Event-by-Event Simulations of Early Gluon Fields in High Energy Nuclear Collisions

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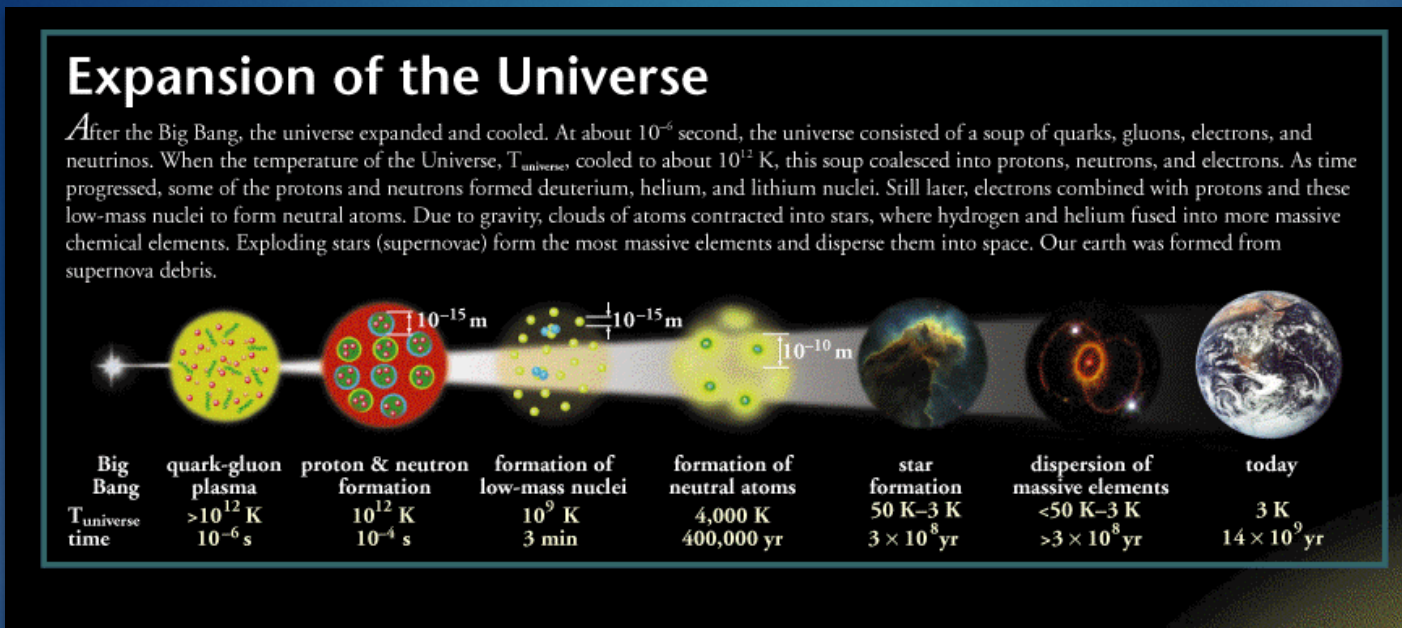
Abstract

- Collisions of heavy ions are carried out at ultra relativistic speeds at the Relativistic Heavy Ion Collider and the Large Hadron Collider to create Quark Gluon Plasma. The earliest stages of such collisions are dominated by the dynamics of classical gluon fields. The McLerran-Venugopalan (MV) model of color glass condensate provides a model for this process. Previous research has provided an analytic solution for event averaged observables in the MV model. Using the High Performance Research Computing Center (HPRC) at Texas A&M, we have developed a C++ code to explicitly calculate the initial gluon fields and energy momentum tensor event by event using the analytic recursive solution. The code has been tested against previously known analytic results up to fourth order. We have also have been able to test the convergence of the recursive solution at high orders in time and studied the time evolution of color glass condensate.

