CLUSTERING AND THE NUCLEAR MANY-BODY PROBLEM

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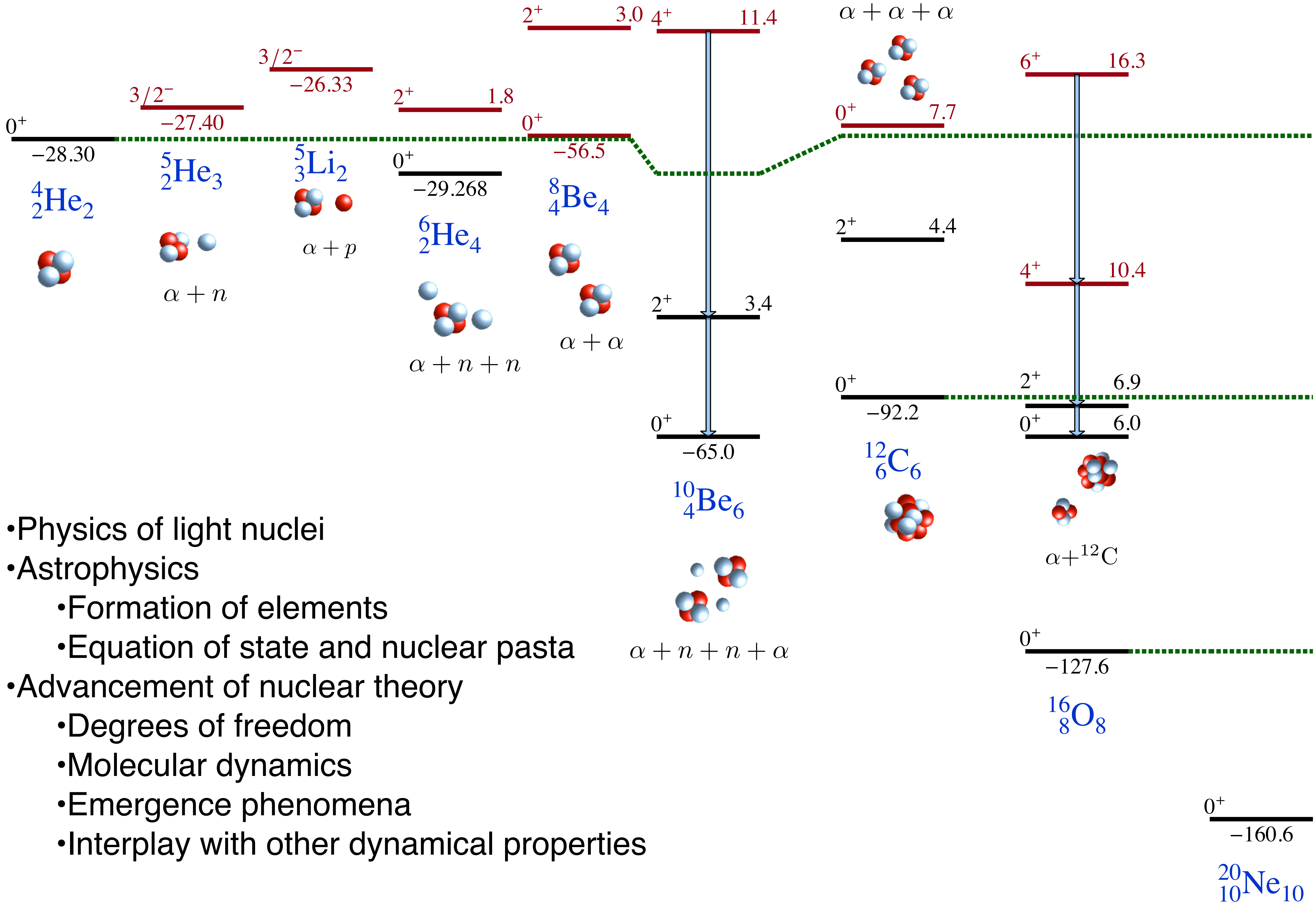
In collaboration with K. Kravvaris
DOE support: DE-SC0009883

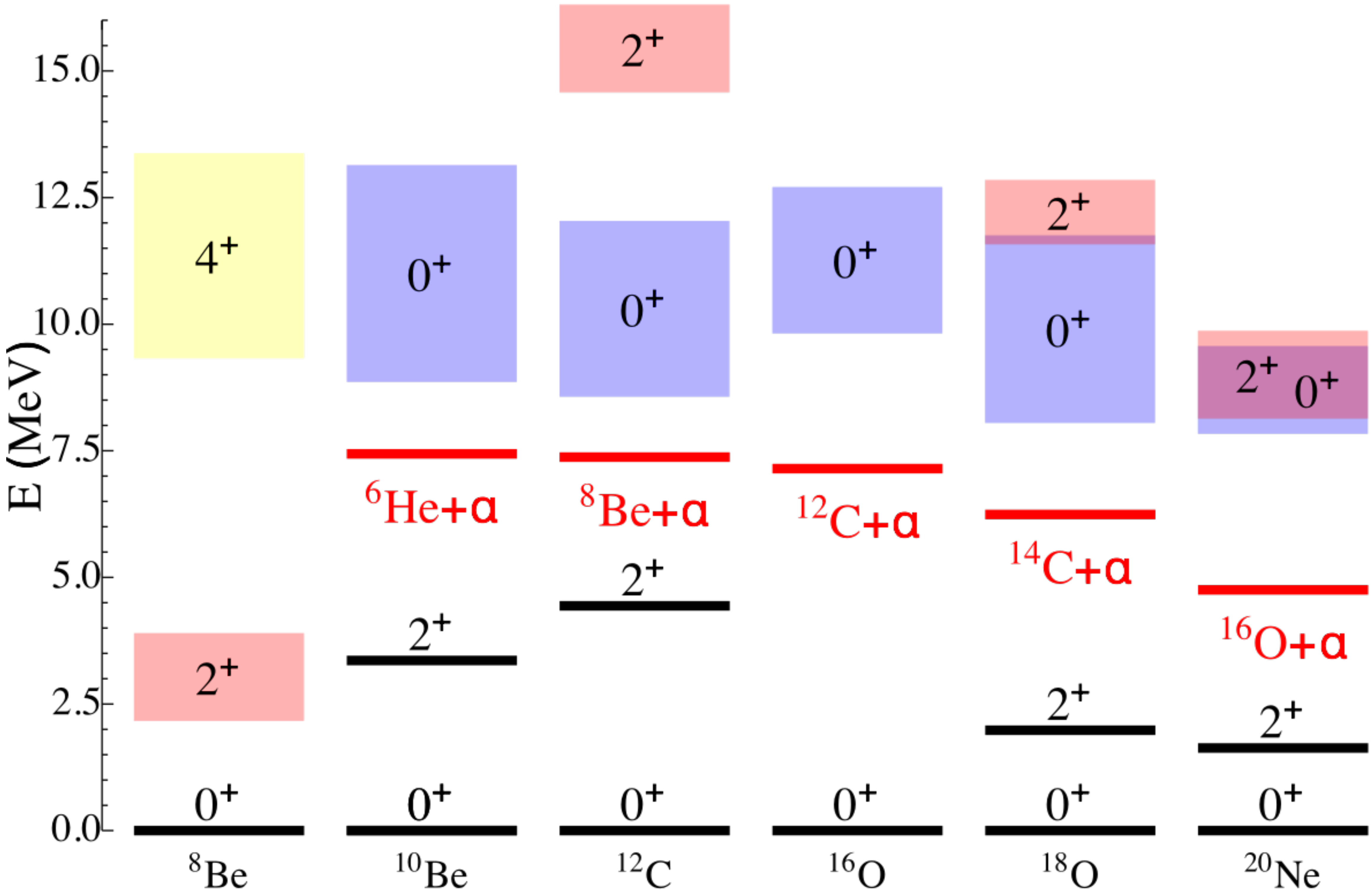
SOTANCP4, TX

Clustering in light nuclei



- Physics of light nuclei
- Astrophysics
  - Formation of elements
  - Equation of state and nuclear pasta
- Advancement of nuclear theory
  - Degrees of freedom
  - Molecular dynamics
  - Emergence phenomena
  - Interplay with other dynamical properties

Clustering and continuum



Key elements of discussion

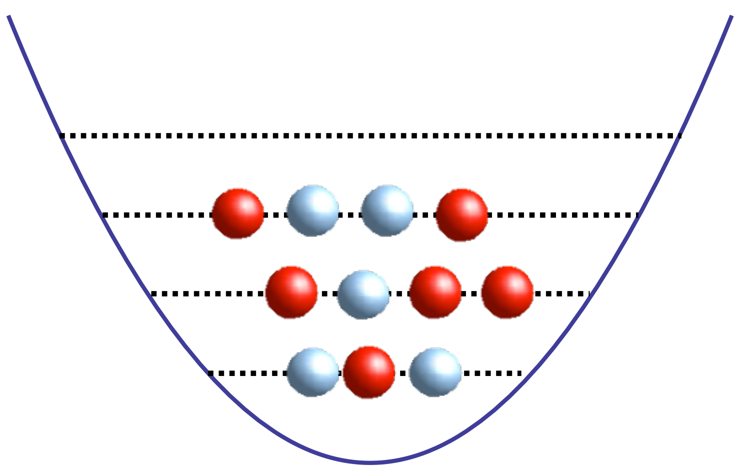
- Configuration interaction approach and clustering
  - CI approach
  - Center of mass boost
  - Relation to SU(3) limit
  - Recoupling CM motion and cluster channels
  - Examples
- Assessing clustering characteristics
  - Traditional (old) spectroscopic factors
  - Orthonormalized (Fliessbach) spectroscopic factors
  - Resonating Group Method (RGM) solutions
  - J-matrix and phase shifts
- Examples
  - Traditional shell model successes and problems
  - Clustering in models from ab-initio principles

Configuration interaction approach and clustering

Traditional shell model configuration
m-scheme

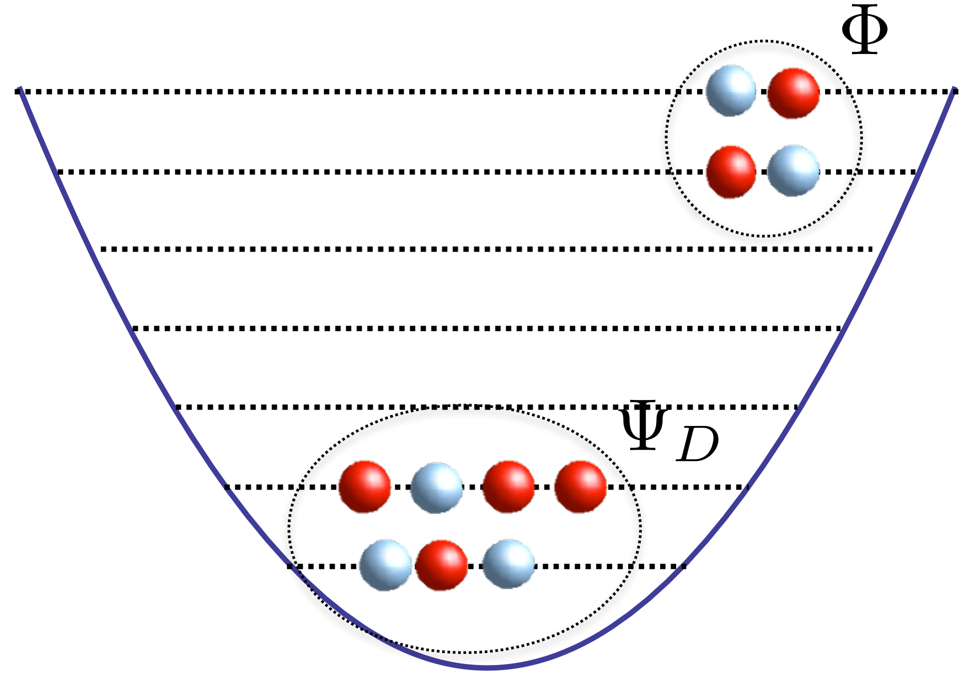
Cluster configuration
SU(3)-symmetry basis

$$|\Psi\rangle = \Psi^\dagger|0\rangle \sim a_1^\dagger a_2^\dagger \dots a_A^\dagger |0\rangle$$



