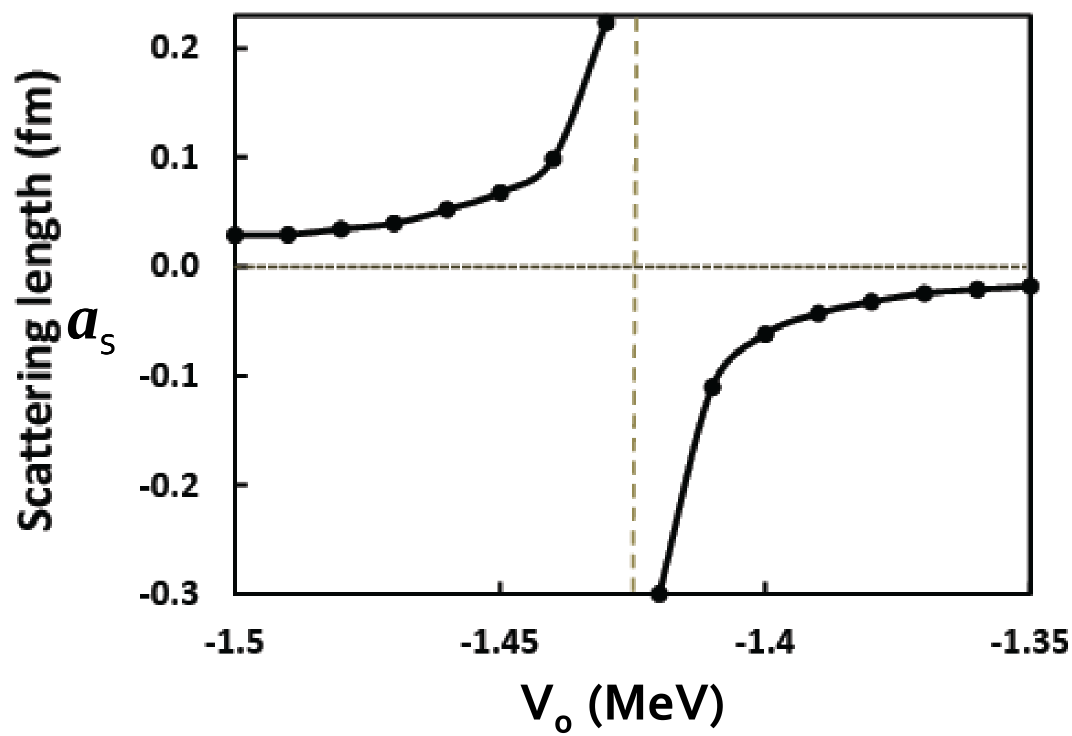
$$a_s = -\lim_{k\to 0} \tan \delta_\theta^N(E)/k,$$