http://www-alice.gsi.de/fsp201/qgp.html



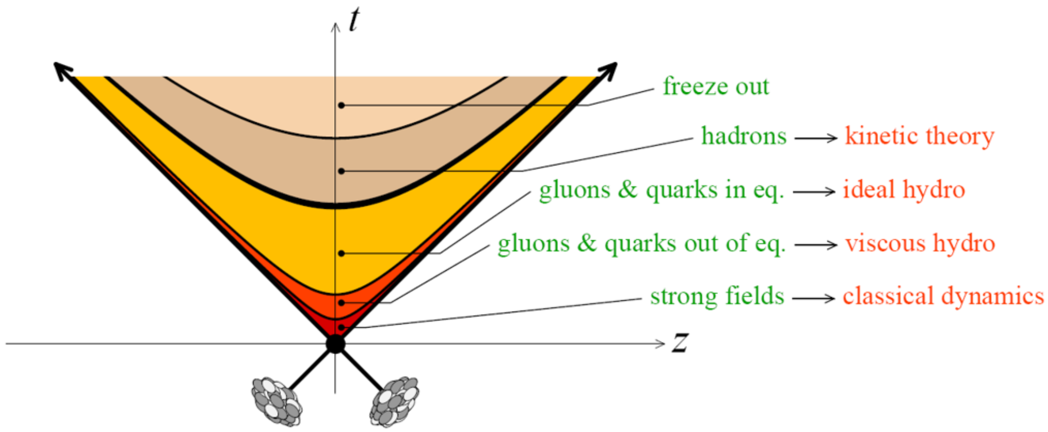
The Grand Scheme

Color Glass Condensate

- Collisions at RHIC and LHC produced a deconfined phase of partons known as Quark Gluon Plasma
- Color Glass Condensate (CGC) models the early times of these high energy collisions[1][2]
- CGC gives the initial conditions to the later, fluid stage of QGP

CGC

- With an Ultra-relativistic approximation the system becomes boost invariant
- Our Simulation models CGC up to time $\frac{1}{Q_s}$
- Q_s^2 is the initial nuclear gluon density



https://arxiv.org/pdf/1312.5497.pdf

The Mathematics of the Model

- At early times, the gluon fields are strong enough to allow for classical approximations
- Our model uses a series expansion to Solve the Yang-Mills Equations
- J^{ν} is the color current

Energy Momentum Tensor

$$T^{\mu\nu} = \sum_{n=0}^{\infty} \tau^{n} T^{\mu\nu}_{(n)}$$

Gauge Gluon Potential

$$A = \sum_{n=0}^{\infty} \tau^{n} A_{(n)}$$

Yang Mills Equation

$$D_{\mu} F^{\mu\nu} = J^{\nu}$$

Math Cont.

- The Boundary Conditions at $\tau = 0$
- A_1 , A_2 are the initial nuclear fields
- The recursive solution

$$A^i_{\perp(0)} = A^i_1 + A^i_2$$

$$A_{(0)} = -\frac{ig}{2}[A^i_1, A^i_2]$$

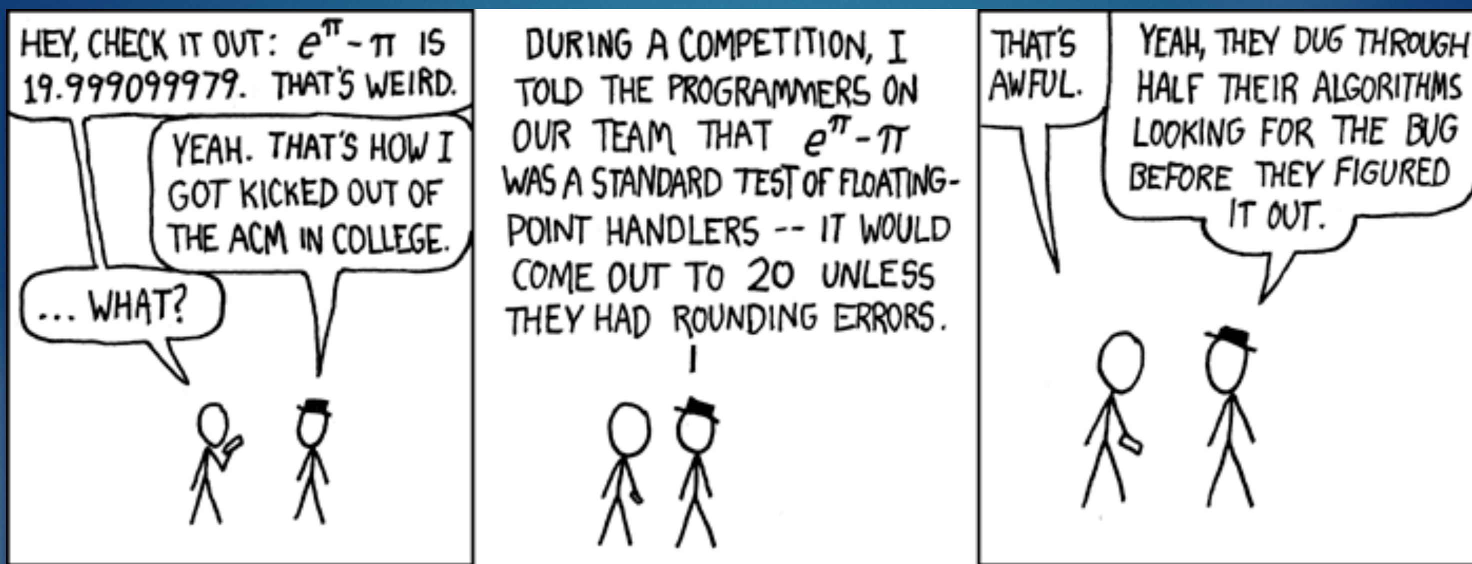
$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n} [D^i_{(k)}, [D^i_{(l)}, A_{(m)}]]$$