+

$$|\mathrm{channel}\rangle \sim |\Phi\Psi_D\rangle \equiv \Phi^\dagger \Psi_D^\dagger |0\rangle$$



$|\Psi\rangle$ + $\Phi^\dagger|\Psi_D\rangle$

Antisymmetrization
state-operator polymorphism

State, equivalent to operator (polymorphism)

$$|\Psi\rangle \equiv \hat{\Psi}^\dagger|0\rangle = \sum_{\{1,2,3,\ldots A\}} \langle 1,2\ldots A|\Psi\rangle\, \hat{a}_1^\dagger \hat{a}_2^\dagger \ldots \hat{a}_A^\dagger|0\rangle$$

$$|\Psi_\alpha\rangle = \Psi_\alpha^\dagger|\rangle = \sum_{\{m\}} X_m^\alpha a_{m_1}^\dagger a_{m_2}^\dagger a_{m_3}^\dagger a_{m_4}^\dagger|\rangle$$

$$|\Psi_{\rm D}\rangle = \Psi_{\rm D}^\dagger|\rangle = \sum_{\{m\}} X_m^{\rm D} a_{m_1}^\dagger a_{m_2}^\dagger \ldots a_{m_{A_{\rm D}}}^\dagger|\rangle$$

Anti-symmetrized channel wave function components are generated by acting with state creation operator and forward ordering.

$$|\Psi_{\rm C}\rangle = \Psi_\alpha^\dagger \Psi_{\rm D}^\dagger|\rangle$$

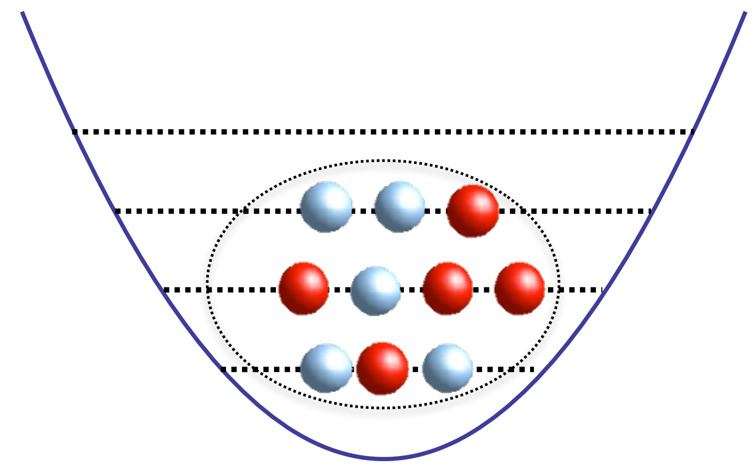
Code at http://www.volya.net [cosmo]

Translational invariance and Center of Mass (CM)

Shell model, Glockner-Lawson procedure

$$\Psi_D = \phi_{000}(\mathbf{R}_D)\,\Psi'_D$$

SM state — Center-of-mass vibration — Intrinsic state



Controlling CM

$$D_\mu = \sqrt{\frac{4\pi}{3}} R_\mu$$

Control only CM quanta

$$R_\mu = \sqrt{\frac{\hbar}{2Am\omega}} (\mathcal{B}^\dagger_\mu + \mathcal{B}_\mu)$$

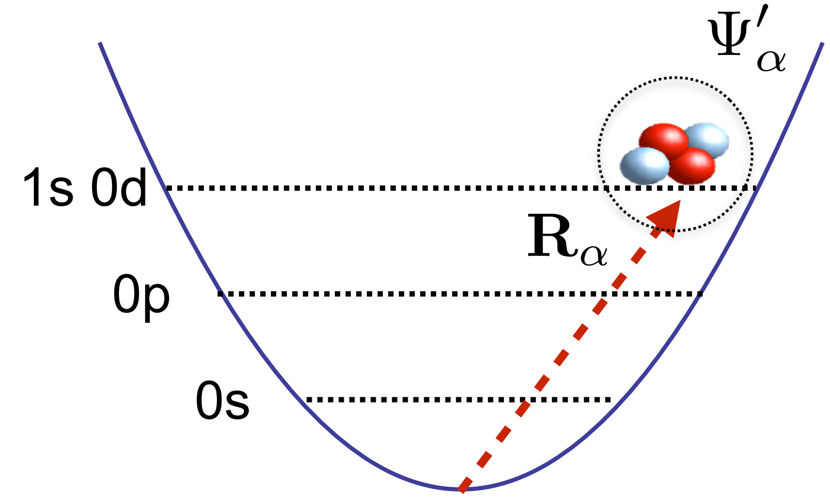
Center-of-Mass boosts

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

$\boldsymbol{\mathcal{B}}^\dagger$ and $\boldsymbol{\mathcal{B}}$ CM quanta creation and annihilation (vectors)

$$\Psi_{n+1\ell m} \propto \boldsymbol{\mathcal{B}}^\dagger \cdot \boldsymbol{\mathcal{B}}^\dagger \Psi_{n\ell m}$$

$\boldsymbol{\mathcal{B}}^\dagger \times \boldsymbol{\mathcal{B}}$ CM angular momentum operator



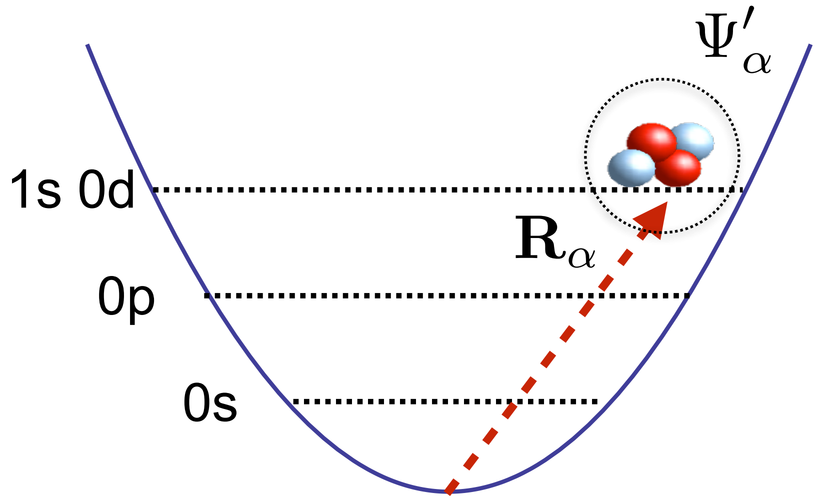
$$N = 2n + \ell$$

	$\alpha[0]$	$\alpha[4]$
Configuration	$N_{\max} = 0$	$N_{\max} = 4$
$(sd)^4$	0.038	0.035
$(p)(sd)^2(pf)$	0.308	0.282
$(p)^2(pf)^2$	0.103	0.094
$(p)^2(sd)(sdg)$	0.154	0.141
$(s)^2(sd)(sdgi)$	0.000	0.005
$(p)(sd)(pf)(sdg)$	0.000	0.009

Select configuration content of NCSM wave functions for ^{4}He with $\Omega = 20$ MeV boosted by 8 quanta (L = 0).

K Kravvaris and A. Volya, Journal of Phys, Conf. Proc. 863, 012016 (2017)

Approximation of N_{max}=0 (s^4) Cluster coefficients for SU(3) components



Expand SU(3) 4-nucleon structure in intrinsic+ relative all oscillator quanta of excitation are in relative motion.

$$\phi_{n\ell m}(\mathbf{R}_\alpha)\Psi'_\alpha = \sum_\eta X^\eta_{n\ell}\,\Phi^\eta_{(n,0):\ell m}$$

$$X^\eta_{n\ell} \equiv \langle \Phi^\eta_{(n,0):\ell m} | \phi_{n\ell m}(\mathbf{R}_\alpha)\,\Psi'_\alpha\rangle = \sqrt{\frac{1}{4^n}\frac{n!}{\prod_i (n_i!)^{\alpha_i}}\frac{4!}{\prod_i \alpha_i!}}$$

Volya and Yu. M. Tchuvil'sky, Phys. Rev. C 91, 044319 (2015).
Yu. F. Smirnov and Yu. M. Tchuvil'sky, Phys. Rev. C 15, 84 (1977).
M. Ichimura, A. Arima, E. C. Halbert, and T. Terasawa, Nucl. Phys. A 204, 225 (1973).
O. F. Nemetz, V. G. Neudatchin, A. T. Rudchik, Yu. F. Smirnov, and Yu. M. Tchuvil'sky, Nucleon Clusters in Atomic Nuclei and Multi-Nucleon Transfer Reactions (Naukova Dumka, Kiev, 1988), p. 295.

Center-of-Mass boosts

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

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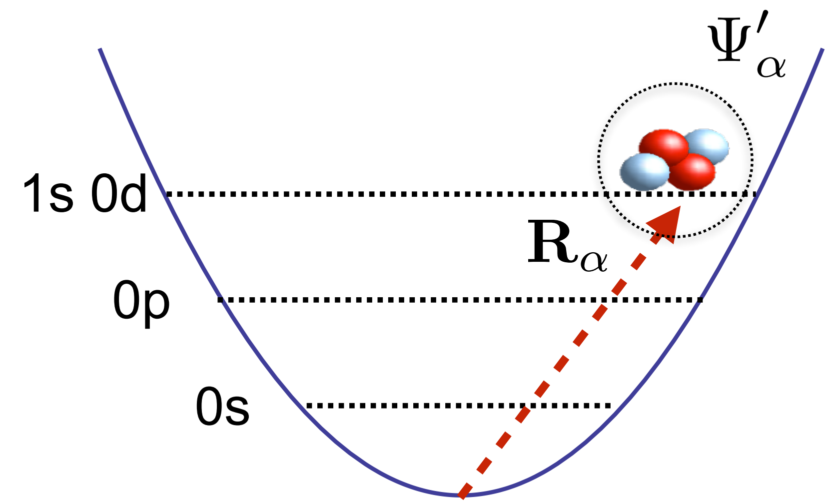
$\boldsymbol{\mathcal{B}}^\dagger \times \boldsymbol{\mathcal{B}}$ CM angular momentum operator



$$\Psi_\alpha = \phi_{n\ell m}(\mathbf{R})\,\Psi'_\alpha = \sum_\eta X^\eta_{n\ell}\,\Phi^\eta_{(n,0):\ell m}$$