Phase shift

$$\Delta(E) = \frac{1}{\pi}\frac{d\delta(E)}{dE}.$$

Continue level density

$$\Delta(E) = \rho(E) - \rho_o(E)$$

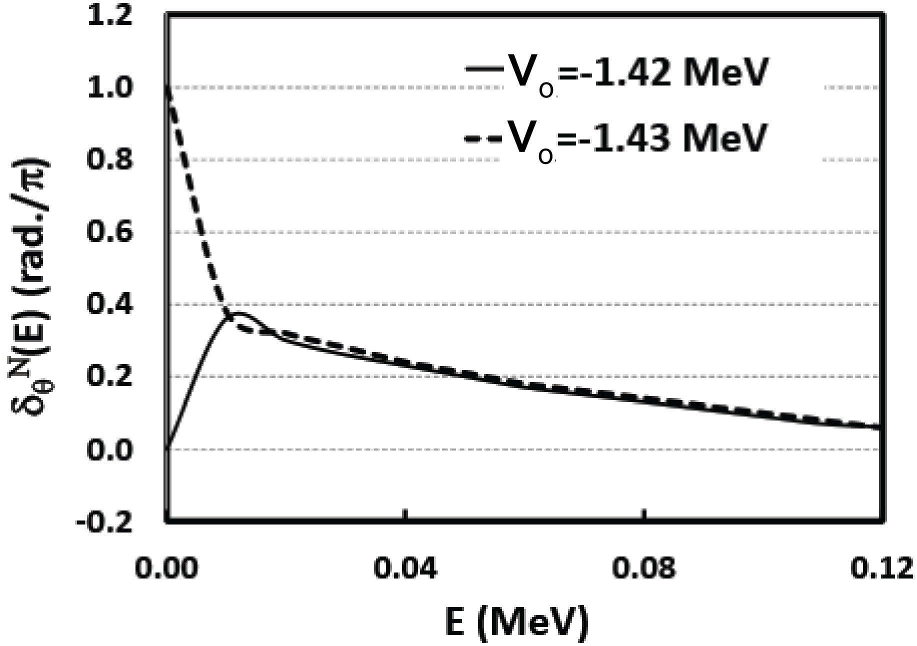


Level density

$$\rho(E) = \int \delta(E-E')dE' = -\frac{1}{\pi}\mathrm{Im}\,Tr\left[\frac{1}{E-H}\right] \qquad \rho_0(E) = -\frac{1}{\pi}\mathrm{Im}\,Tr\left[\frac{1}{E-H_0}\right]$$

$V_o \leq -1.43\ MeV,\quad a_s > 0$ → Bound state

$V_o \geq -1.42\ MeV,\quad a_s < 0$ → Virtual state



V_o=-1.43 MeV

$\delta(E=0)=\pi$ (Levinson theorem)

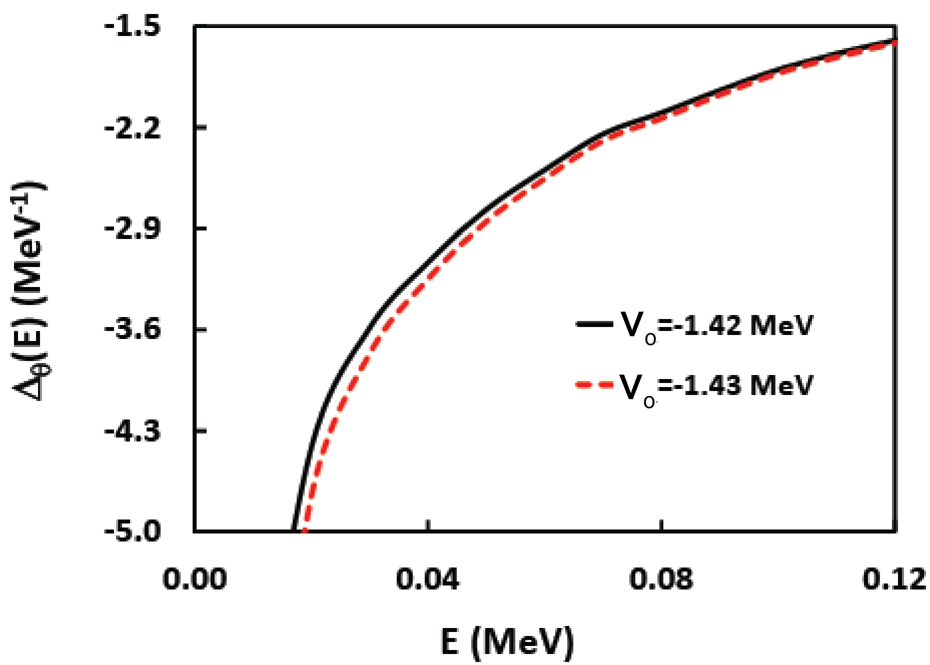
V_o=-1.42 MeV

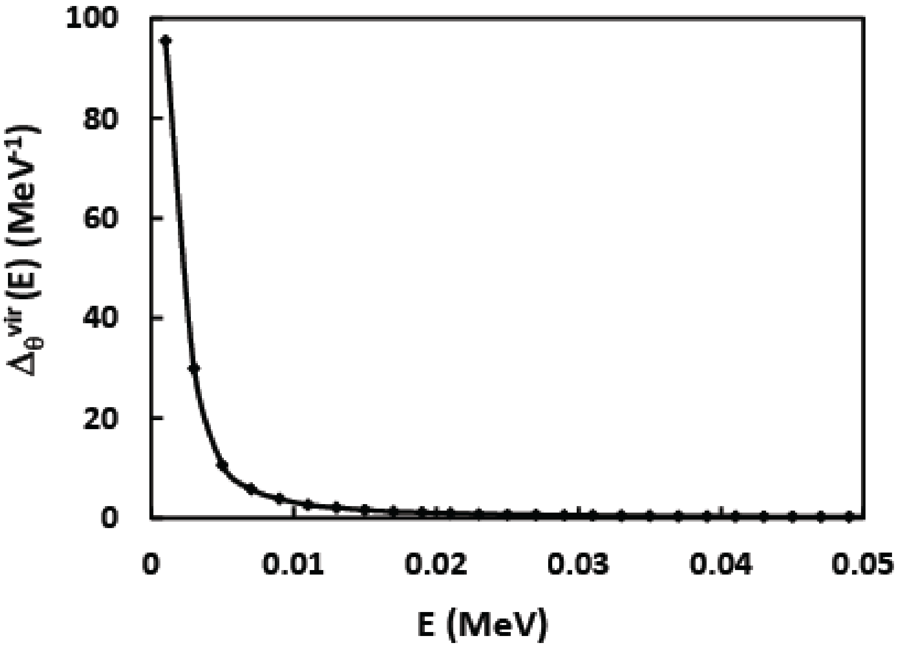
Peak value of phase shift is π/3 approximately