$$A^i_{\perp(n)} = \frac{1}{n^2}\left(\sum_{k+l=n-2} \left[D^j_{(k)}, F^{ij}_{(l)}\right] + ig \sum_{k+l+m=n-4} [A_{(k)}, [D^i_{(l)}, A_{(m)}]]\right)$$

Event Generator Code

- Monte Carlo code for event-by-event calculations
  - Samples the transverse color charge density ρ_1 and ρ_2 for each nucleus
  - Computes the nuclear fields A_1^i, A_2^i
  - Calculated the fields after the collision using the recursion relation
  - Computes the Energy Momentum Tensor

https://xkcd.com/217/



The Coding Process

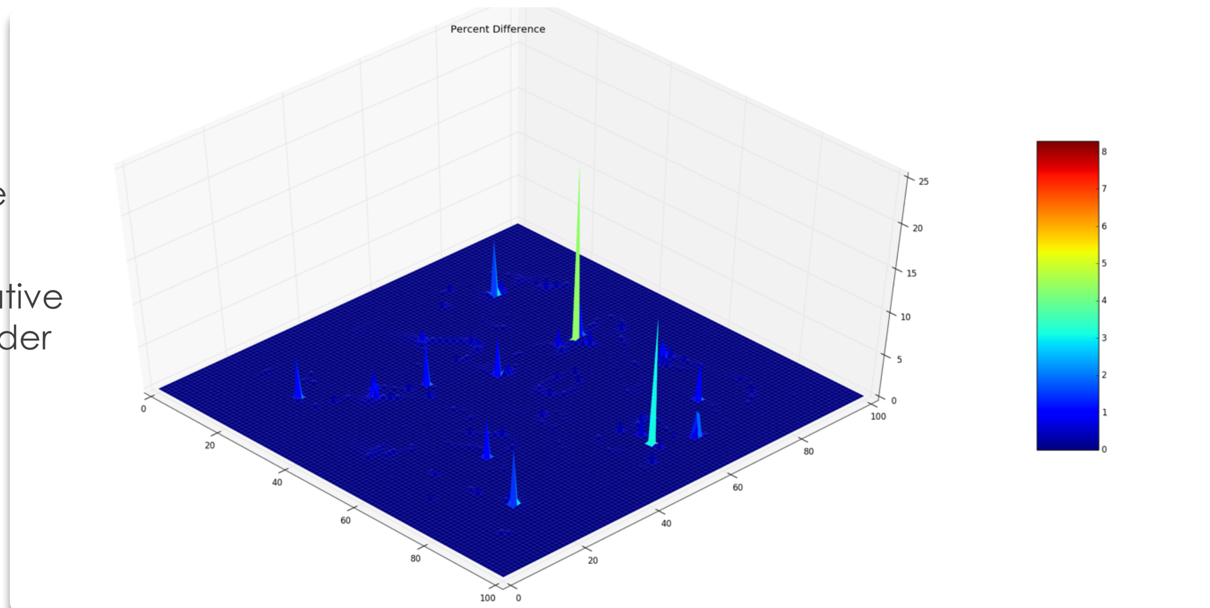
Validation of the Energy Momentum Tensor

- Previous Research by Chen [3] Provided analytic solutions to the Energy Momentum Tensor up to fourth Order
- The analytic solutions were dependent on the initial fields while the recursive solution is not.
- Comparison up to 4th order tests all features of the recursion relation

- Analytic
- $T^{3i}_{(1)} = 2Tr(\epsilon^{ij}([D_i, B_0]E_0 - [D^j, E_0]B_0))$
- Recursive
- $T^{3i}_{(1)} = 2Tr(E_0 E^i_{(1)} - B_0 B^i_{(1)})$