Configuration	$N_{\max}=0$	$N_{\max}=4$
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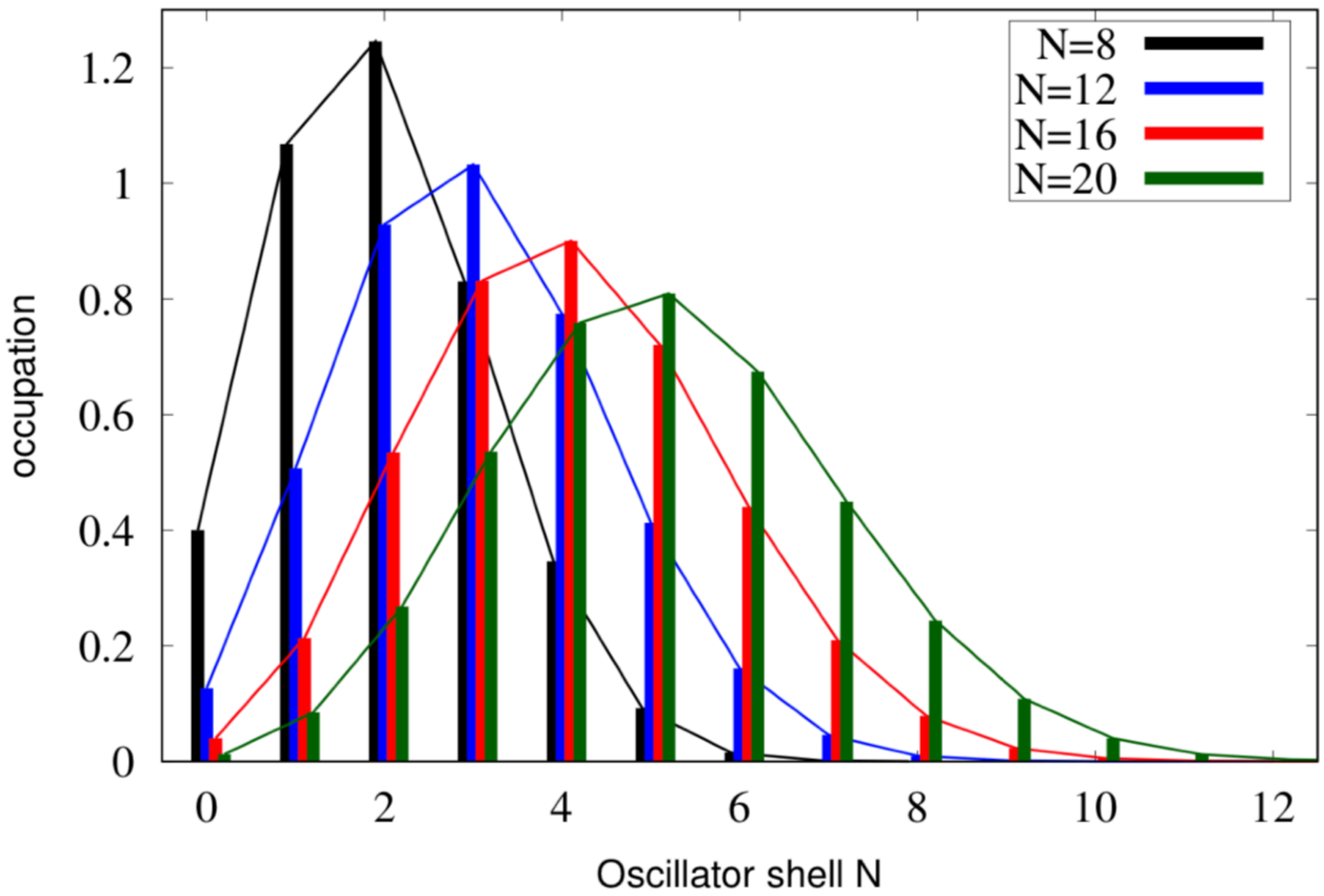
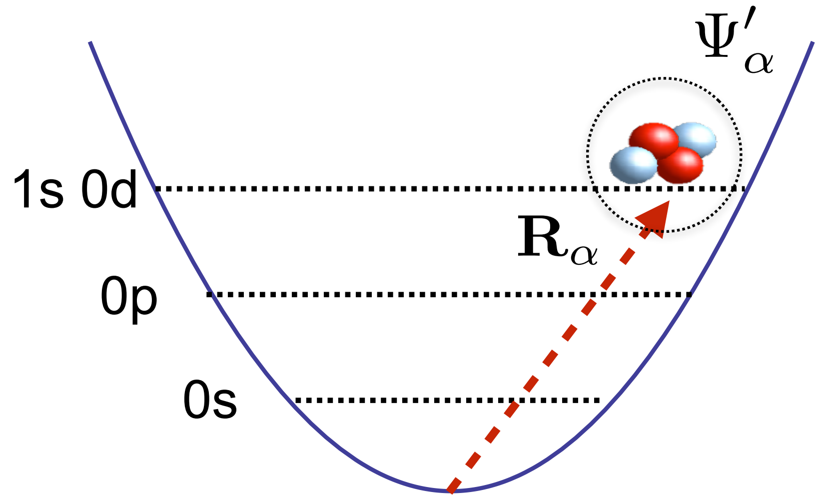
K Kravvaris and A. Volya, Journal of Phys, Conf. Proc. 863, 012016 (2017)

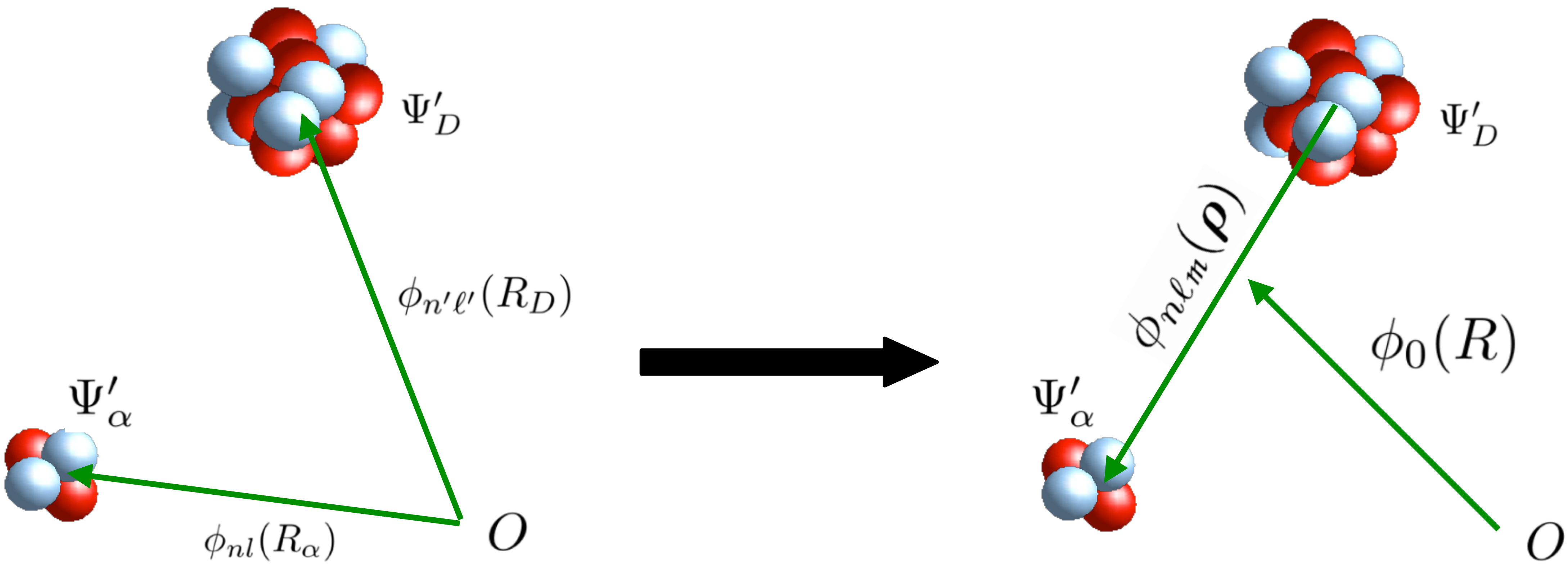
CM-boosted configuration from shell model perspective





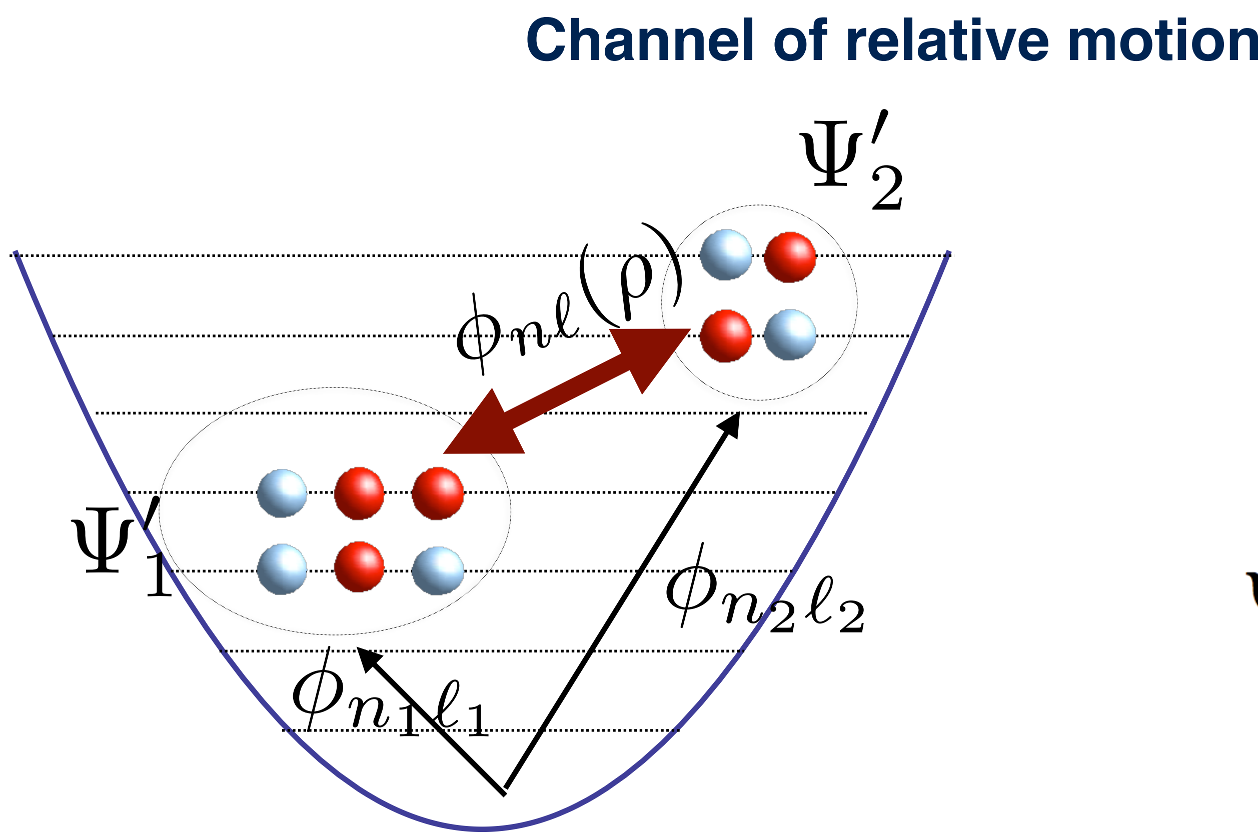
K Kravvaris and A. Volya, Journal of Phys, Conf. Proc. 863, 012016 (2017)

