Clustering in A=11-~~13~~ Nuclei

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Cluster States in the Neighborhood of ^{12}C

- The Hoyle state in ^{12}C has been one of the central topics of previous SOTANCP conferences and the cluster community in general
- alpha-cluster models for ^{12}C are successful in describing many properties of the ground state band and the cluster states including the Hoyle state

Is ^{12}C and the Hoyle state special ?

- The lowest threshold in ^{12}C is the three-alpha threshold, and alphas are “good" clusters, ^{8}Be is essentially a two-alpha system
- The decay of cluster states however is happening mainly through the ^{8}Be+alpha channel, ^{8}Be+alpha configurations can be used to describe cluster states in ^{12}C

What about ^{11}C and ^{13}N ?

- alpha-cluster models obviously do not work, we will combine FMD with explicit cluster configurations
- In ^{11}C the $^{7}Be+^{4}He$ is the first open channel, $^{8}Be+^{3}He$ is not far away
- Can we understand the low-lying positive parity states?
- in ^{13}N it gets even more complicated due to the low-lying proton threshold, alpha-clustered states might be expected close to the $^{9}B+^{4}He$ threshold

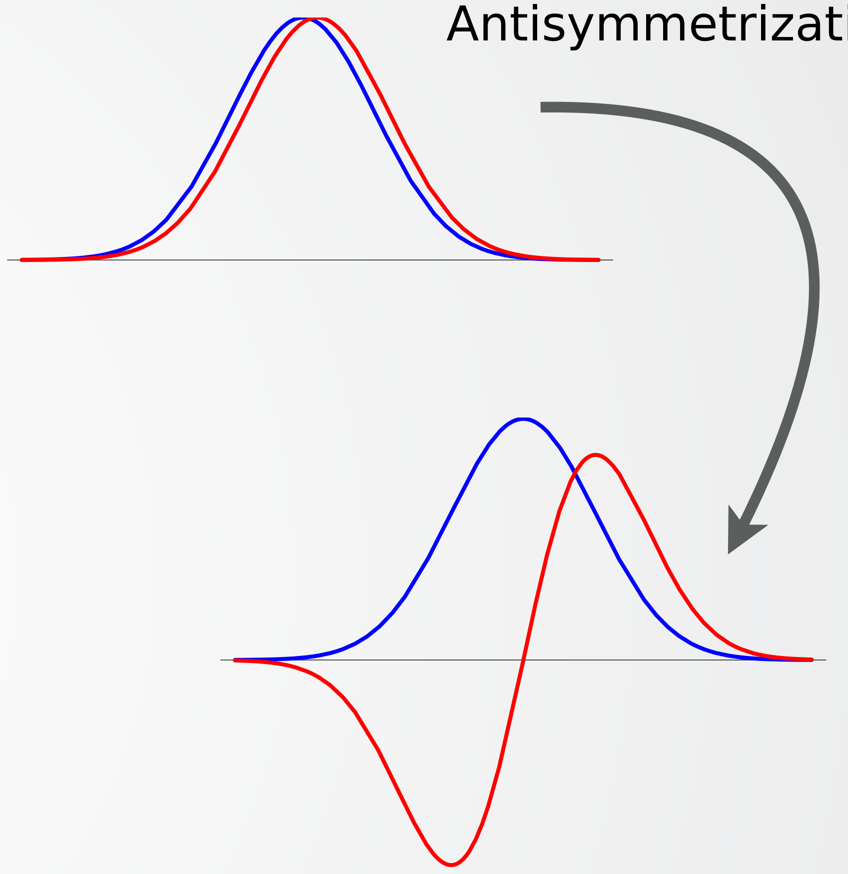
Fermionic Molecular Dynamics

Fermionic

Intrinsic many-body states

$$\left|Q\right\rangle = \hat{\mathcal{A}}\left\{\left|q_1\right\rangle \otimes \cdots \otimes \left|q_A\right\rangle\right\}$$

are antisymmetrized A-body states



Molecular

Single-particle states

$$\left\langle \mathbf{x} \middle| q \right\rangle = \sum_i c_i \exp\left\{-\frac{(\mathbf{x}-\mathbf{b}_i)^2}{2a_i}\right\} \otimes \left|\chi_i^\uparrow, \chi_i^\downarrow\right\rangle \otimes \left|\xi\right\rangle$$

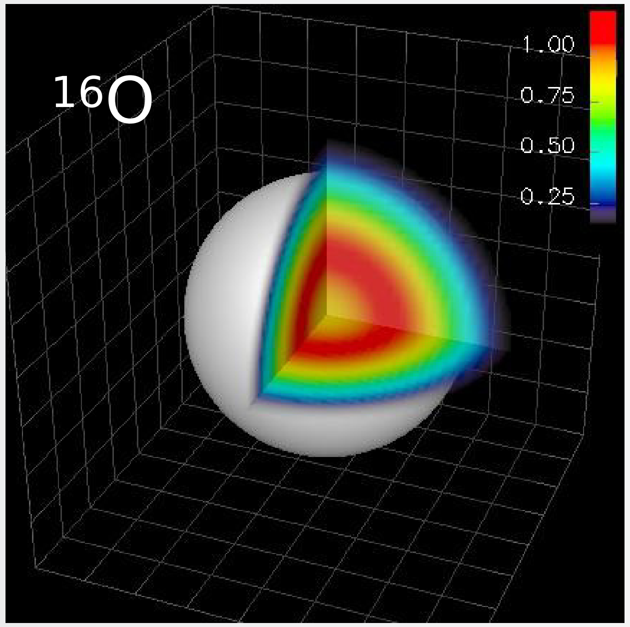
- Gaussian wave-packets in phase-space (complex parameter $\mathbf{b}_i$ encodes mean position and mean momentum), spin is free, isospin is fixed
- width a_i is an independent variational parameter for each wave packet
- use one or two wave packets for each single particle state

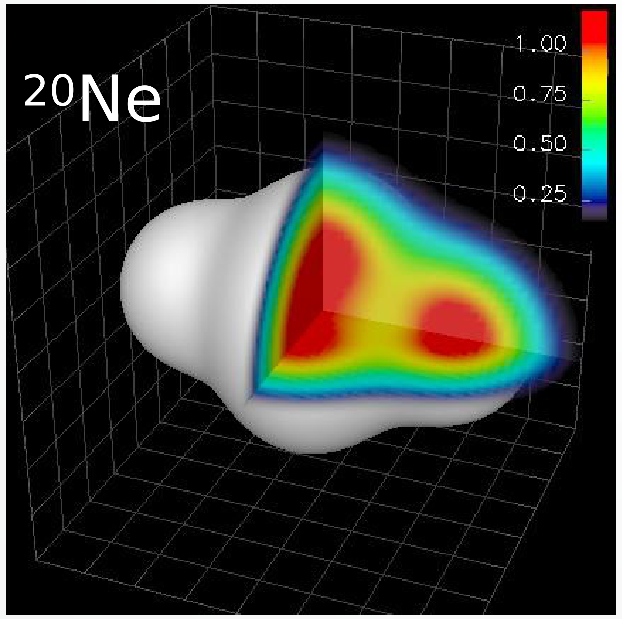
FMD basis contains **harmonic oscillator shell model** and **Brink-type cluster** configurations as limiting cases

Projection after Variation

Variation and Projection

- minimize the energy of the intrinsic state
- intrinsic state may break symmetries of Hamiltonian
- restore inversion, translational and rotational symmetry by **projection on parity**, **angular (and linear) momentum**





Generator coordinates

- use generator coordinates (radii, quadrupole or octupole deformation, strength of spin-orbit force) to create additional basis states

Variation

$$\min_{\{q_\nu\}} \frac{\langle Q | \hat{H} - \hat{T}_{\mathrm{cm}} | Q \rangle}{\langle Q | Q \rangle}$$

Projection

$$\hat{P}^{\pi} = \frac{1}{2}(1 + \pi \hat{\Pi})$$

$$\hat{P}^{J}{}_{MK} = \frac{2J+1}{8\pi^2} \int d^3\Omega \, D^{J}{}_{MK}{}^{\star}(\Omega)\, \hat{R}(\Omega)$$

$$\hat{P}^{\mathbf{P}} = \frac{1}{(2\pi)^3} \int d^3X \; \exp\{-i(\hat{\mathbf{P}} - \mathbf{P}) \cdot \mathbf{X}\}$$

Variation after Projection

Variation after Projection

- Correlation energies can be quite large for well deformed and/or clustered states
- For light nuclei it is possible to perform real variation after projection
- Can be combined with generator coordinate method

Multiconfiguration Mixing

- Set of N intrinsic states optimized for different spins and parities and for different values of generator coordinates are used as basis states
- Diagonalize in set of projected basis states

Variation

$$\min_{\{q_\nu\}} \frac{\left\langle Q \middle| \hat{H} - \hat{T}_{\mathrm{cm}} \middle| Q \right\rangle}{\left\langle Q \middle| Q \right\rangle}$$