E-M Tensor

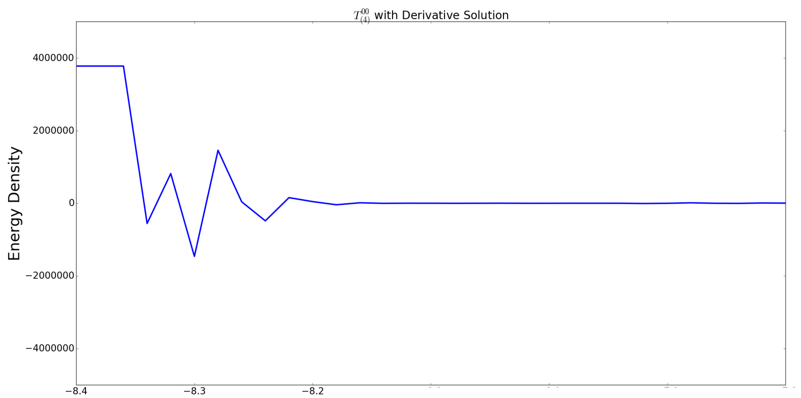
- Final Results showed agreements within one percent
- The graph shows the relative difference of the fourth order energy density

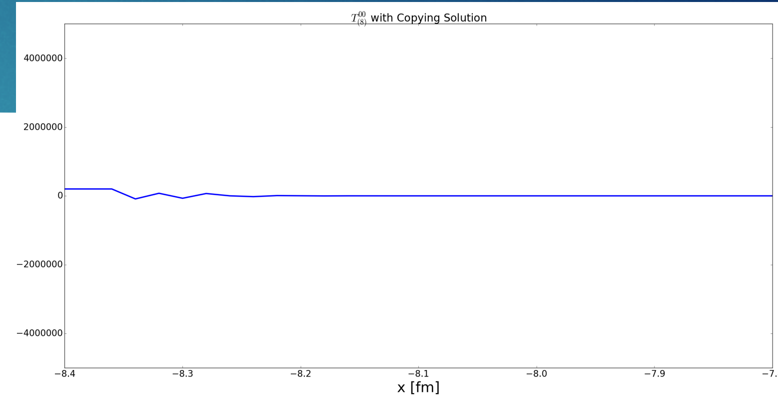


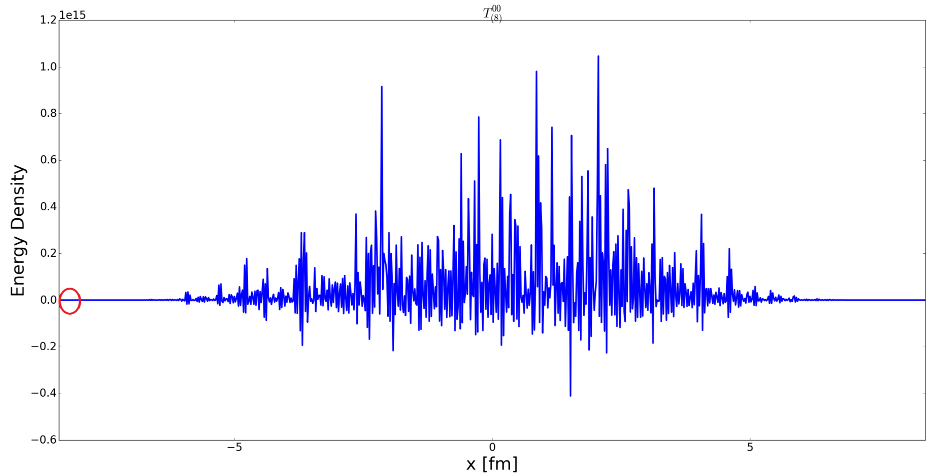
Improving the Boundary

- Our code uses a 4th order accurate derivative.
- $\frac{df(i)}{dx} = \frac{f(i-2)-8f(i-1)+8f(i+1)-f(i+2)}{12h}$
- This lead to a shrinking grid size
- Two potential solutions were tested
- One used a one sided derivative along the edge
- The other simply replaced the boundary points with the first valid point

Boundary Cont.