Extraction of the virtual state contribution from difference between CLD's for v_o=-1.43 MeV and -1.42 MeV.

$$\Delta(E:-1.43)=\delta(E-E_{BS})+\Delta_C(E:-1.43)$$

$$\Delta(E:-1.42)=\Delta_{VS}(E)+\Delta_C(E:-142)$$





If $\Delta_C(E{:}\text{-}143) \approx \Delta_C(E{:}-142)$

CLD of the virtual state

$$\Delta_{VS}(E) = \Delta(E{:}\text{-}142) - \Delta(E{:}\text{-}143)$$

$$\propto \frac{1}{E - E_v}.$$

We can evaluate E_v from Δ_{VS}

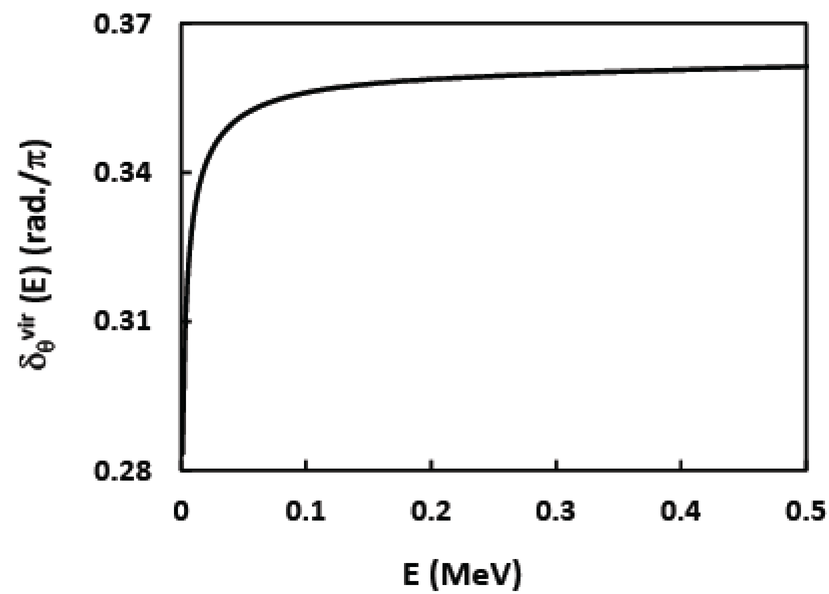
$$E_v \approx -0.001 \text{ MeV}$$

Ref. $E_v = -4.97 \times 10^{-6}$ MeV

Phase shift of the virtual state

$$\delta^{virt}(E) = \pi \int_0^E \Delta^{virt}(E')dE'.$$

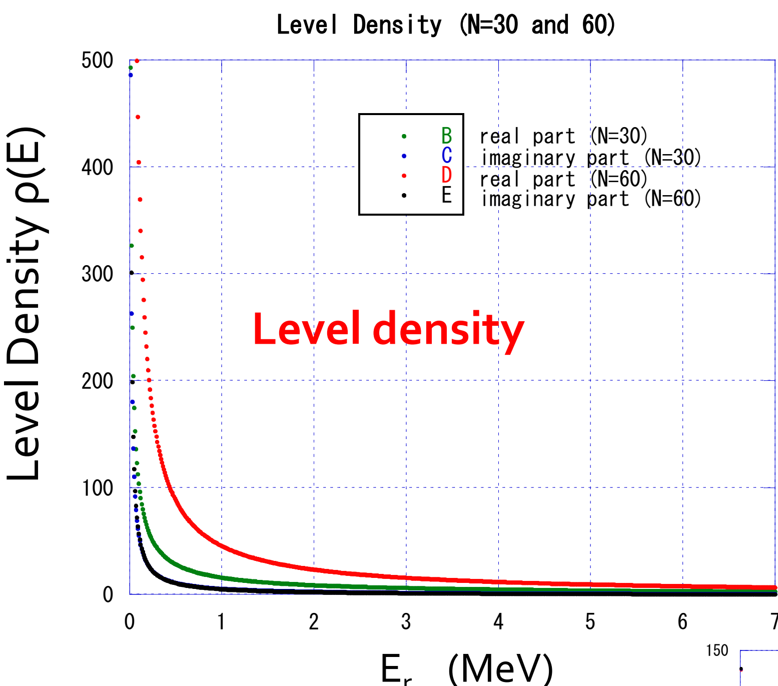
It does not reach π, and its maximum is smaller than π/2.