Variation after Projection

$$\min_{\{q_\nu, c^\alpha{}_K\}} \frac{\sum_{KK'} c^\alpha{}_K{}^\star \left\langle Q \middle| (\hat{H} - \hat{T}_{\mathrm{cm}}) \hat{P}^\pi \hat{P}^J{}_{KK'} \middle| Q \right\rangle c^\alpha{}_{K'}}{\sum_{KK'} c^\alpha{}_K{}^\star \left\langle Q \middle| \hat{P}^\pi \hat{P}^J{}_{KK'} \middle| Q \right\rangle c^\alpha{}_{K'}}$$

(Intrinsic) Basis States

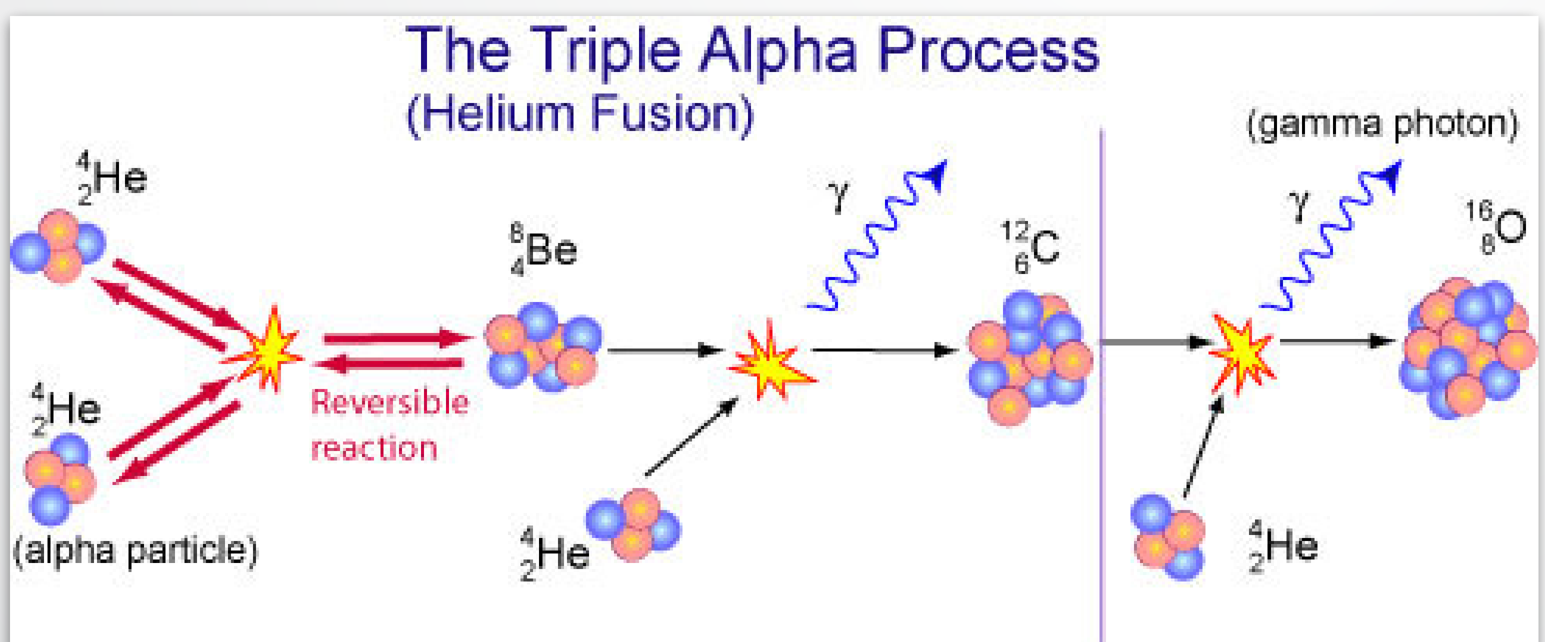
$$\left\{ \left| Q^{(a)} \right\rangle, a = 1, \ldots, N \right\}$$

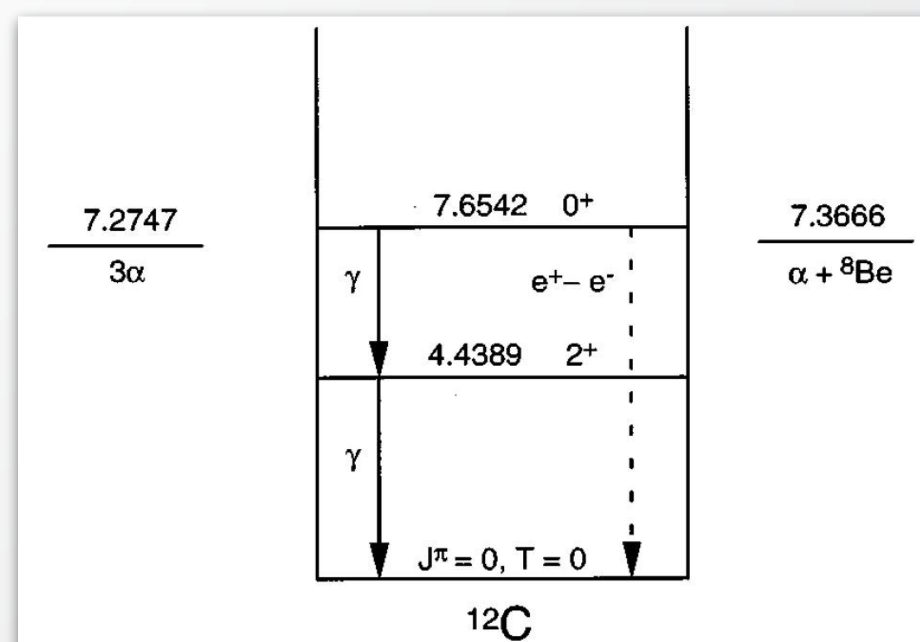
Generalized Eigenvalue Problem

$$\sum_{K'b} \underbrace{\left\langle Q^{(a)} \middle| \hat{H} \hat{P}^\pi \hat{P}^J{}_{KK'} \hat{P}^{\mathbf{P}=0} \middle| Q^{(b)} \right\rangle}_{\text{Hamiltonian kernel}} c^\alpha{}_{K'b} = E^{J^\pi \alpha} \sum_{K'b} \underbrace{\left\langle Q^{(a)} \middle| \hat{P}^\pi \hat{P}^J{}_{KK'} \hat{P}^{\mathbf{P}=0} \middle| Q^{(b)} \right\rangle}_{\text{norm kernel}} c^\alpha{}_{K'b}$$