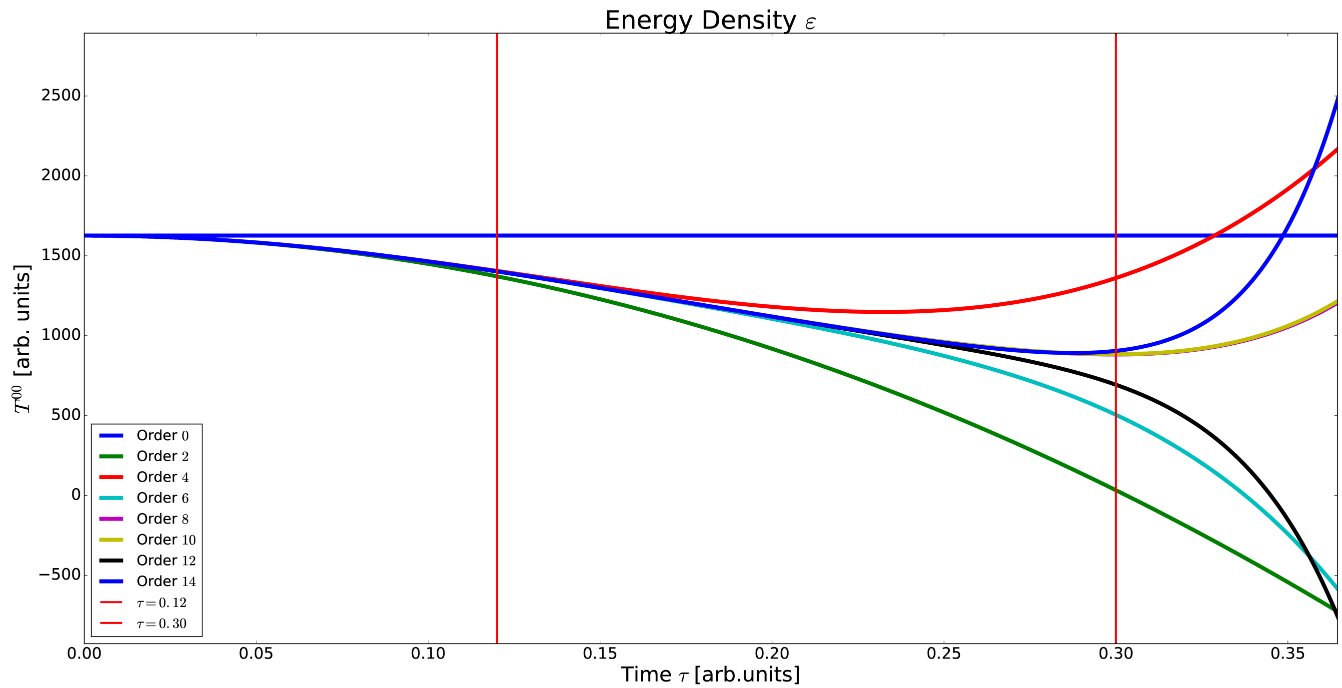




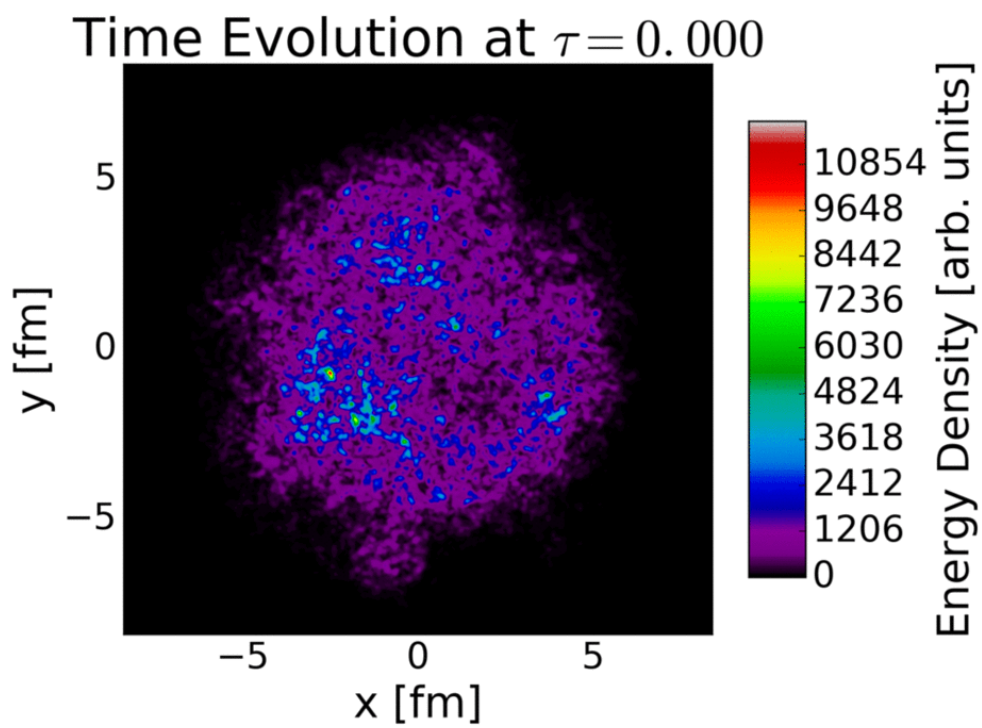


Convergence of Recursion Relation

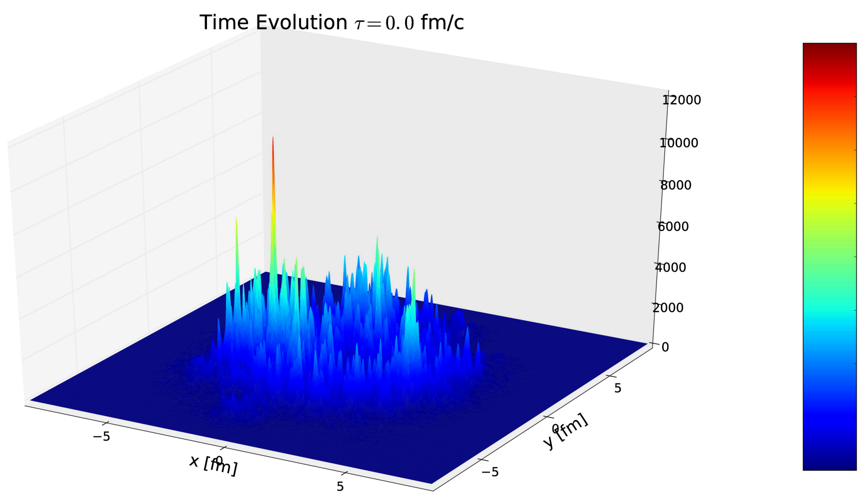
- Going from 4th to 14th order pushes convergence by a factor 2-3 in further in time.