Recoil Recoupling



- Recoupling is done with Talmi-Moshinsky brackets
- Diagonalization

Center-of-mass recoil correction



$$\Psi = \phi_{000}(\mathbf{R})\Psi'$$

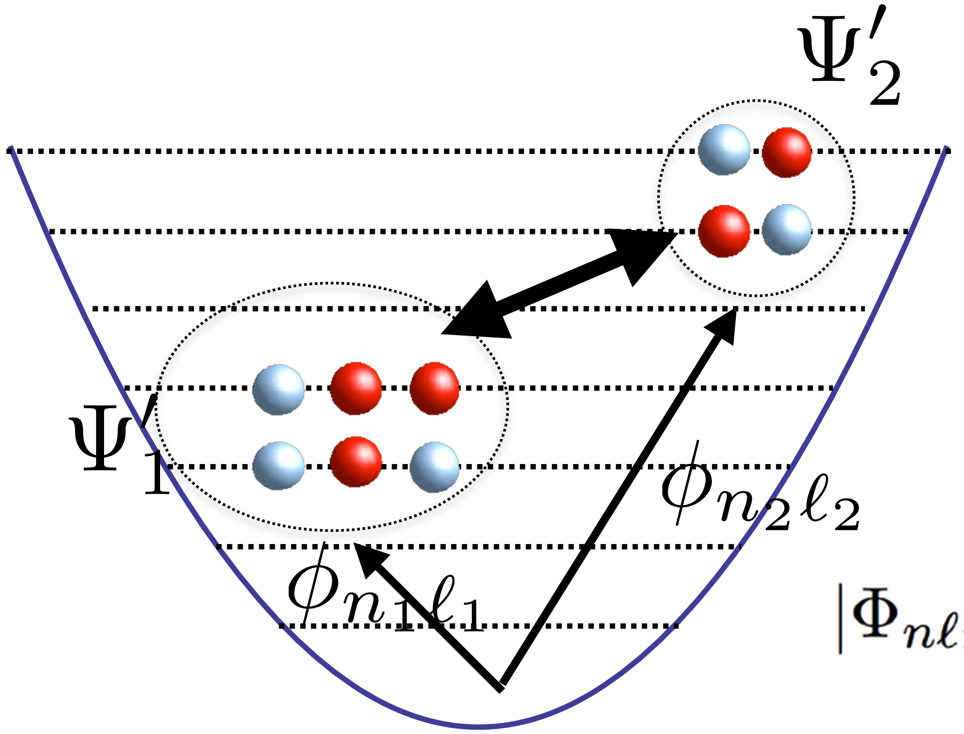
Boost

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

CM-Recouple

$$\Psi_C = \left\{ \{\phi_{n_1\ell_1}(1)\phi_{n_2\ell_2}(2)\}_{n\ell}\ \{\Psi'(1)\Psi'(2)\}_J \right\}_{J_c}$$

Examples and tests



Channel of relative motion

$$^{16}\mathrm{O} \rightarrow \, ^{12}\mathrm{C} + \alpha$$

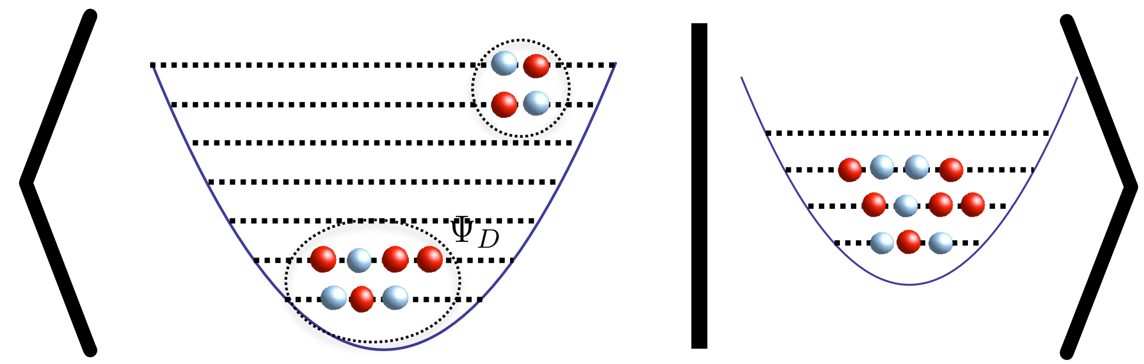
$$|\Phi_{n\ell m}\rangle = \sum_{\substack{n_1\ell_1 m_1 \\ n_2\ell_2 m_2}} \mathcal{M}^{n\ell 00}_{n_1\ell_1 n_2\ell_2} C^{\ell m}_{\ell_1 m_1 \ell_2 m_2} |\Psi_{n_1\ell_1 m_1}(1)\, \Psi_{n_2\ell_2 m_2}(2)\rangle$$

n_1	ℓ_1	n_2	ℓ_2	$\langle 00, n\ell\|1,2\rangle_\ell$	$(0p_{3/2})^8$	SU(3)
0	0	2	0	$\frac{9}{16} \approx 0.563$	0.0761	$\sqrt{3/32}$
0	1	1	1	$-\frac{3\sqrt{3}}{8} \approx$ -0.650	-0.0878	$-1/\sqrt{8}$
0	2	0	2	$\sqrt{\frac{3}{32}} \approx 0.306$	0.0414	$1/\sqrt{32}$
1	0	1	0	$\sqrt{\frac{15}{128}} \approx 0.342$	0.0463	$\sqrt{5/12}$
1	1	0	1	$-\frac{\sqrt{3}}{8} \approx$ -0.217	-0.0293	$-1/\sqrt{72}$
2	0	0	0	$\frac{1}{16} \approx 0.063$	0.0085	$1/\sqrt{864}$
				Exact SF	**0.0183**	**8/27=0.296**

Cluster Spectroscopic Characteristics

Traditional (old) spectroscopic factor

$$\langle \phi_{n\ell} | \varphi_\ell \rangle = \langle \hat{\mathcal{A}} \{ \phi_{n\ell m}(\boldsymbol{\rho})\, \Psi'_\alpha\, \Psi'_D \} | \Psi'_P \rangle \; =$$



$$\langle \phi_{n\ell} | \varphi_\ell \rangle = \mathcal{R}_{n\ell} \sum_\eta X^\eta_{n\ell}\, \mathcal{F}^\eta_{n\ell}$$

Recoil Factor — Cluster Coefficient — Fractional Parentage Coefficient

Normalized (new) spectroscopic factor

$$\psi_\ell(\rho) \equiv \hat{\mathcal{N}}_\ell^{-1/2} \varphi_\ell(\rho)$$

$$S_\ell^{(\mathrm{new})} \equiv \langle \psi_\ell | \psi_\ell \rangle = \int \rho^2 d\rho\, |\psi_\ell(\rho)|^2$$

Sum of all new SF from all parent states to a given final state equals to the number of channels

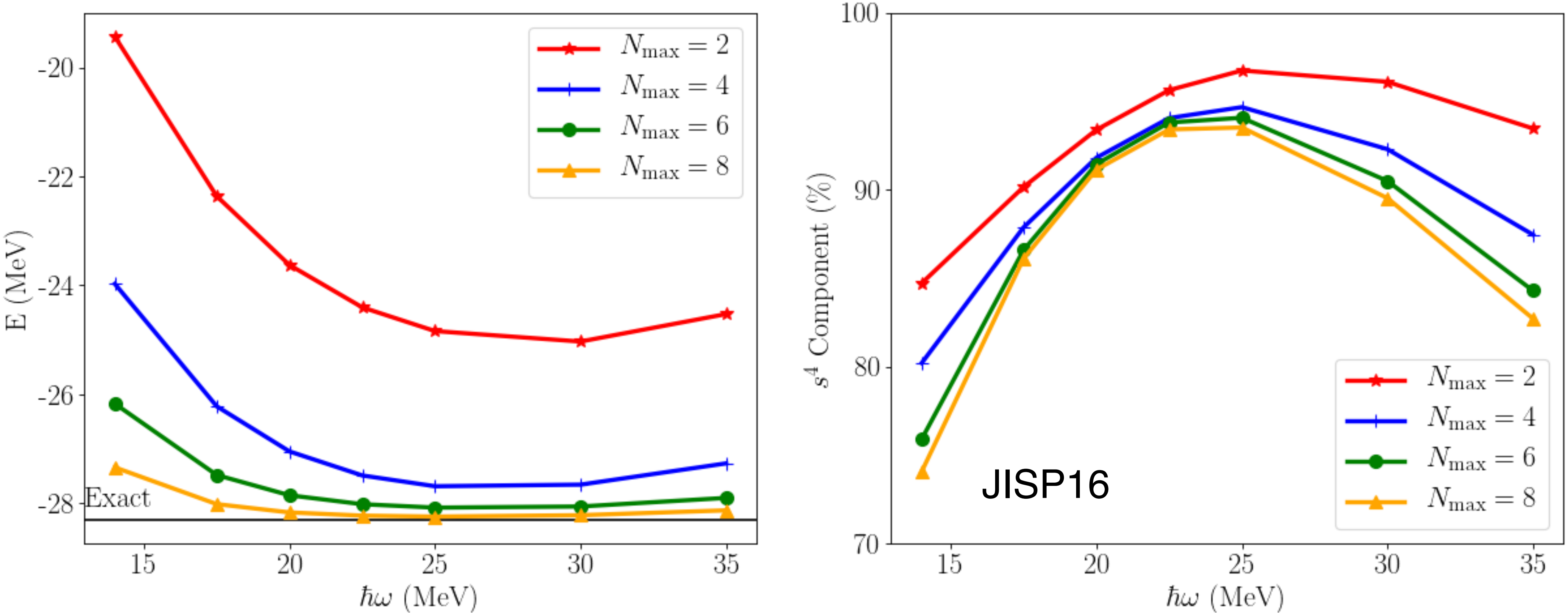
R. Id Betan and W. Nazarewicz Phys. Rev. C 86, 034338 (2012)
S. G. Kadmenskya, S. D. Kurgalina, and Yu. M. Tchuvil'sky Phys. Part. Nucl., 38, 699–742 (2007).
R. Lovas et al. Phys. Rep. 294, No. 5 (1998) 265 – 362.
T. Fliessbach and H. J. Mang, Nucl. Phys. A **263**, 75–85 (1976).
H. Feschbach et al. Ann. Phys. 41 (1967) 230 – 286