Cluster States in ^{12}C

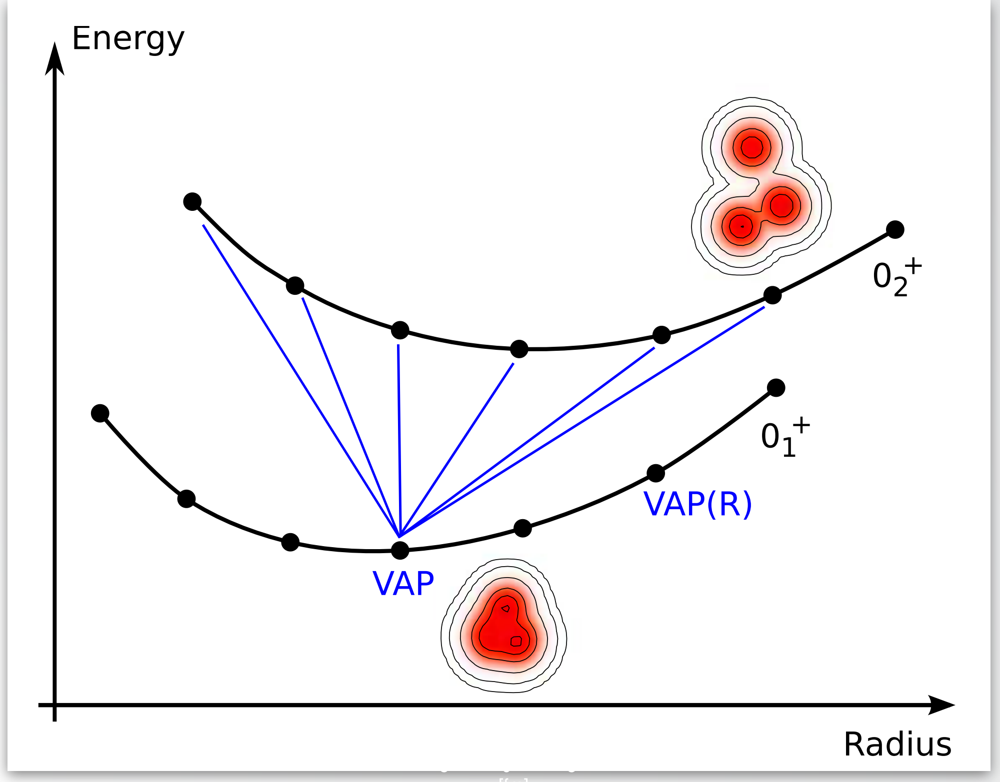
FMD and Cluster Model Calculations

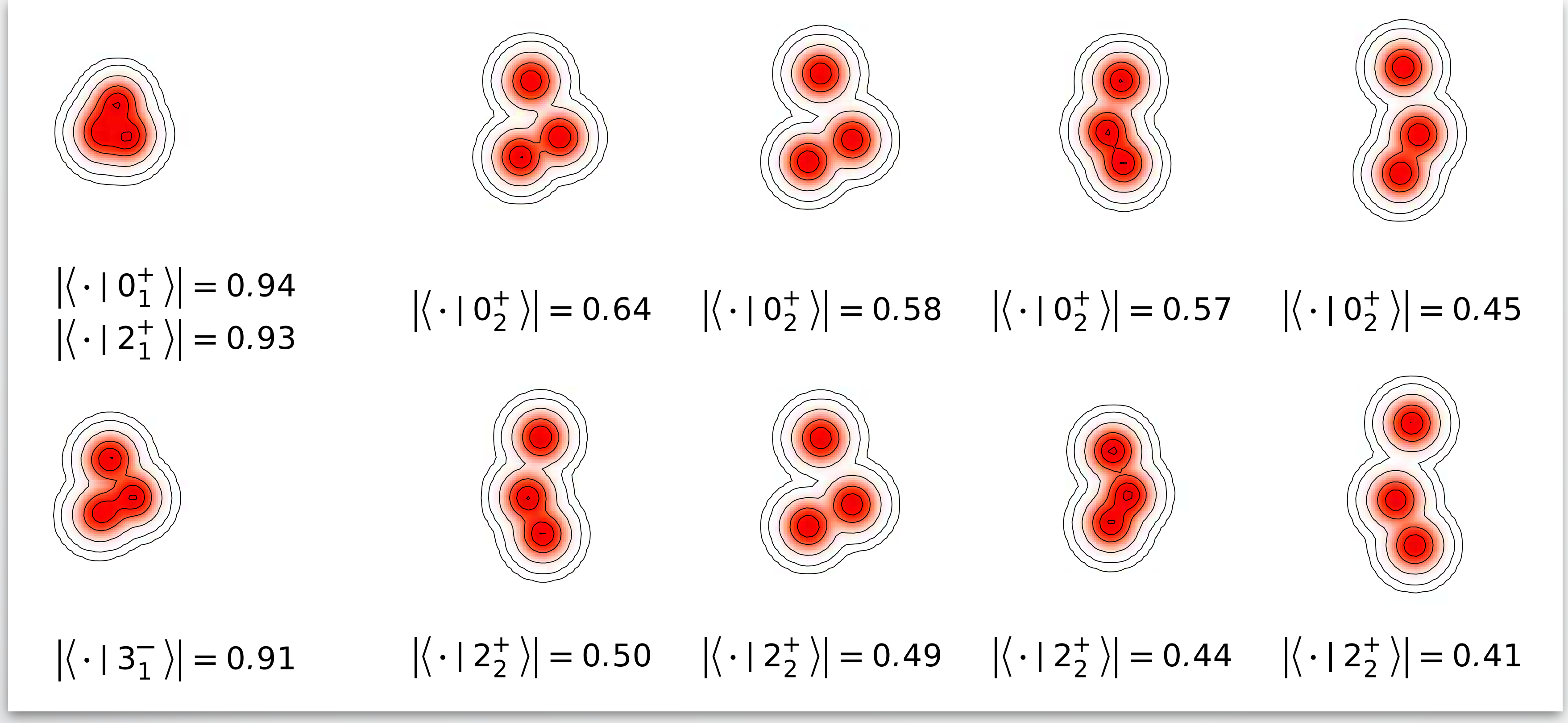




^{12}C: FMD + ^{8}Be-^{4}He Cluster Configurations

- **AV18 UCOM(SRG)** (α=0.20 fm^4) interaction — Increase strength of spin-orbit force by a factor of two to partially account for omitted three-body forces
- Internal region: FMD basis states obtained by **VAP** with radius as generator coordinate for **first 0^+, 1^+, 2^+, …**, perform VAP for **second 0^+, 1^+, 2^+, …** with radius as generator coordinate
- External region: **$^{8}Be(0^+,2^+,4^+)$-α configurations**, polarization effects in ^{8}Be are important

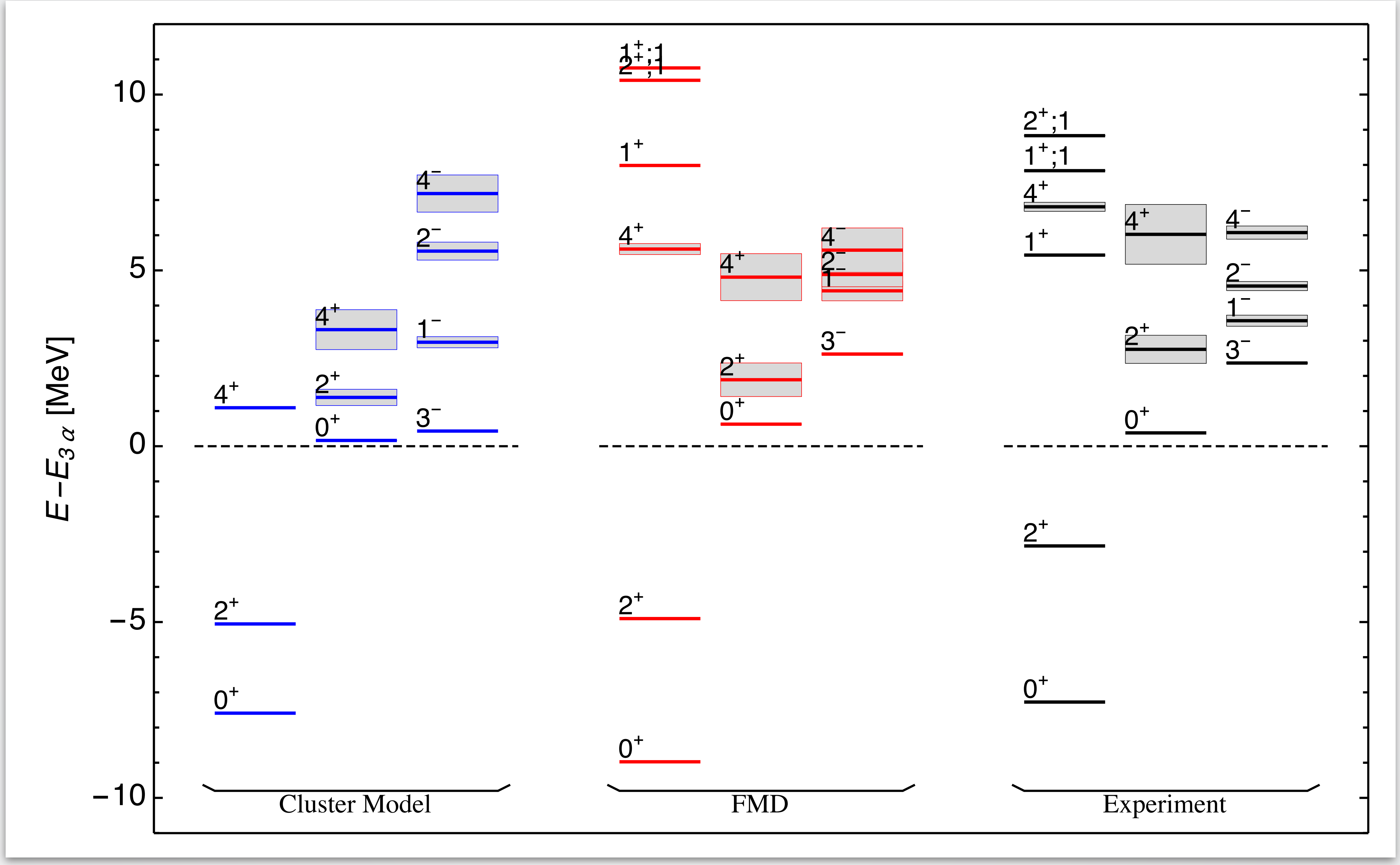




Basis states are not orthogonal !