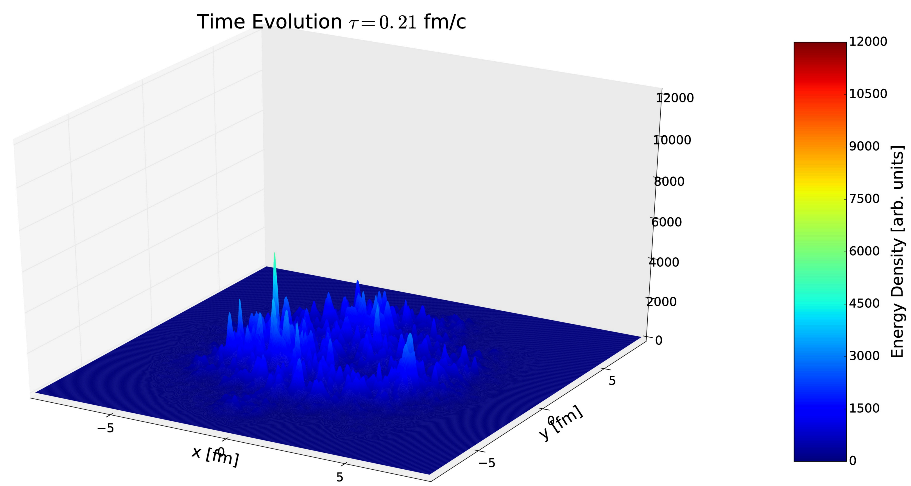


Time Evolution of Energy Density



Time Evolution of Energy Density





Summary and Outlook

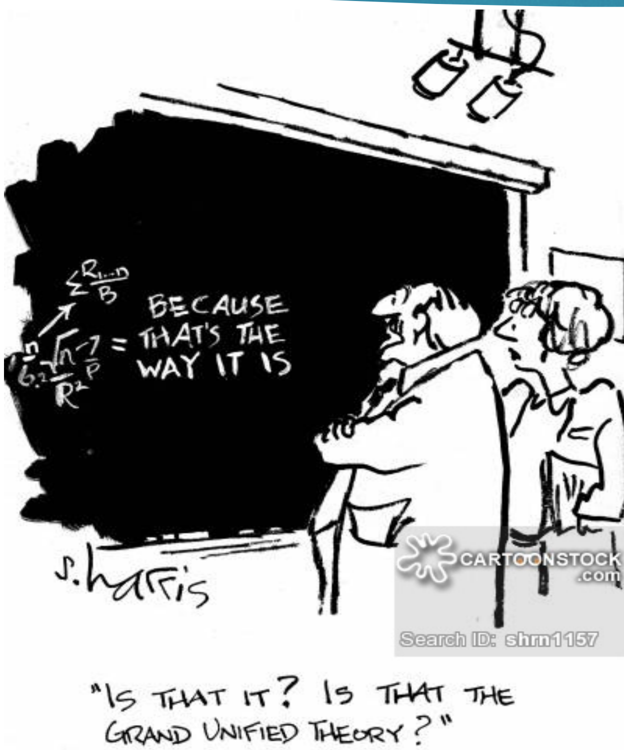
- Recursive solution correctly implemented
- Pushing to higher order improves convergence significantly
- Reduction of the 3D problem to 2D reduces run time compared to previous implementations
- Resulting energy momentum tensor can be input into fluid dynamics in the near future.

References

- [1] McLerran et al Phys. Rev. D 49, 2233 (1994)
- [2] Kovner et al Phys. Rev. D 52, 6231 (1995)
- [3] Chen et al Phys. Rev. C 92, 064912 (2015)