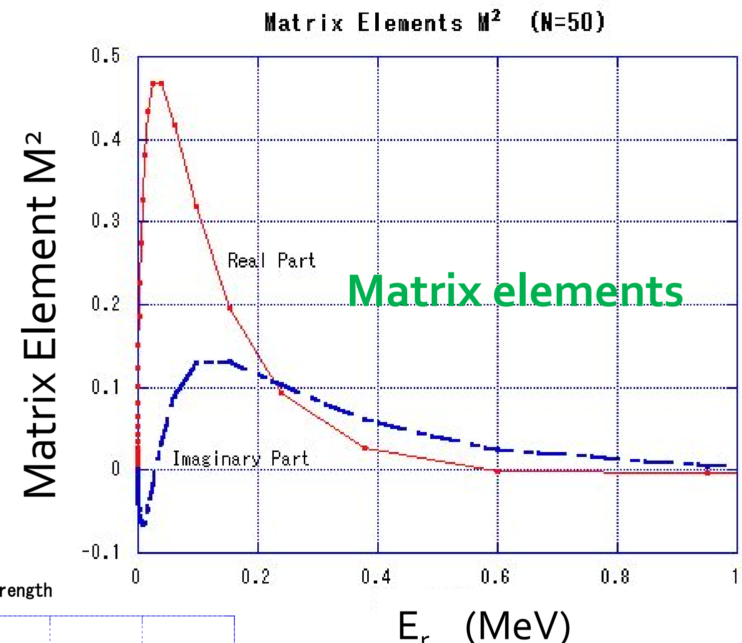


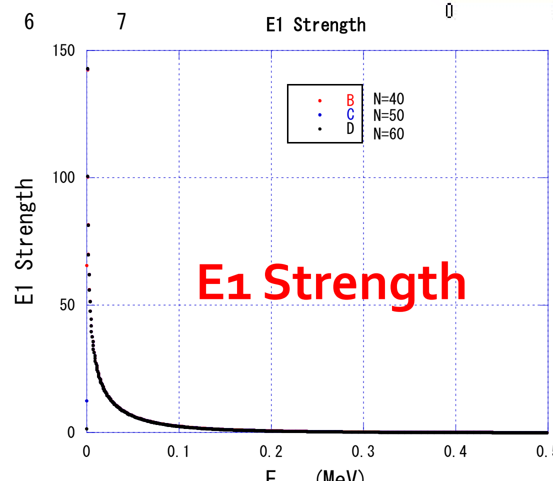
Origin of the peak at energies above threshold

$$\frac{dB(E1,E)}{dE} = -\frac{1}{\pi}\frac{1}{2J_{gr}+1}\, Im \sum_{\upsilon} \left\langle \tilde{\Psi}^{gr\theta}_{J_{gr}} \| (\hat{O}^{\theta}(E1))^{\dagger} \| \Psi^{\upsilon\theta}_{J_{\upsilon}} \right\rangle \frac{1}{E-E^{\theta}_{\upsilon}} \left\langle \tilde{\Psi}^{\upsilon\theta}_{J_{\upsilon}} \| \hat{O}^{\theta}(E1) \| \Psi^{gr\theta}_{J_{gr}} \right\rangle$$