0^+_2 and 2^+_2 states have no rigid intrinsic structure

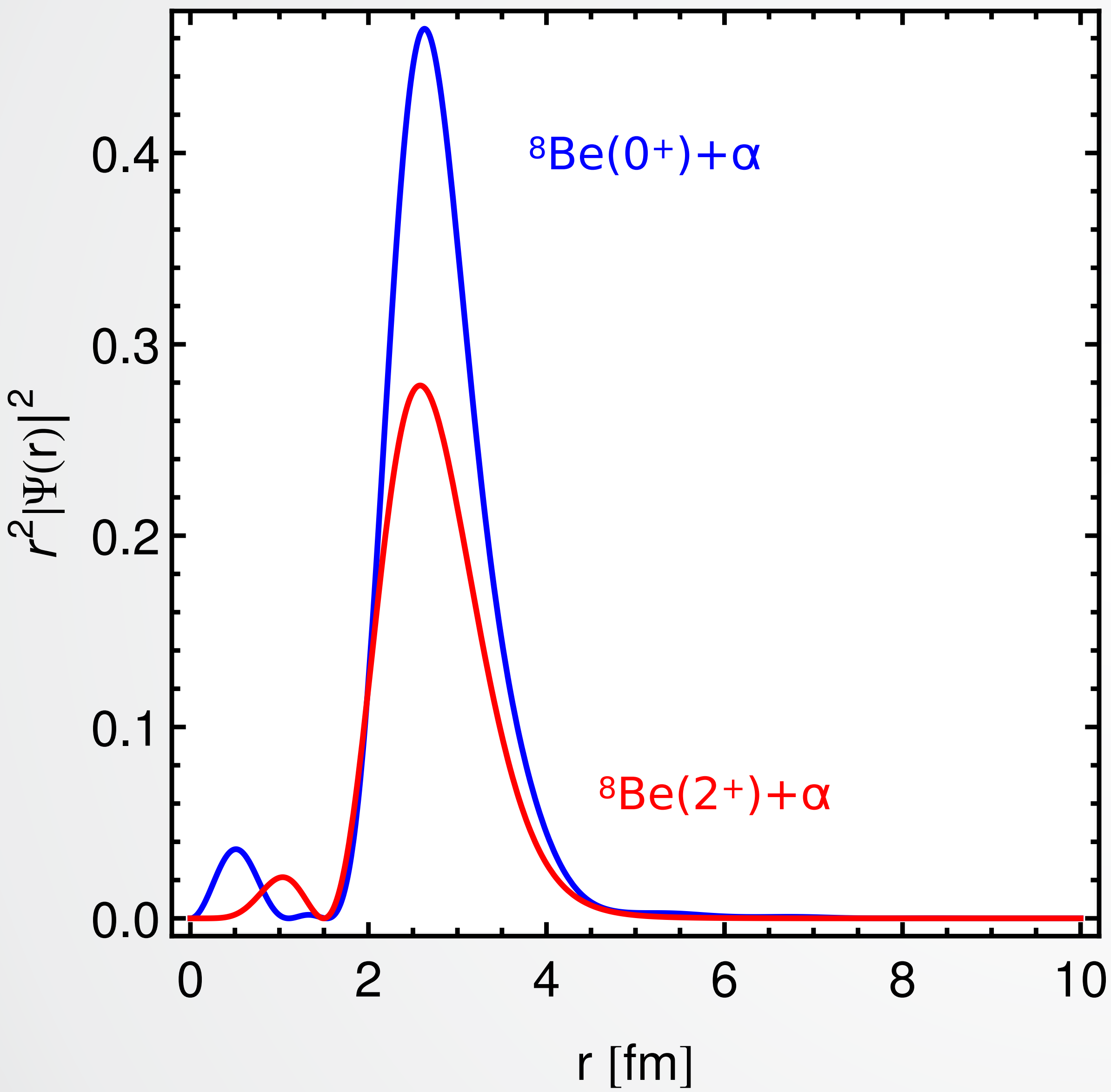
^{12}C: Spectrum including Continuum



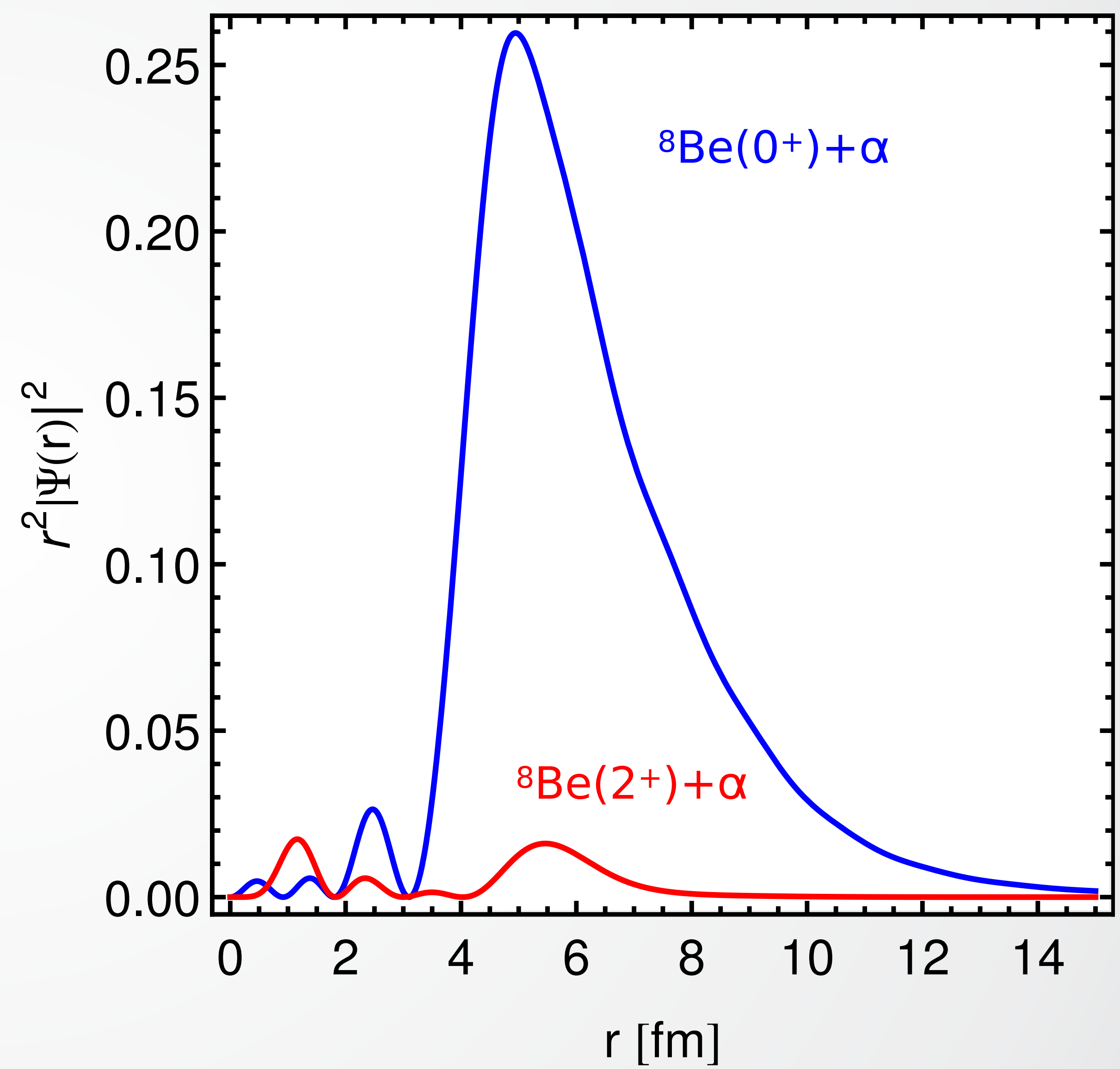
- FMD provides a consistent description of *p*-shell states, negative parity states and cluster states

^{12}C: ^{8}Be-α Spectroscopic Amplitudes

Ground State



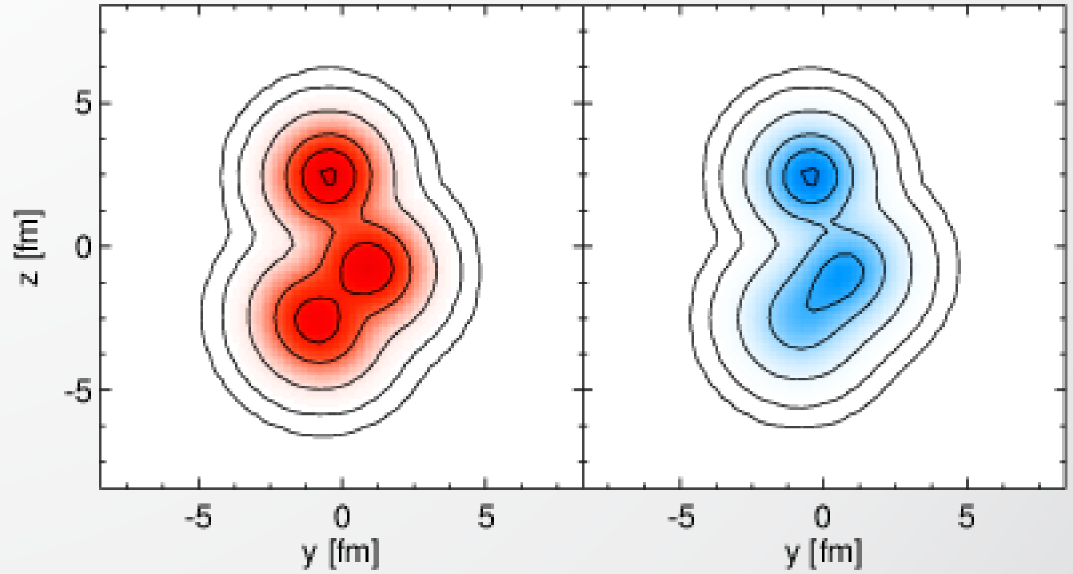
Hoyle State



- Ground state overlap with $^{8}Be(0^+)+\alpha$ and $^{8}Be(2^+)+\alpha$ configurations of similar magnitude
- Hoyle state overlap dominated by $^{8}Be(0^+)+\alpha$ configurations, large spatial extension