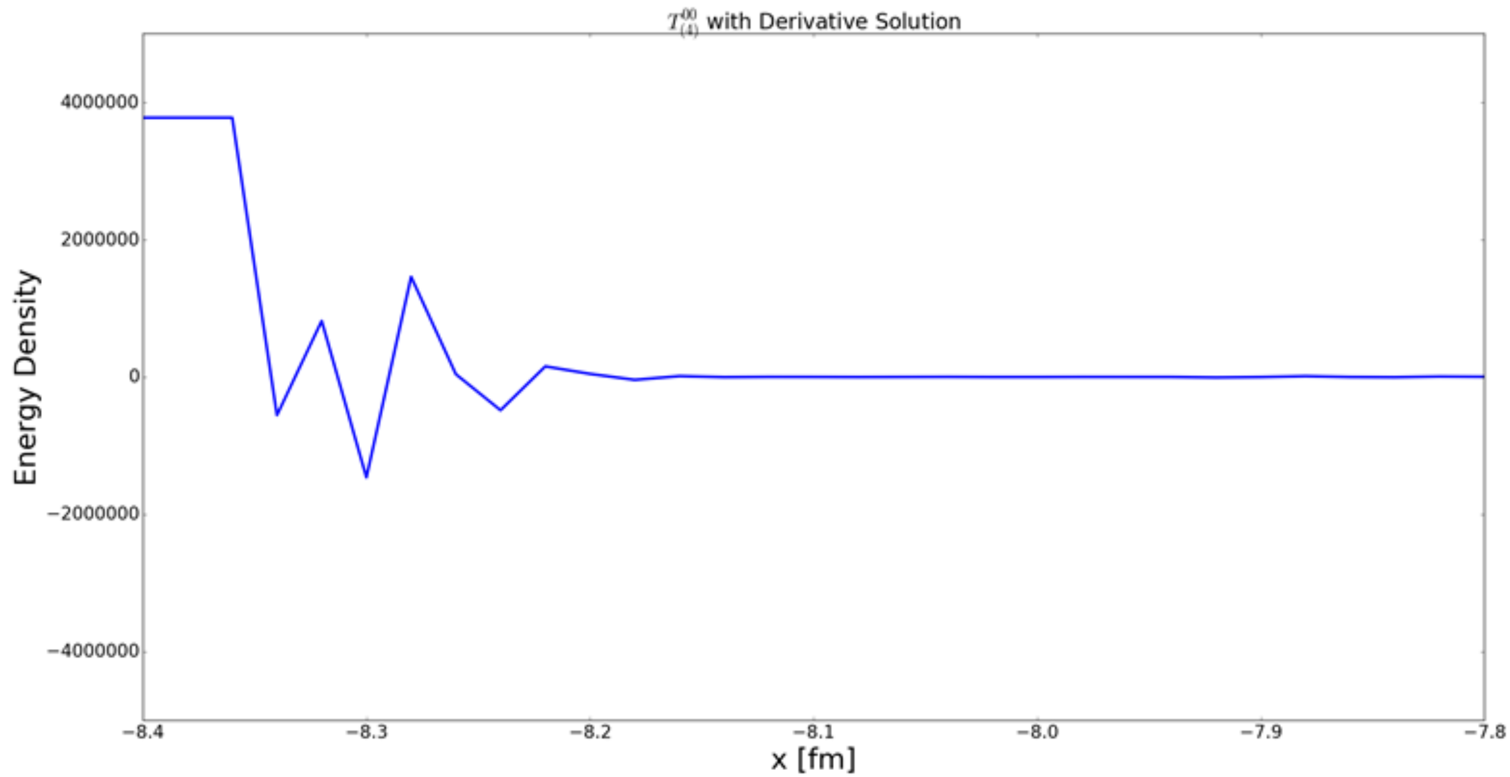
Acknowledgements

Question I happen