Channels, spectroscopic factors examples

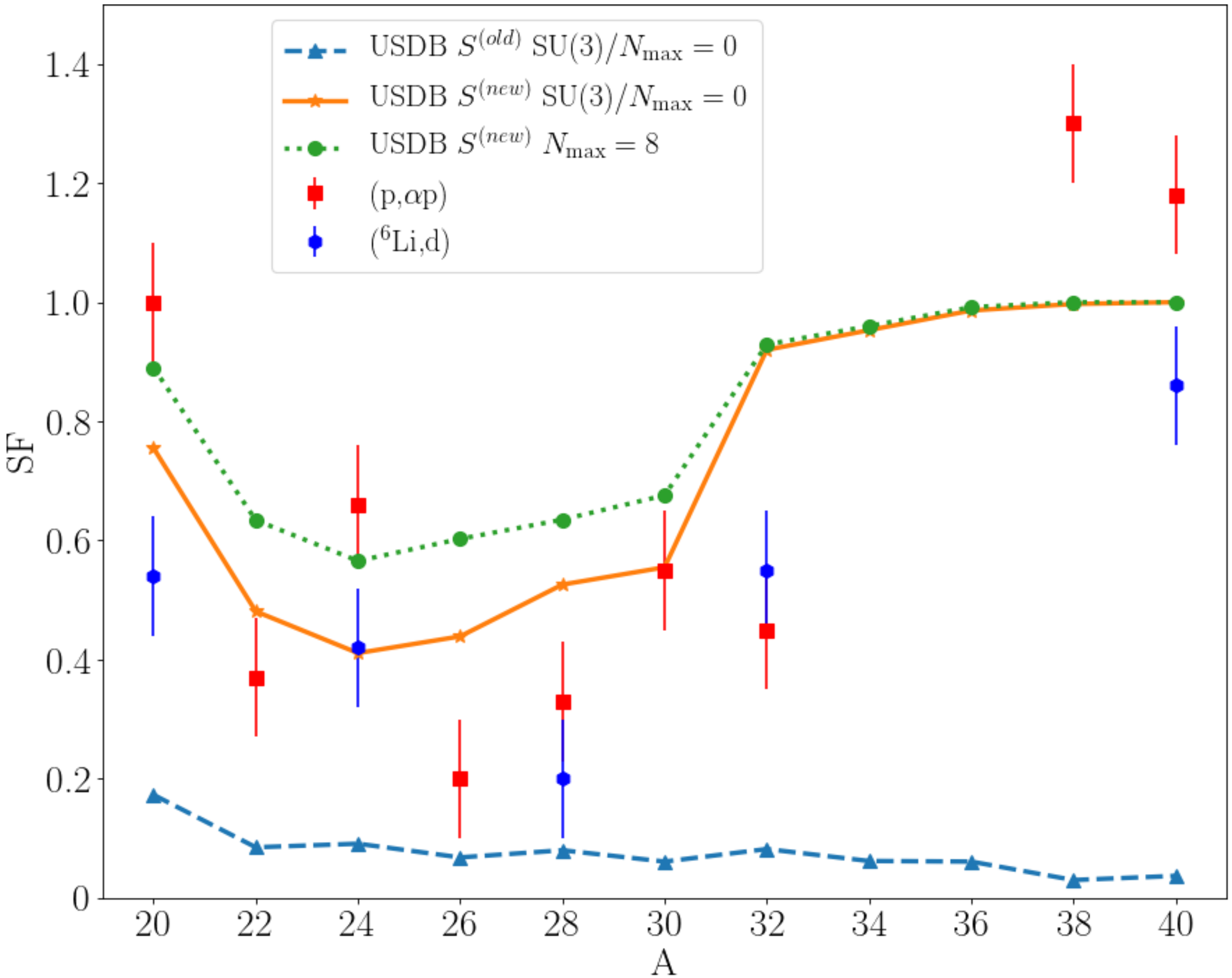
parent	channel	N_c	$\|\langle\Psi\|\Phi_C\rangle\|$	$\langle\Phi_C\|\Phi_C\rangle$
$^{16}\mathrm{O}[0]$	$^{12}\mathrm{C}[(0,4)]+\alpha[0]$	4	$\sqrt{8/27}$	$8/27$
$^{16}\mathrm{O}[0]$	$^{12}\mathrm{C}[\mathrm{p}^8_{3/2}]+\alpha[0]$	4	0.135	0.018
$^{16}\mathrm{O}[0]$	$^{12}\mathrm{C}[\mathrm{p}^8_{3/2}]+\alpha[4]$	4	0.130	0.017
$^{8}\mathrm{Be}[(4,0)]$	$\alpha[0]+\alpha[0]$	4	$\sqrt{3/2}$	$3/2$
$^{8}\mathrm{Be}[0]$	$\alpha[0]+\alpha[0]$	4	1.160	$3/2$
$^{8}\mathrm{Be}[4]$	$\alpha[0]+\alpha[0]$	4	0.984	$3/2$
$^{8}\mathrm{Be}[4]$	$\alpha[0]+\alpha[0]$	6	0.644	$15/8$
$^{8}\mathrm{Be}[4]$	$\alpha[2]+\alpha[2]$	4	0.981	1.492
$^{12}\mathrm{C}[\mathrm{p}^8_{3/2}]$	$\alpha[0]+\alpha[0]+\alpha[0]$	8	$1/4$	$81/80$
$^{16}\mathrm{O}[0]$	$(\alpha[0])^4$	12	$\sqrt{3/10}$	$3/10$

l=0 spectroscopic amplitudes of base

Structure of the alpha particle in NCSM



A. M. Shirokov, J. P. Vary, A. I. Mazur, and T. A. Weber. Realistic nuclear hamiltonian: Ab exitu approach. Physics Letters B, 644(1):33, 2007.

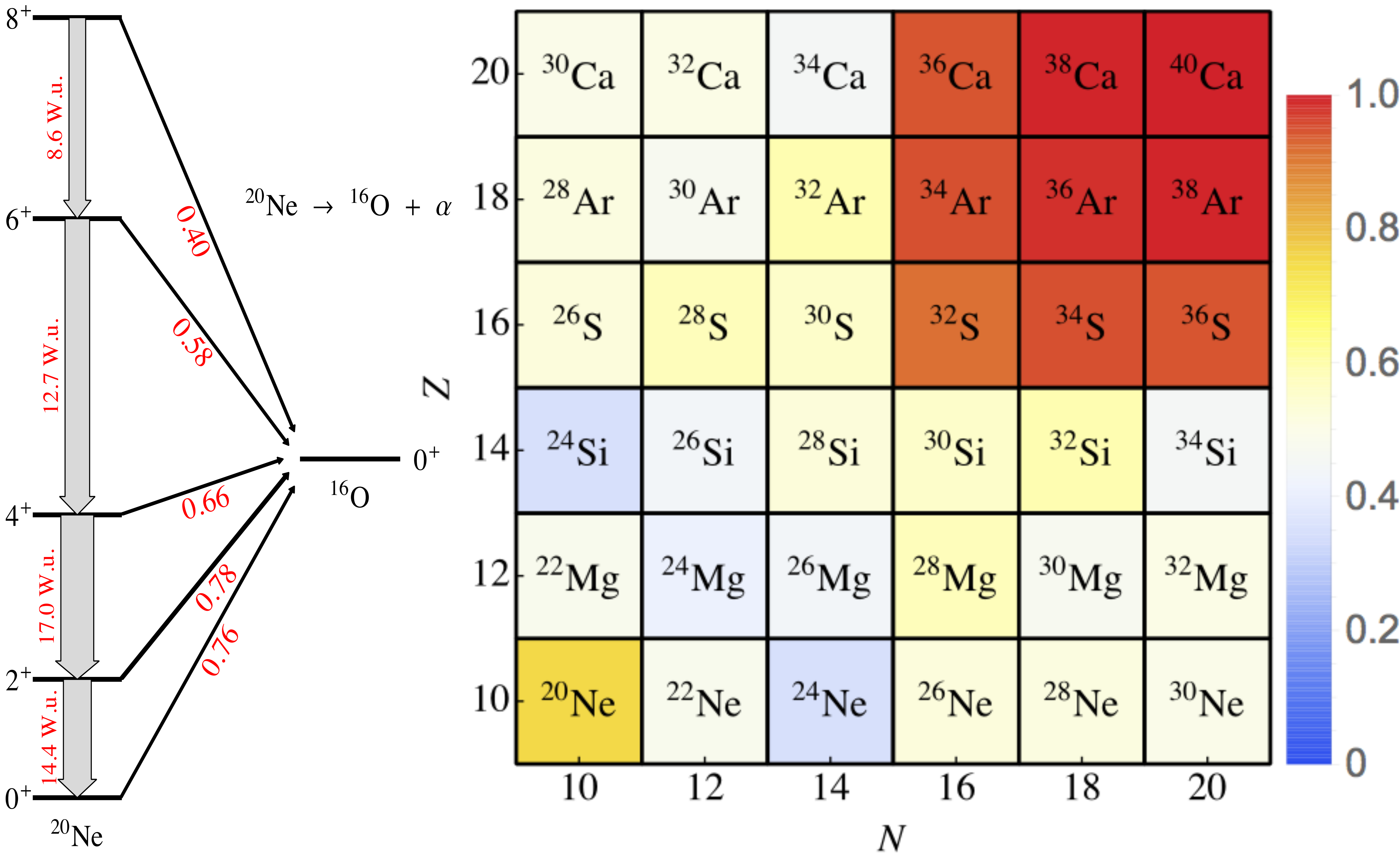


Experiment:

[1] T.A. Carey, P.G. Roos, N.S. Chant, A. Nadsen, H.L. Chen, Phys. Rev. C 23,576(R) (1981)

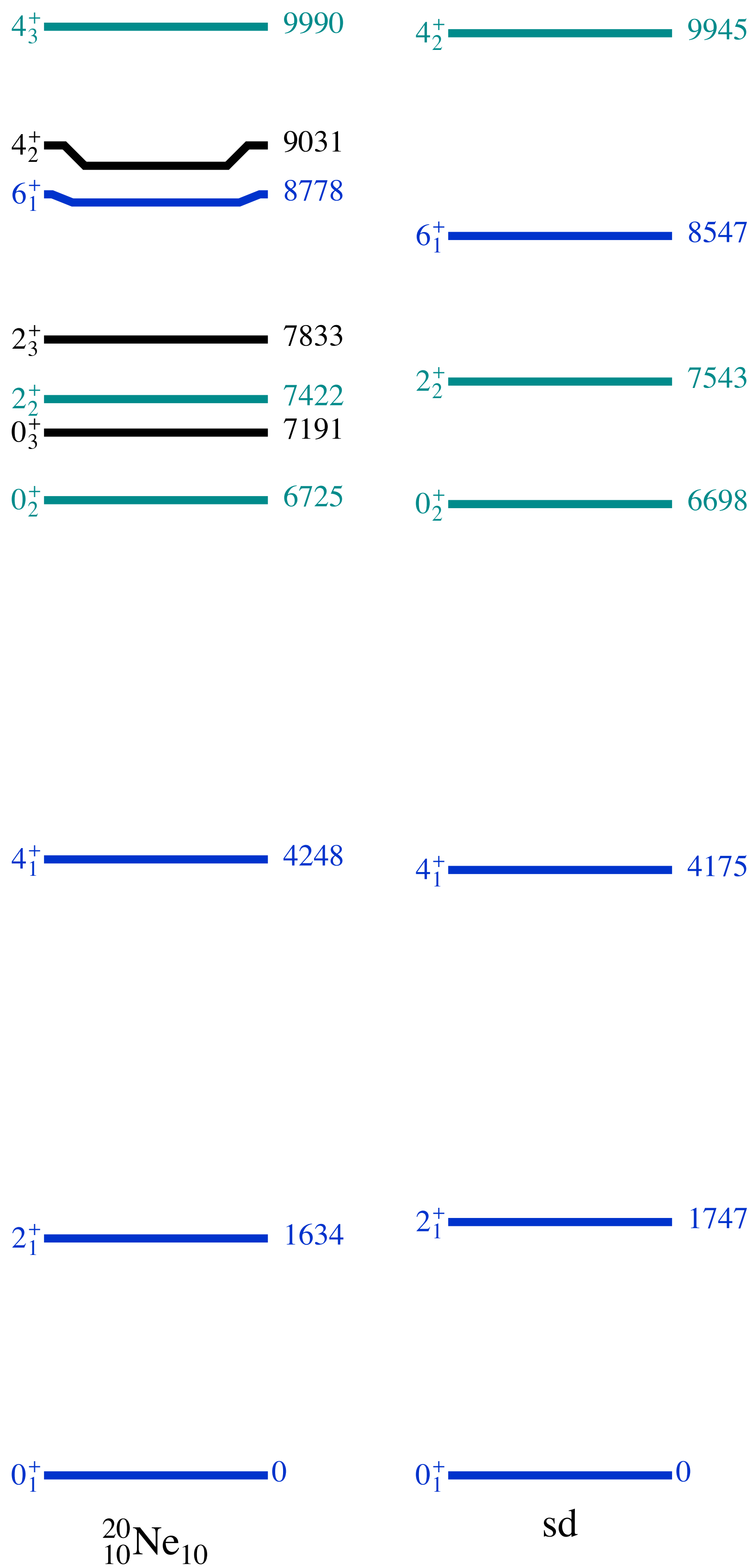
[2] N. Anantaraman et al. Phys. Rev. Lett. 35, 1131 (1975)