Cluster States in ^{11}C

FMD + Cluster Configurations



^{11}C: Outline of Calculation

I) FMD Calculation using VAP basis states

- Perform VAP calculations for the first couple of eigenstates for each spin and parity
- Can we observe the appearance of cluster structures?
- This provides only a relatively small set of basis states especially for loosely bound and spatially extended states

II) Cluster model calculations with ^{7}Be-^{4}He and ^{8}Be-^{3}He configs

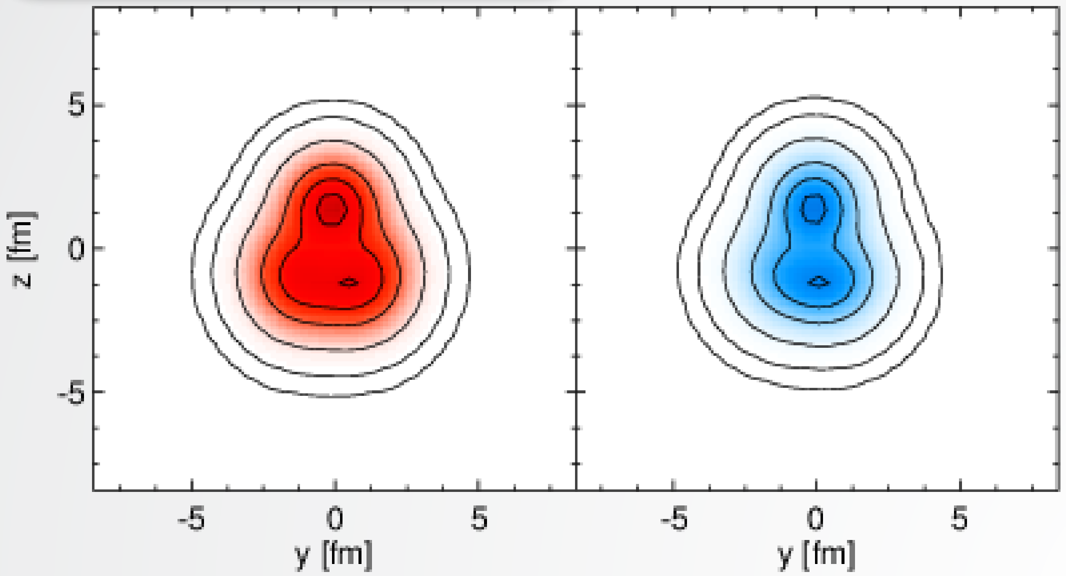
- $^{7}Be(3/2-,1/2-)$ clusters described using a superposition of $^{7}Be(3/2^{-})$ VAP state and an extended ^{4}He-^{3}He config
- $^{8}Be(0+,2+)$ clusters described using a superposition of $^{8}Be(0^{+})$ VAP state and an extended ^{4}He-^{4}He config
- Double-projection of ^{7}Be-^{4}He and ^{8}Be-^{3}He configs at distances of D=1.5, …, 9.0 fm

III) Full calculation with combined FMD and Cluster basis states

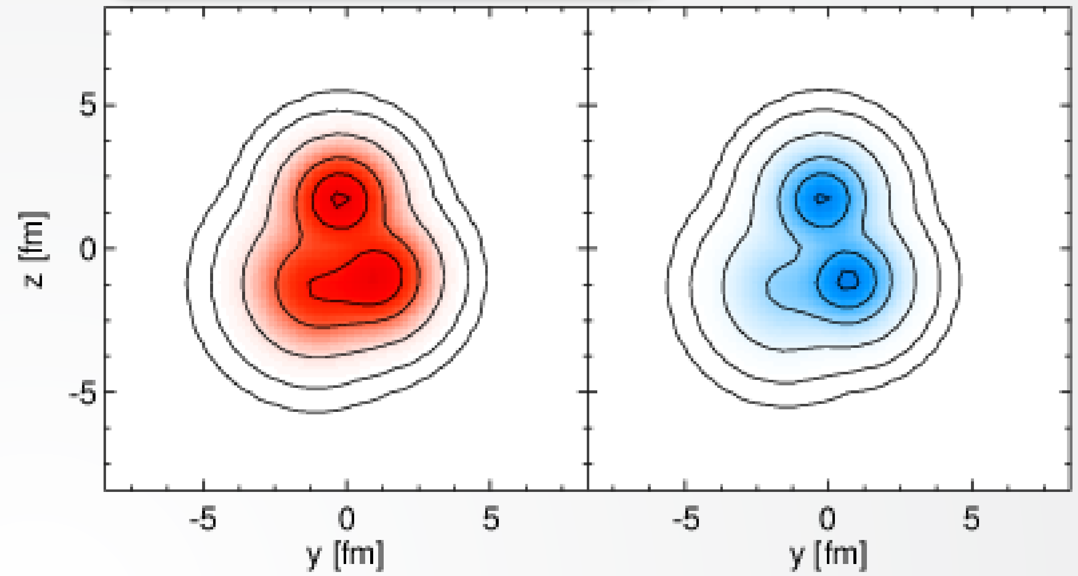
- Basis is overcomplete
- Cluster configs become orthogonal at large distances where the overlap between the clusters vanishes