Matrix elements Level density







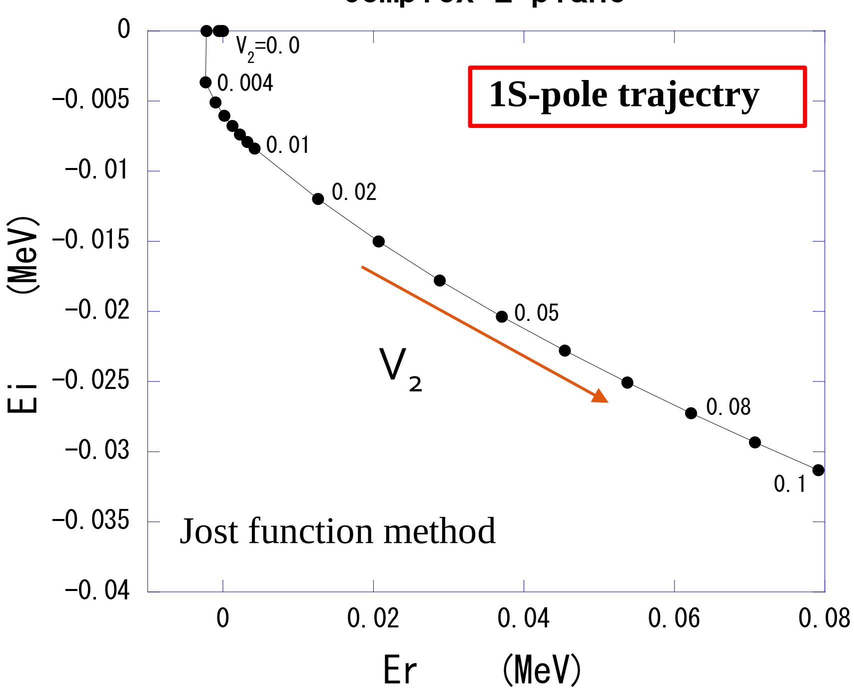
Shape of the peak of virtual and resonant states

Virtual state changes to Resonant state
by switching on a barrier potential at a large distance