Our results tabulated: https://www.volya.net/ (see research, clustering)

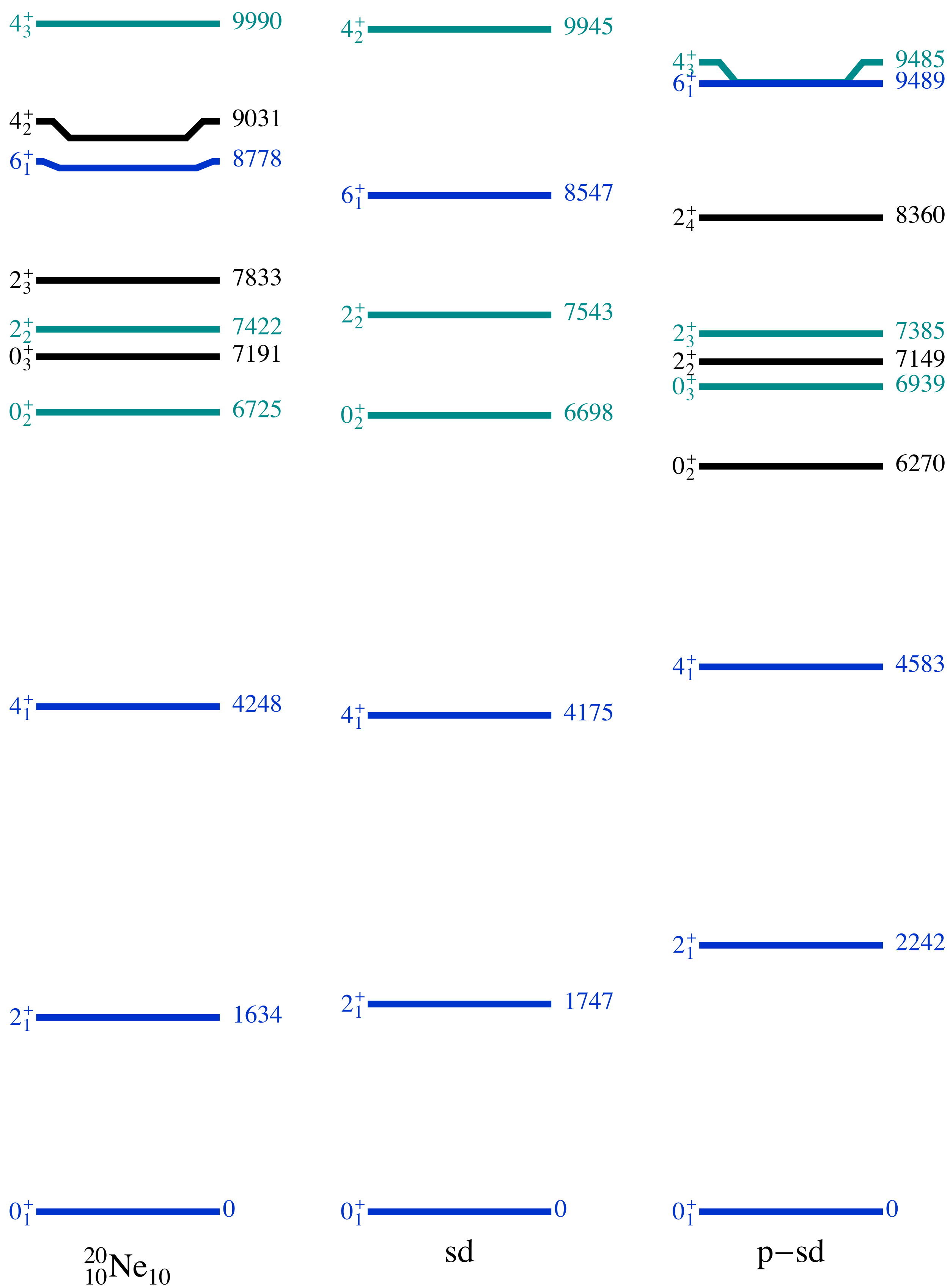


8+
6+
4+
2+
0+
8.6 W.u.
12.7 W.u.
17.0 W.u.
14.4 W.u.
0.40
0.58
0.66
0.78
0.76
20Ne → 16O + α
0+
16O
20Ne
Z
N
20
18
16
14
12
10
30Ca
32Ca
34Ca
36Ca
38Ca
40Ca
28Ar
30Ar
32Ar
34Ar
36Ar
38Ar
26S
28S
30S
32S
34S
36S
24Si
26Si
28Si
30Si
32Si
34Si
22Mg
24Mg
26Mg
28Mg
30Mg
32Mg
20Ne
22Ne
24Ne
26Ne
28Ne
30Ne
1.0
0.8
0.6
0.4
0.2
0

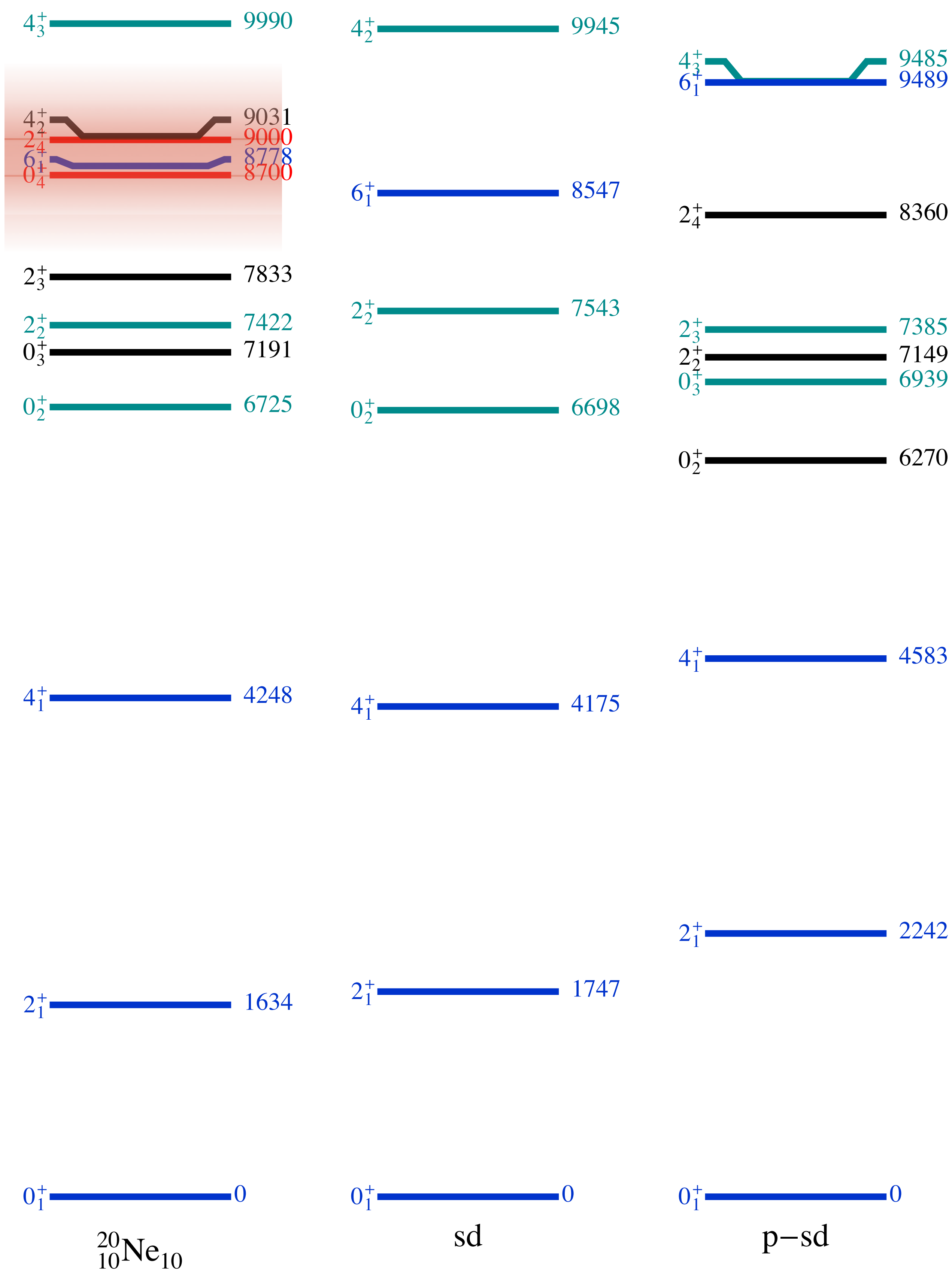
Clustering in ^{20}Ne



Clustering in ^{20}Ne



Clustering in ^{20}Ne



J	E MeV	Γ width	SF ex	SF th.
0+	0	0		0.73
2+	1.63	0		0.67
4+	4.25	0		0.62
0+	6.73	19	0.47	0.46
0+	7.19	3.4	0.02	0.10
2+	7.42	15	0.19	0.12
2+	7.83	2	0.01	0.09
0+	8.7	800	0.3	
6+	8.78	0.11	0.5	0.51
2+	9.00	800	0.86	

D.K. Nauruzbayev et al., Phys. Rev. C **96**, 014322 (2017)

Resonating group method

$$\mathcal{F}_\ell(\rho) = \sum_n \chi_n \Phi_{n\ell}$$

$$\sum_n \mathcal{H}^{(\ell)}_{nn'} \chi_{n'} = E \sum_n \mathcal{N}^{(\ell)}_{nn'} \chi_{n'}$$

$$\mathcal{H}^{(\ell)}_{nn'} = \langle \Phi_{n\ell} | H | \Phi_{n'\ell} \rangle \quad \mathcal{N}^{(\ell)}_{nn'} = \langle \Phi_{n\ell} | \Phi_{n'\ell} \rangle$$