^{11}C: FMD Variation after Projection

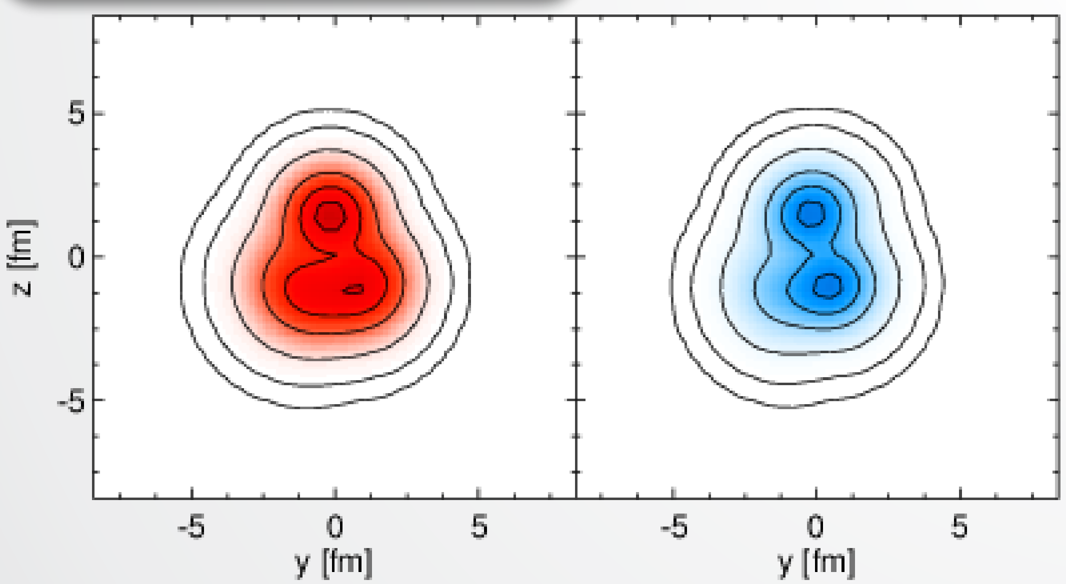
first 3/2^{-}



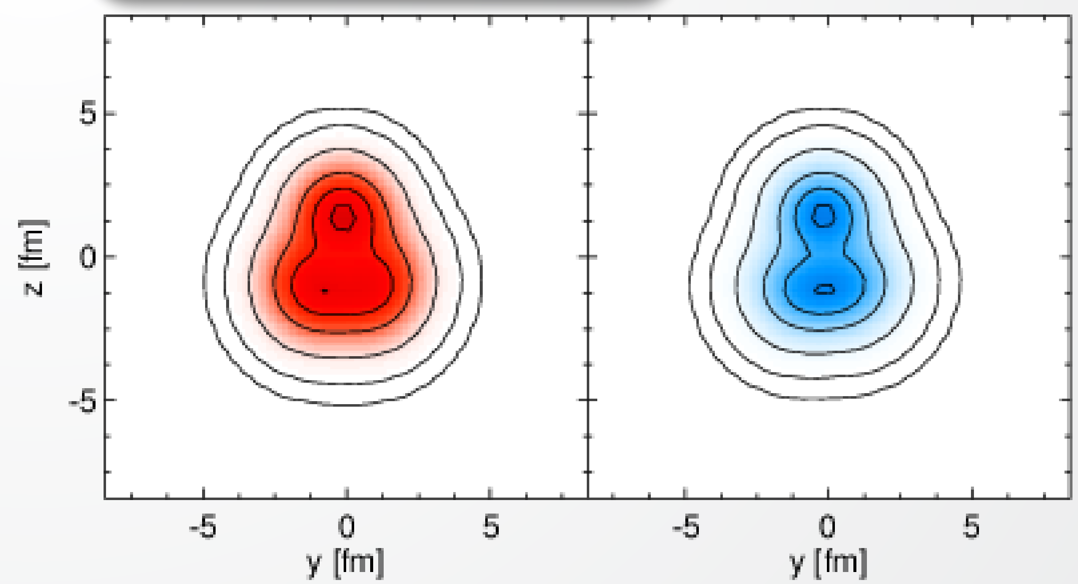
second 3/2^{-}



first 1/2^{-}



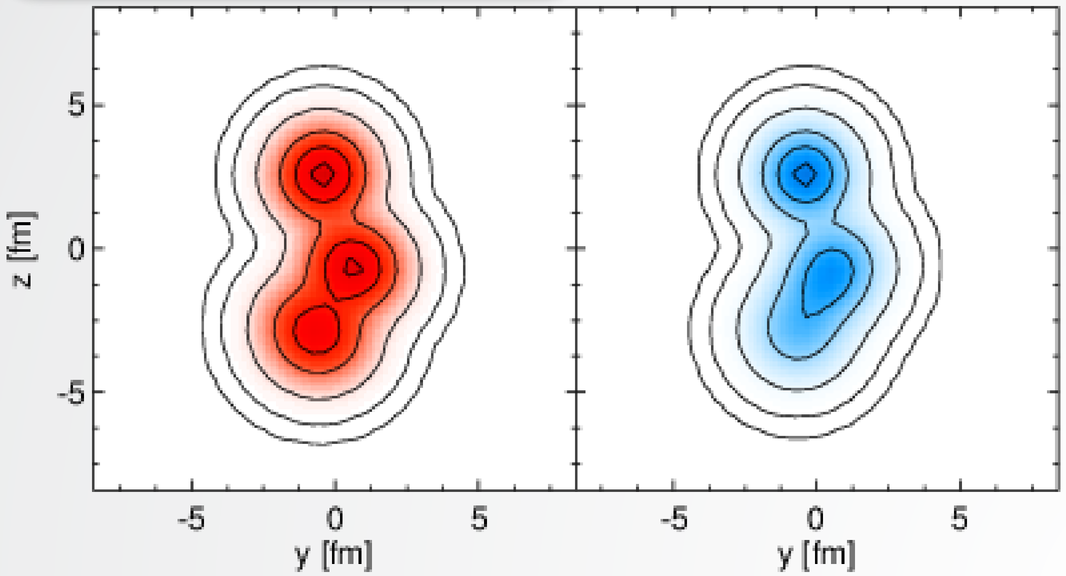
first 5/2^{-}



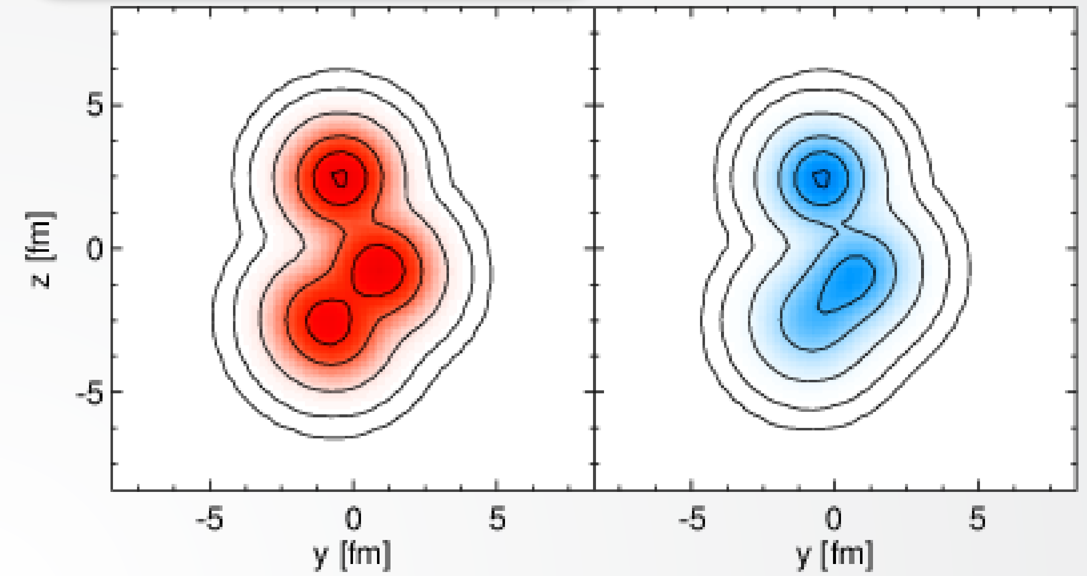
p-shell states with some hint of clustering