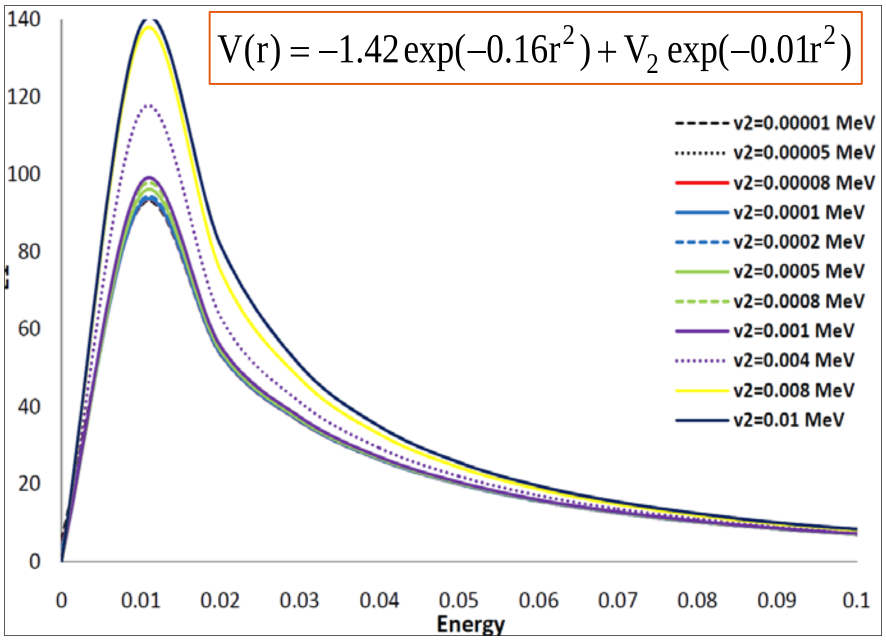
$$V(r)=V_0\exp(-ar^2)+V_2\exp(-br^2)$$

$a = 0.16\ \mathrm{fm}^{-2} \qquad b = 0.01\ \mathrm{fm}^{-2}$

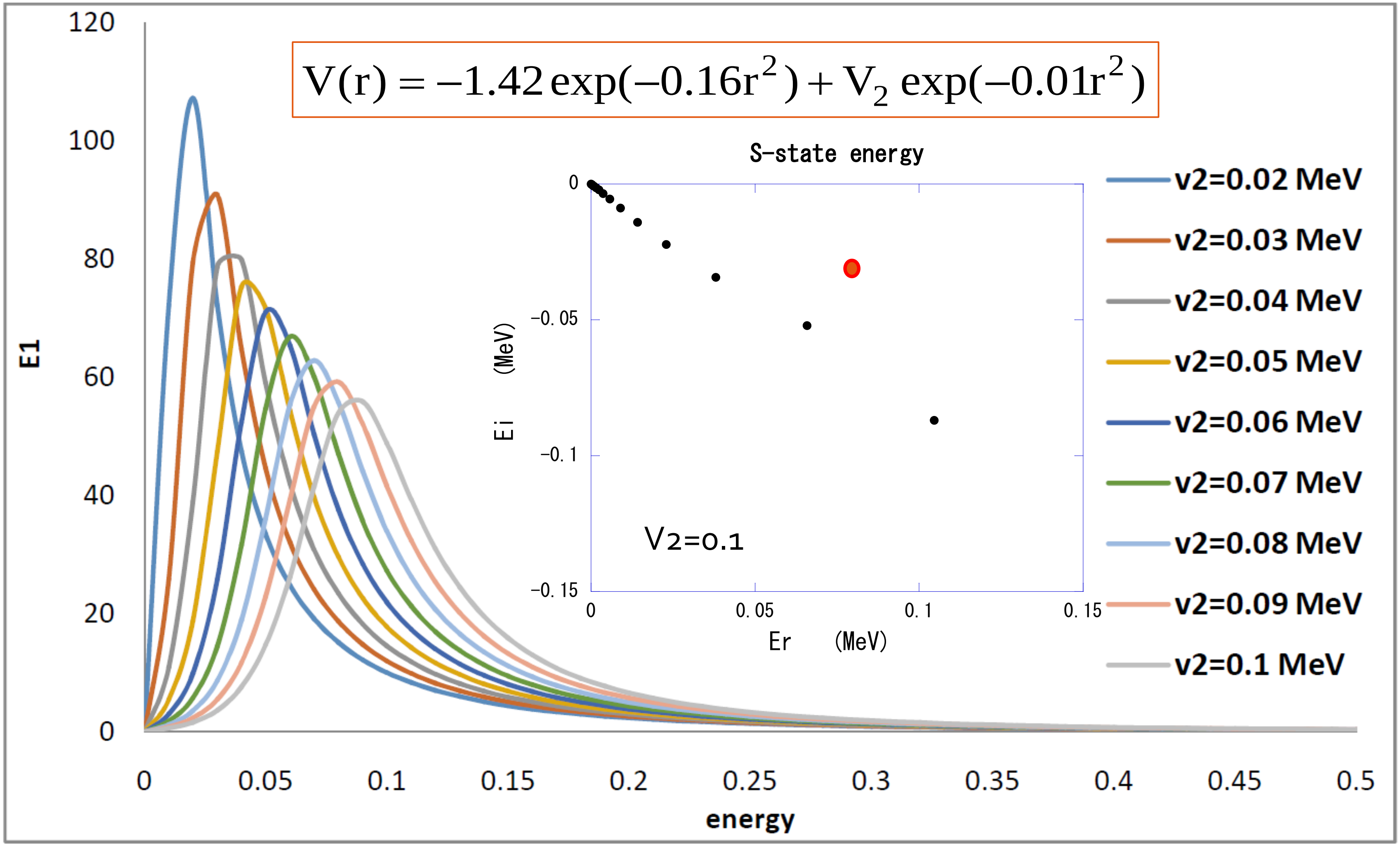
Complex E-plane



E1 strength in CSM



The strength may be changed slightly, but the shapes of peaks in the E1 photo-disintegration are very similar for virtual and resonant states.



As the resonance position is far from the threshold energy, the shape of the cross section peak becomes different from that of the virtual state case.

4. Summary

- An enhancement of nuclear reaction cross sections just above the threshold does not necessarily mean the existence of a resonance.
- It has been a long standing problem, whether the cross section peak of the photodisintegration observed just above the ^{8}Be+n threshold in ^{9}Be causes from the ½$^{+}$ resonant state or a neuron *S*-wave virtual.
- The complex scaled α+α+n model shows no resonant states of $1/2^{+}$, but reproduces the observed peak of the E1 strength due to a neuron *S*-wave virtual state of ^{8}Be(0^{+})+n.
- Using a schematic two-body model simulating the ^{8}Be+n structure in ^{9}Be, the *E1* photodisintegration cross sections have been investigated for the cases of virtual and resonant states.
- It is shown that the virtual state giving a sharp peak in the photodisintegration cross section can be extracted from the continuum level density in CSM.
- The peak shape due to the virtual state is very similar to that of the resonance state. Therefore, from the cross section shape, it is difficult to distinguish whether the origin of the peak structure is virtual or resonant state.

Collaborators

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Thank you ！