Spectroscopic factors we discuss:

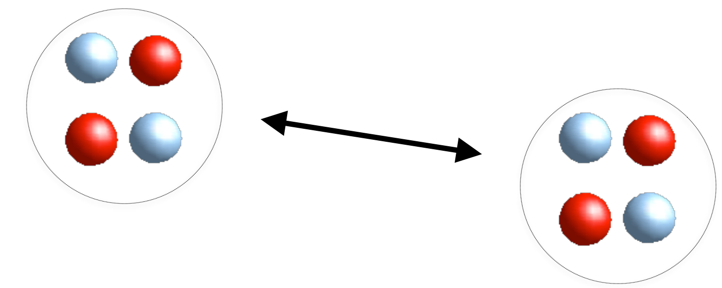
$\Phi_{n\ell}$ Basis channel state (HO relative motion)

$\hat{\mathcal{N}}^{-1/2}\,\Phi_{n\ell}$ Orthonormalized basis channels

$\mathcal{F}_\ell(\rho) = \sum_n \chi_n \Phi_{n\ell}$ RGM solution channels

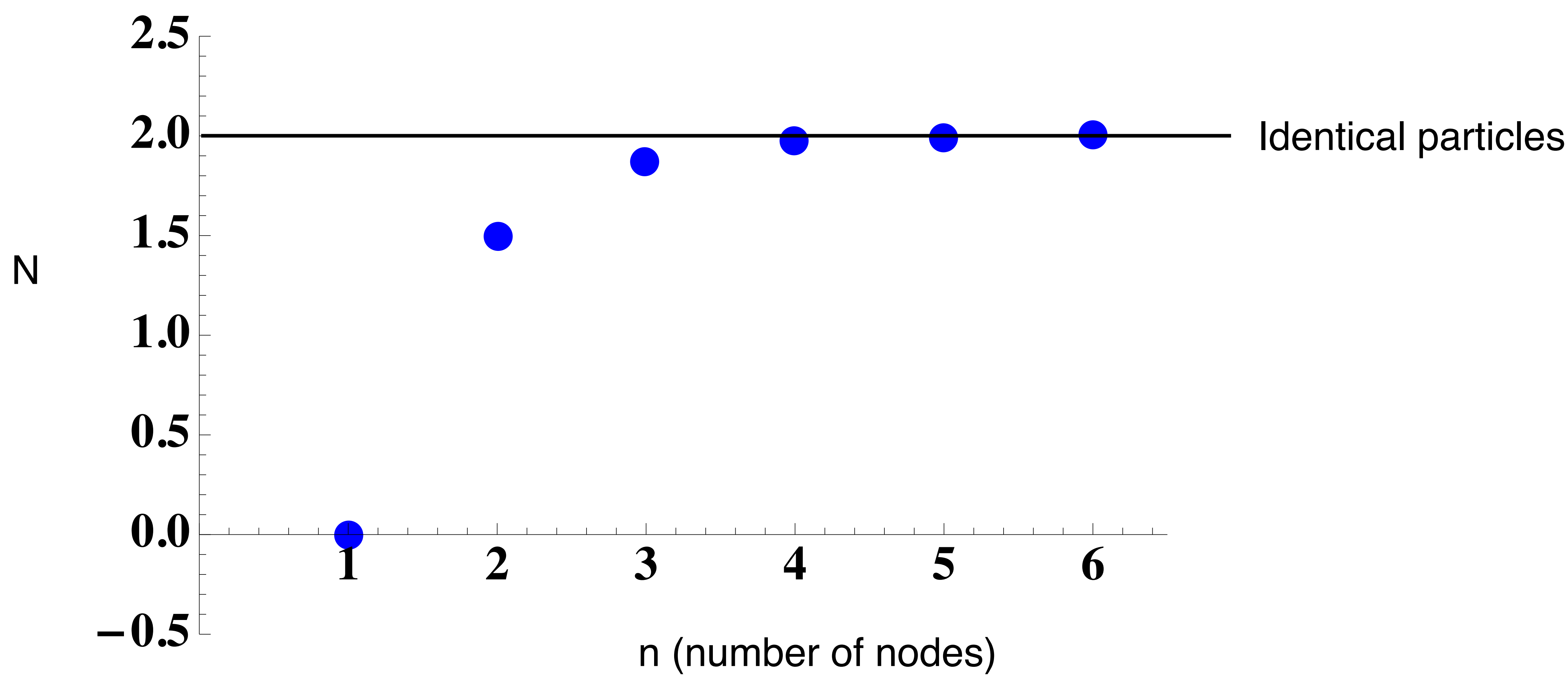
Resonating group method ^{8}Be

$$\mathcal{F}_\ell(\rho) = \sum_n \chi_n \Phi_{n\ell}$$



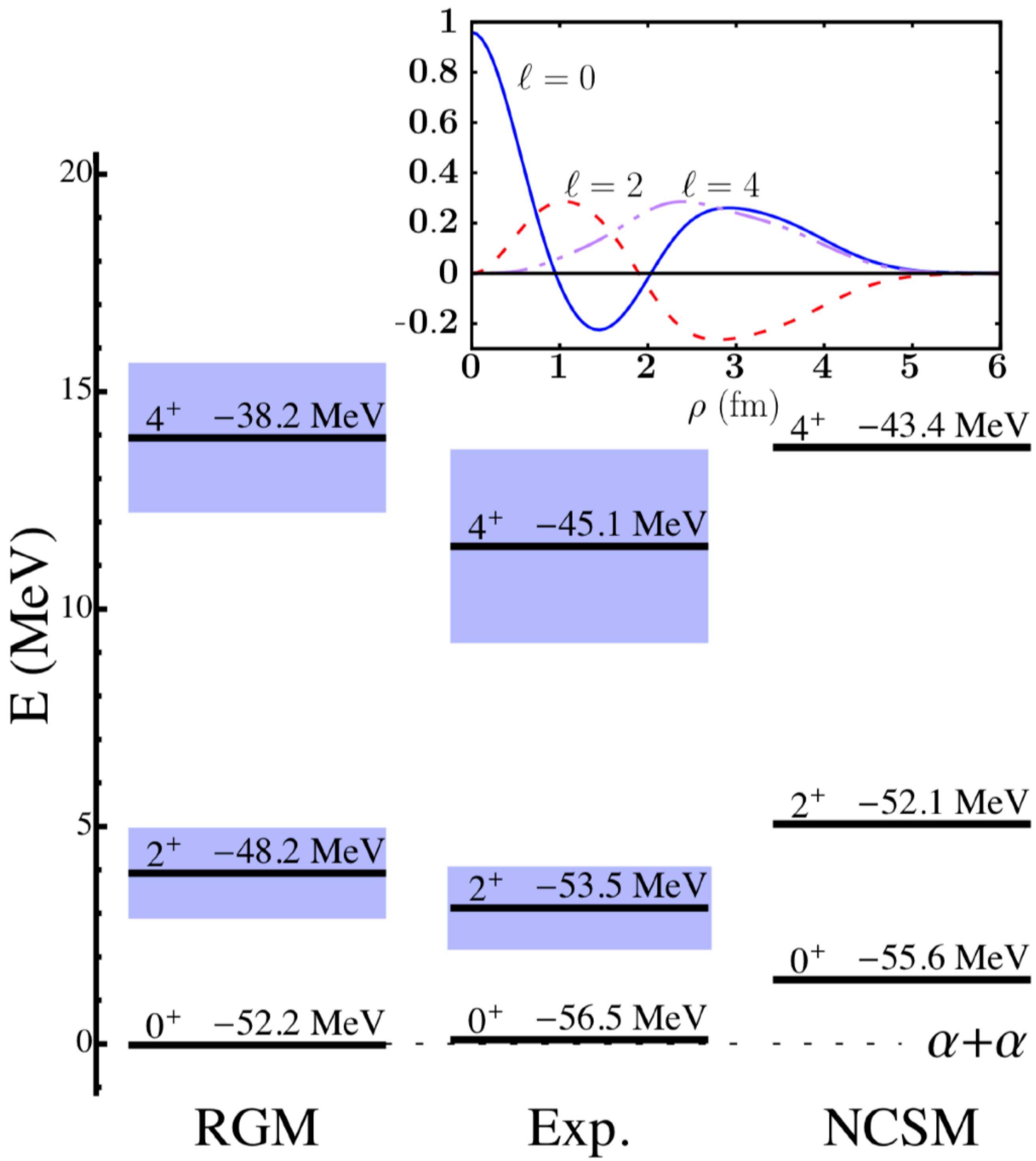
$$\sum_n \mathcal{H}^{(\ell)}_{nn'} \chi_{n'} = E \sum_n \mathcal{N}^{(\ell)}_{nn'} \chi_{n'}$$

$$\mathcal{H}^{(\ell)}_{nn'} = \langle \Phi_{n\ell} | H | \Phi_{n'\ell} \rangle \qquad \mathcal{N}^{(\ell)}_{nn'} = \langle \Phi_{n\ell} | \Phi_{n'\ell} \rangle$$