^{11}C: FMD Variation after Projection

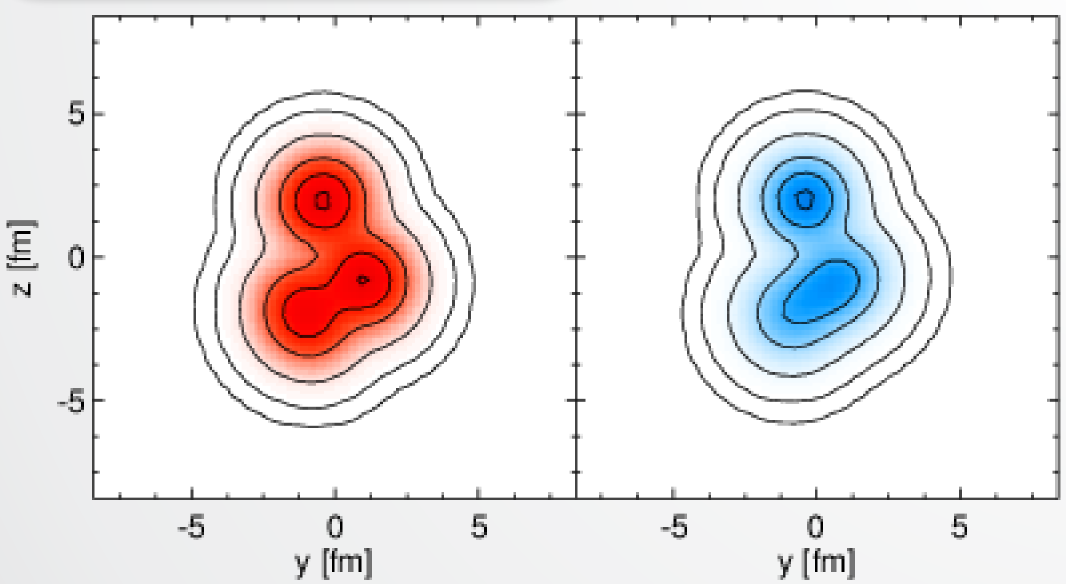
first 1/2+



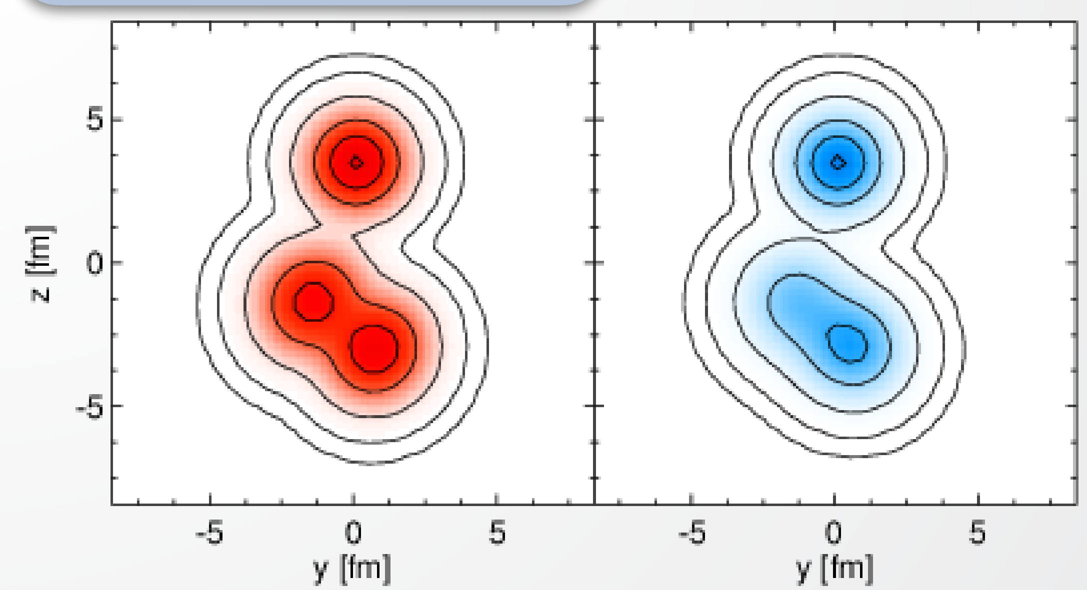
third 3/2-



first 5/2+

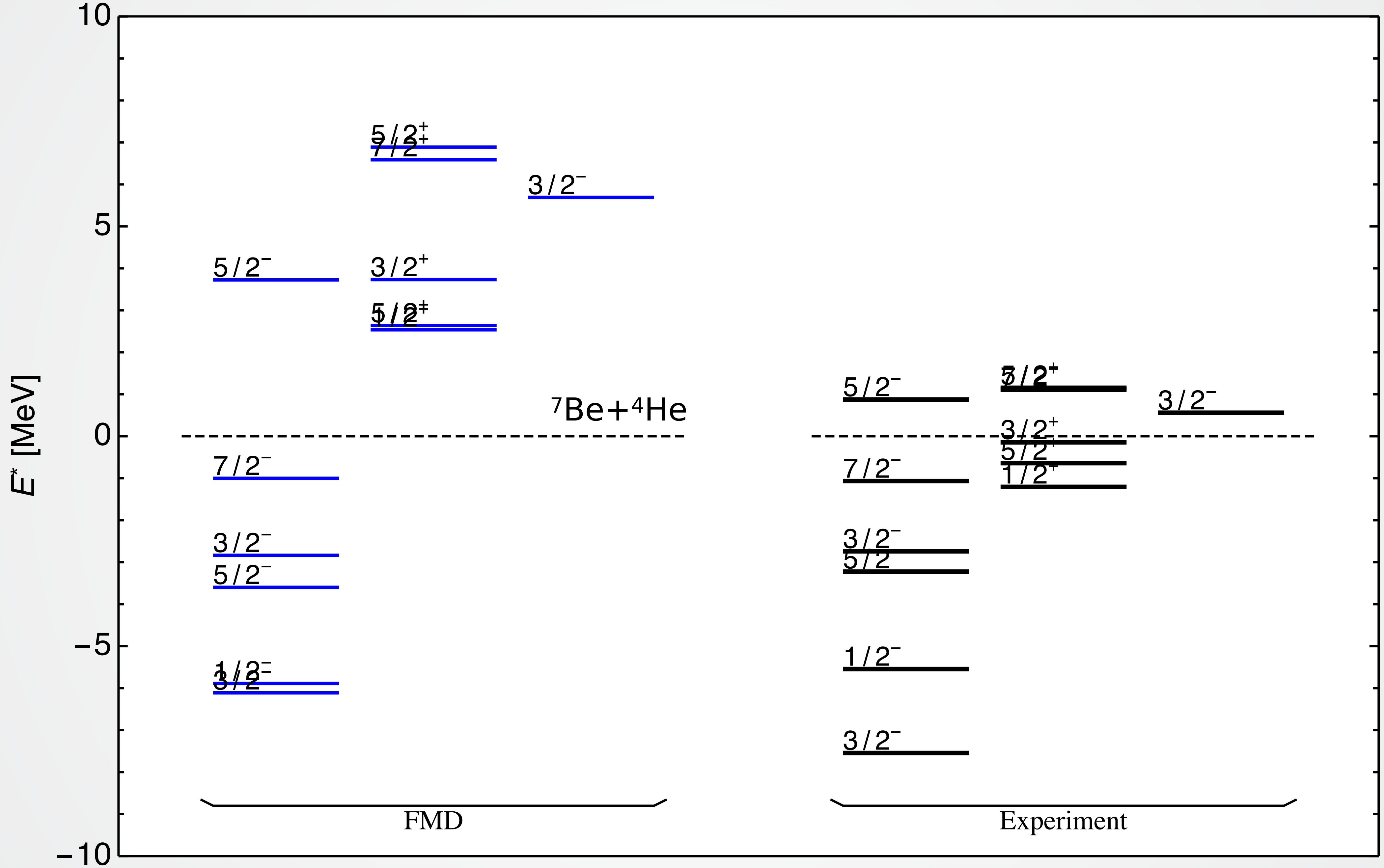


fourth 3/2-



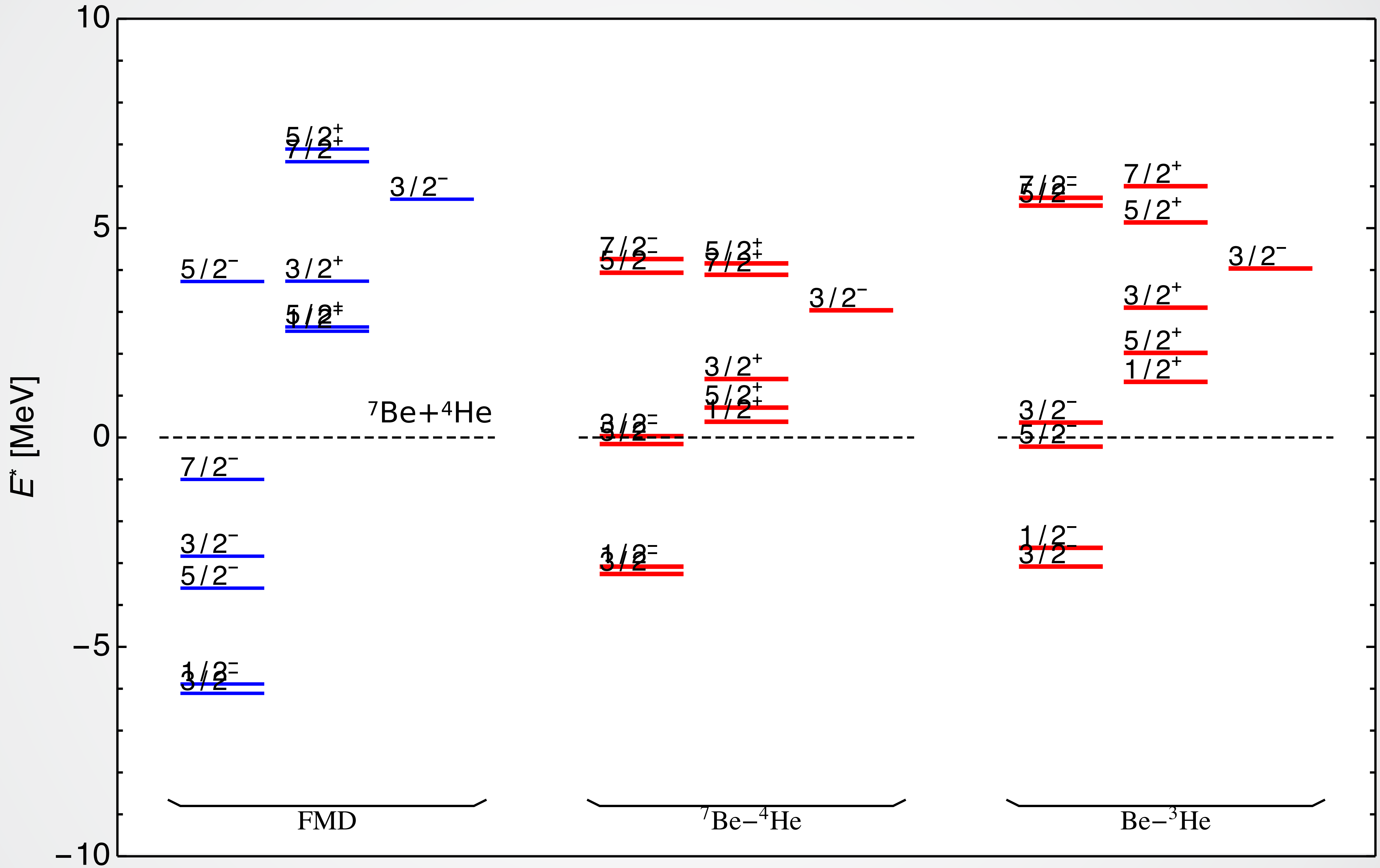
states with well defined cluster structure