SU(3) limit verification: Y Suzuki, K.T Hecht Nuclear Physics A455 (1986) 315

Resonating group method ^{8}Be results



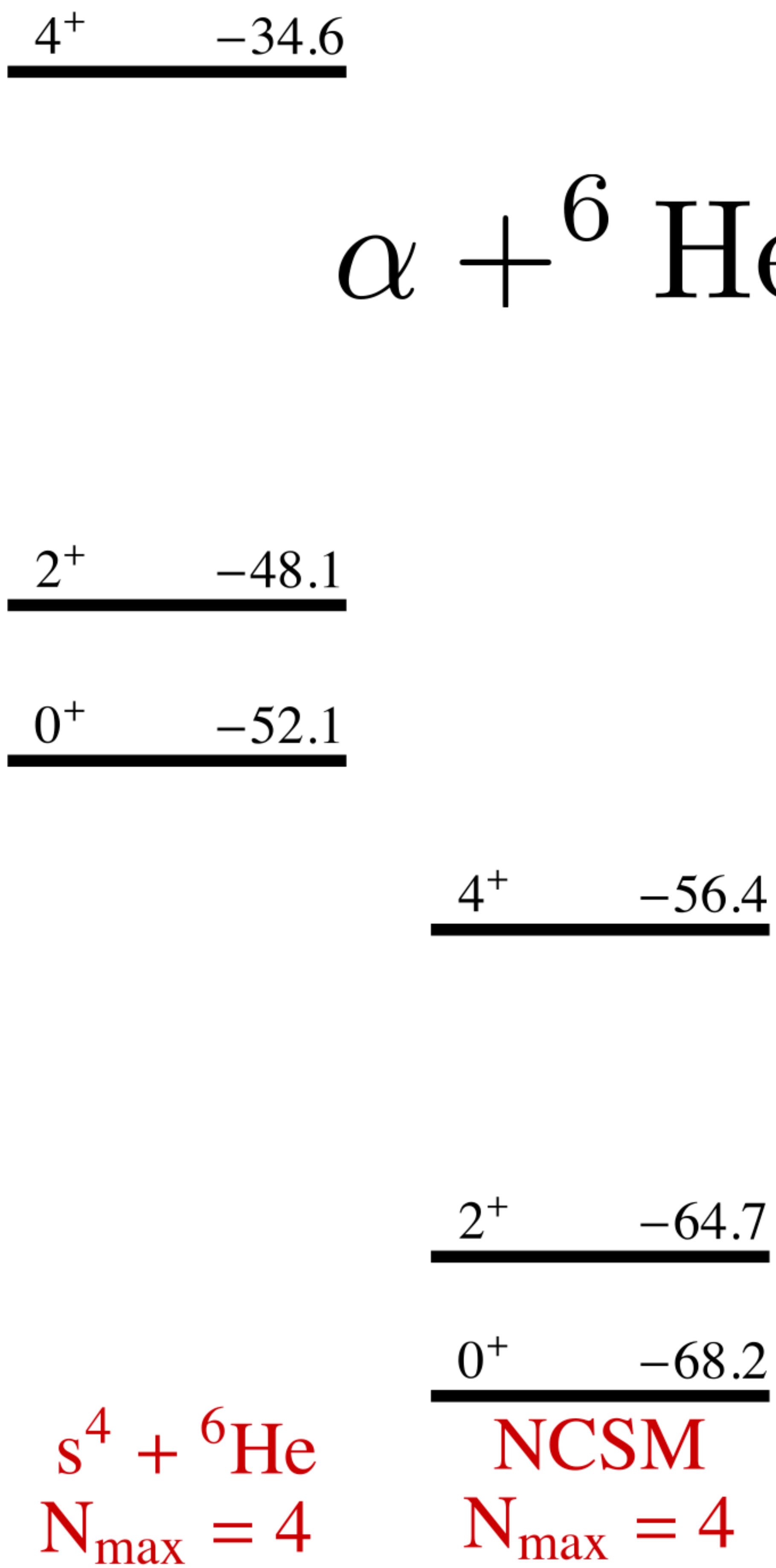
$\hbar\Omega = 25$ MeV

		Theory	Exp.
l=0	ev	8.7	5.6
l=2	MeV	1.3	1.5
l=4	MeV	2.1	3.5

$$\Gamma = 2P_L(\rho_c)|g(\rho_c)|^2$$

K Kravvaris and A. Volya, Phys.Rev.Lett, 119(6), 062501 (2017)

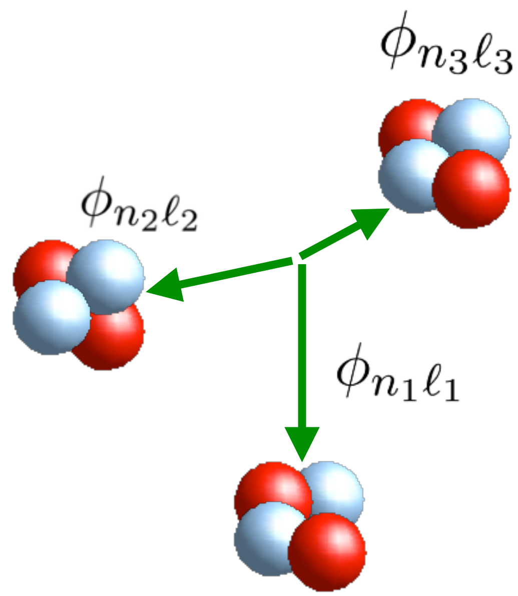
$\alpha + {}^{6}\mathrm{He}$

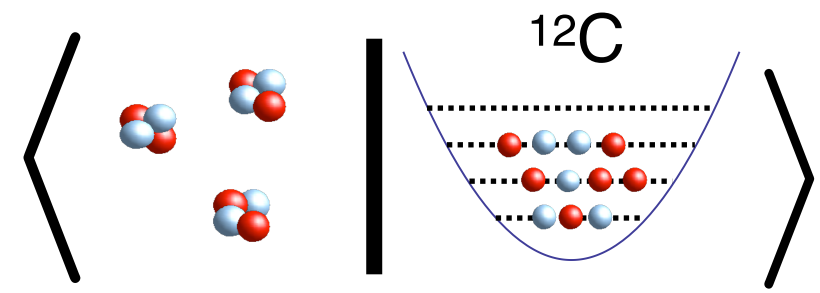


SF comparison for
Nmax = 4 calculation
in ^{10}Be, 4 quanta in relative motion,
hw=25,

	0^+	2^+	4^+
$\mathcal{S}^{(old)}$	0.498	0.404	0.148
$\mathcal{S}^{(new)}$	0.605	0.561	0.303
$\mathcal{S}^{(dyn)}$	0.672	0.633	0.407

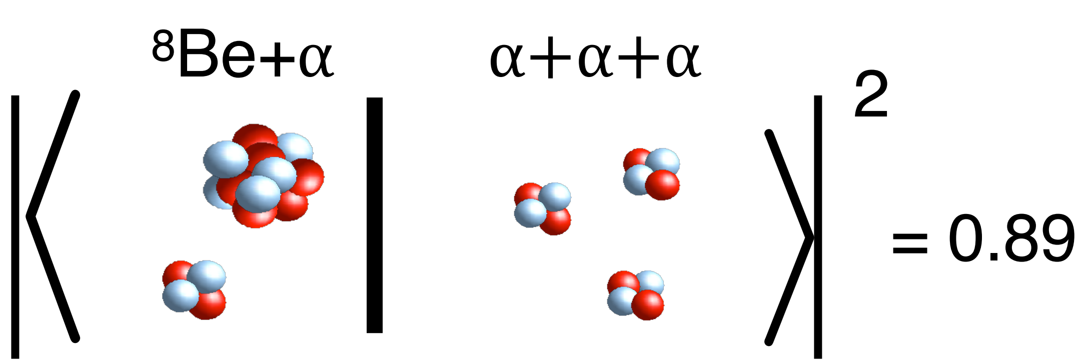
Ttriple-alpha RGM





parent	channel	overlap
$^{12}\mathrm{C}[4](0_1^+)$	$\alpha[0]+\alpha[0]+\alpha[0]$	0.841
$^{12}\mathrm{C}[4](0_2^+)$	$\alpha[0]+\alpha[0]+\alpha[0]$	0.229

N_{max}(rel)=12



Coupling with continuum

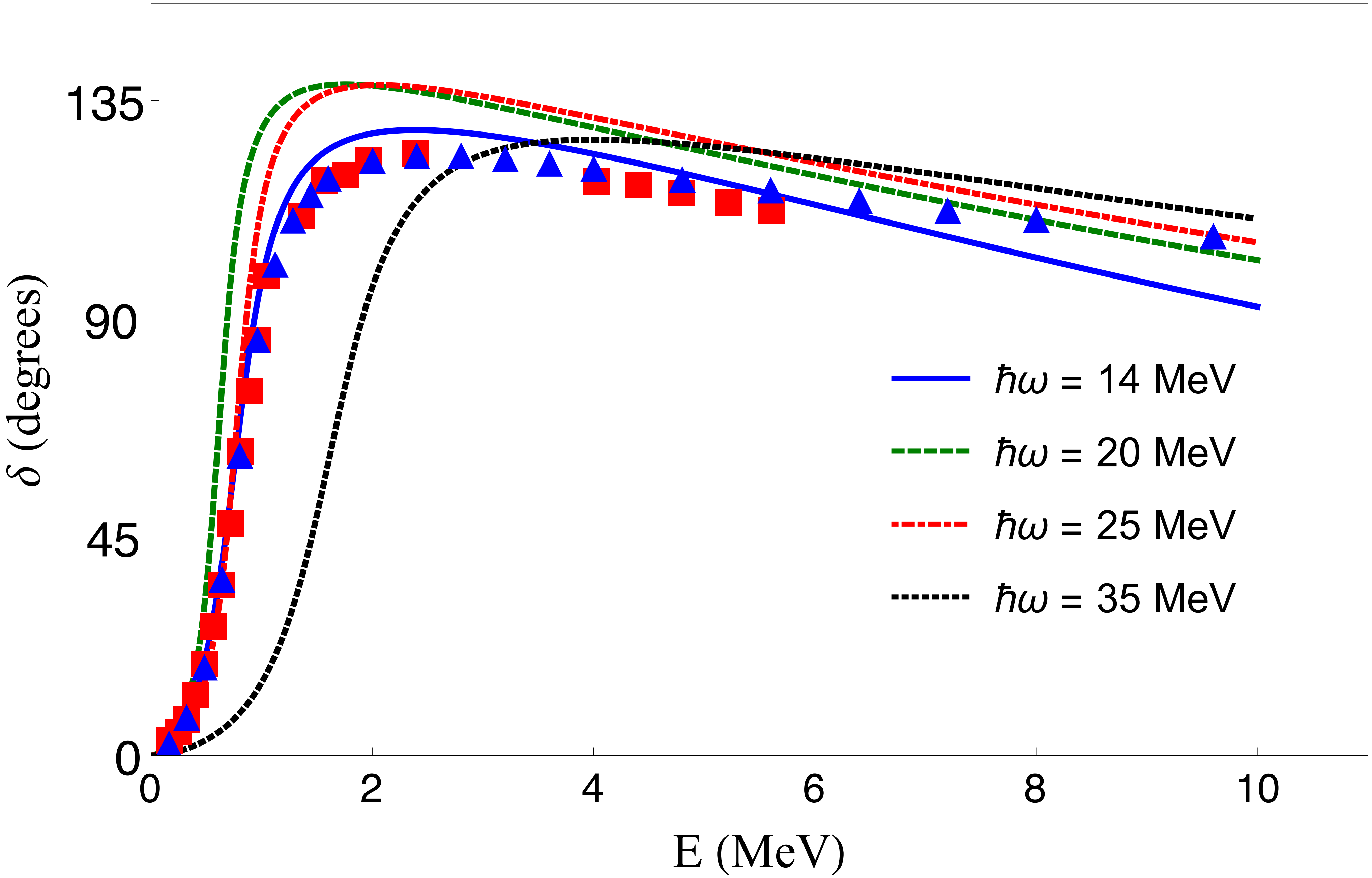
$$\sum_{n} \mathcal{H}^{(\ell)}_{nn'} \chi_{n'} = E \sum_{n} \mathcal{N}^{(\ell)}_{nn'} \chi_{n'}$$

$$\begin{pmatrix} \mathcal{H}_{00} & \dots & \mathcal{H}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{H}_{n0} & \dots & \mathcal{H}_{nn} & T_{nn+1} & 0 & \vdots \\ 0 & 0 & T_{n+1n} & T_{n+1n+1} & T_{n+1n+2} & 0 \\ 0 & \dots & 0 & T_{n+2n+1} & T_{n+2n+2} & \ddots \\ 0 & \dots & \dots & 0 & \ddots & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix} = \mathrm{E} \begin{pmatrix} \mathcal{N}_{00} & \dots & \mathcal{N}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{N}_{n0} & \dots & \mathcal{N}_{nn} & 0 & 0 & \vdots \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \dots & 0 & 0 & 1 & 0 \\ 0 & \dots & \dots & 0 & 0 & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix}$$

Asymptotic solution with phase shift

J-matrix (or HORSE) method: J. M. Bang, Annals of Physics 280, 299 (2000)
Experimental data: Phys. Rev. 168, 1114 (1968); Nucl. Phys. A287, 317 (1977)

n+alpha scattering phase shifts



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Resources: https://www.volya.net/ (see research, clustering)