^{11}C: Diagonalization with FMD VAP States



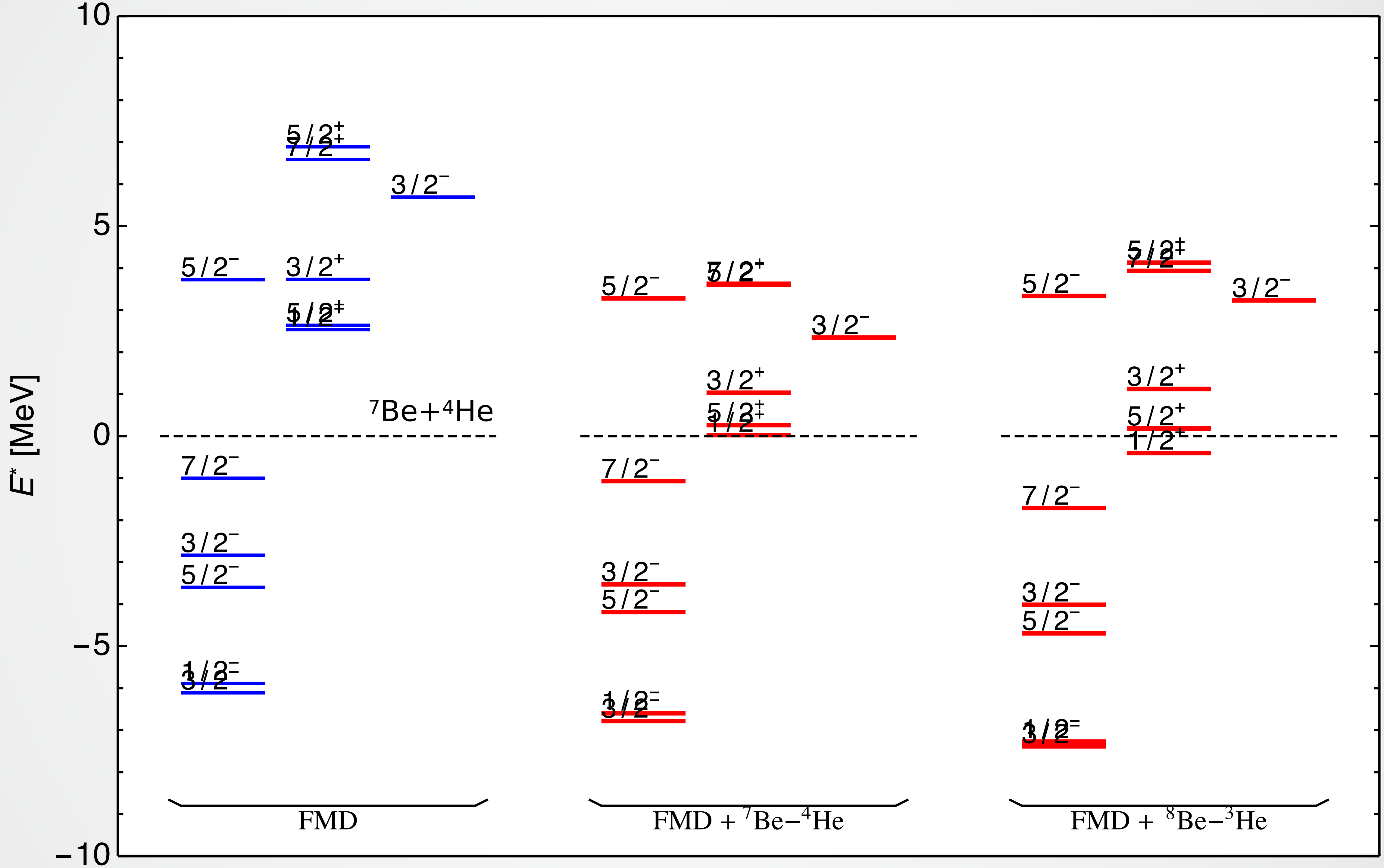
clustered states are well above threshold

^{11}C: FMD vs Cluster Model



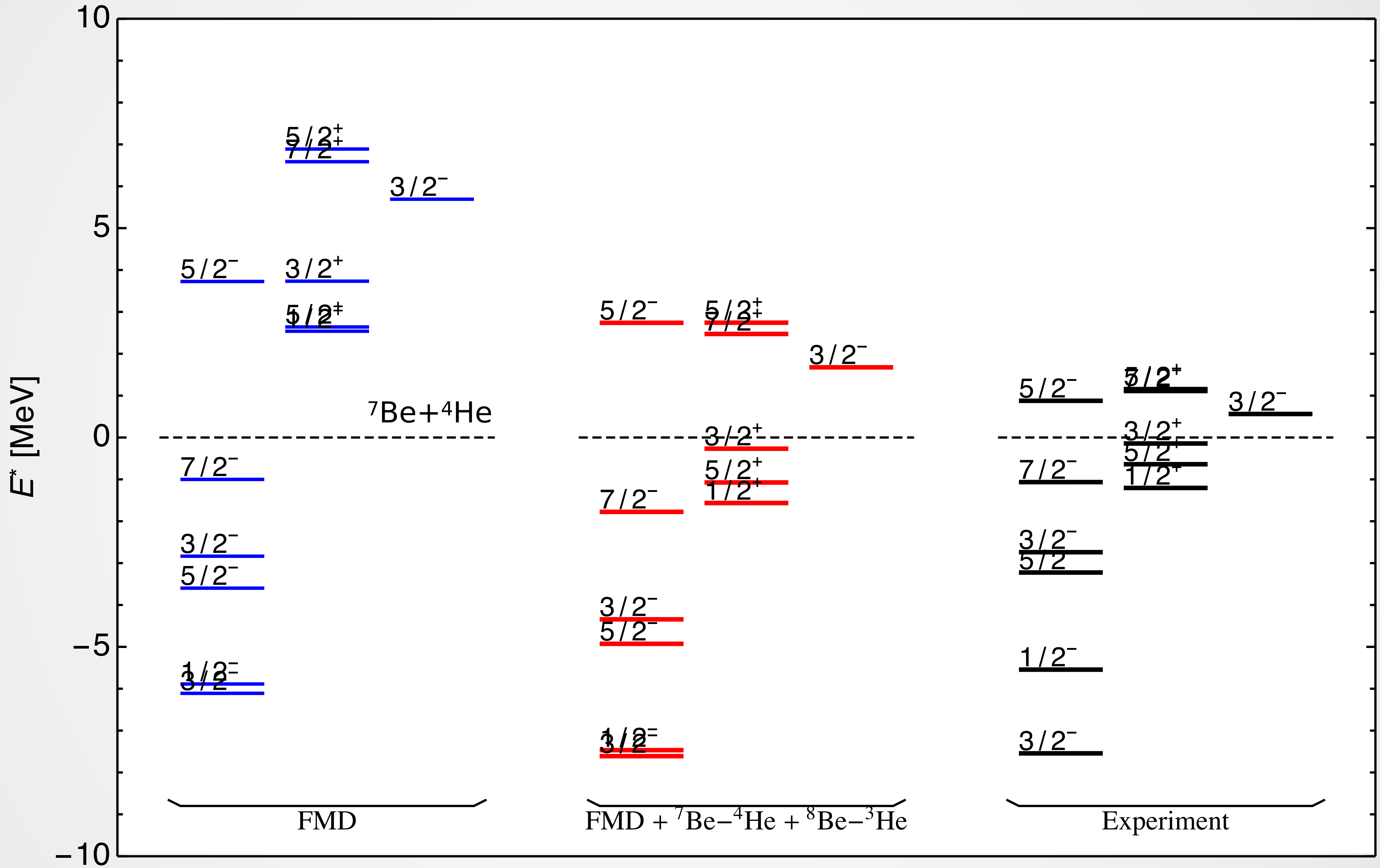
worse for p-shell states, better for clustered states

^{11}C: FMD plus Cluster Model



improves both p-shell and clustered states

^{11}C: FMD vs Full Calculation



consistent picture of p-shell and clustered states

^{11}C: Cluster Ovlaps

	$^7Be(3/2^-)$-^4He	$^7Be(3/2^-,1/2^-)$-^4He	$^8Be(0^+)$-^3He	$^8Be(0^+,2^+)$-^3He
$3/2^-$	0.83	0.85	0.62	0.82
$1/2^-$	0.86	0.88	0.62	0.79
$5/2^-$	0.81	0.82	0.01	0.78
second $3/2^-$	0.76	0.86	0.14	0.82
third $3/2^-$	0.76	0.80	0.11	0.37
$1/2^+$	0.72	0.86	0.56	0.77
$5/2^+$	0.88	0.90	0.54	0.77

- Be careful with interpretation — because of antisymmetrization a large overlap with cluster configurations does not necessarily mean that the state is well clustered
- More interesting than these spectroscopic factors would be spectroscopic amplitudes / ANCs / alpha-widths — work in progress
- What is the Hoyle analogue state — the $1/2^+$ or the third $3/2^-$ or neither ?

Summary and Outlook

Summary

- FMD calculations find pronounced clustering in ^{11}C in positive parity states and the third $3/2^-$ state
- The description of these states around the $^7Be+^4He$ threshold can be significantly improved by combing the FMD basis states with $^7Be+^4He$ and $^8Be+^3He$ cluster configurations using double-projection
- The third $3/2^-$ state above the threshold with its $^7Be(3/2^-)+^4He$ structure looks similar to the Hoyle state with its $^8Be+^4He$ structure
- The $1/2^+$ state below the threshold has a significant $^8Be(0^+)+^3He$ contribution that could be interpreted as the analogue of the Hoyle where three clusters are found mostly in relative S-wave

Outlook

- Investigate the ^{13}N structure — technical difficulties due to large number of states in the interesting region and contributions from $^{12}C+p$ continuum
- Treatment of continuum — exact R-matrix formulation for scattering requires wave functions with wave-packets of equal width (within each cluster) that can be rewritten as RGM wave functions — alternative in GCM picture possible?
- Look at observables like EM and weak transition strengths, ANCs and alpha-widths