not to know

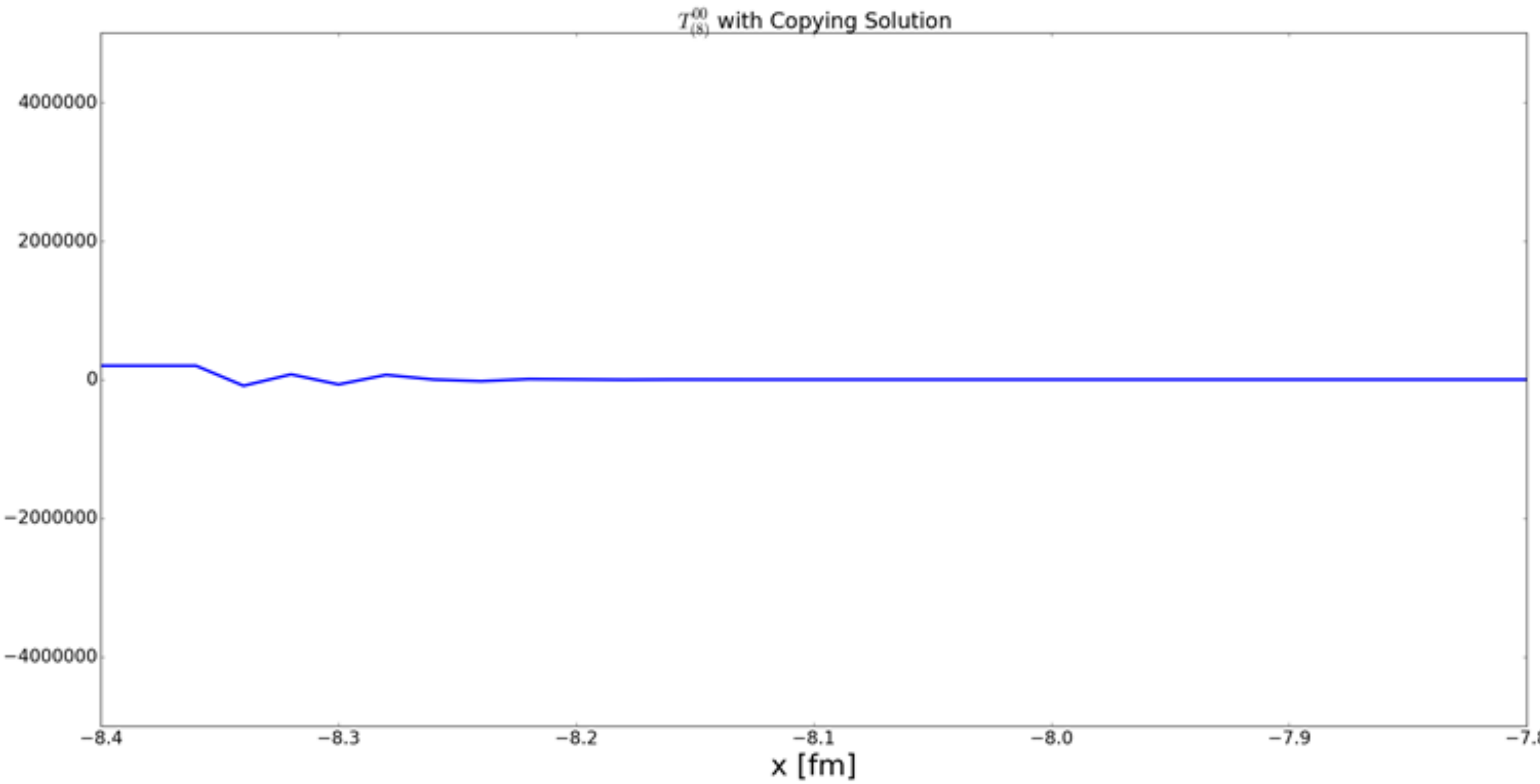


https://www.cartoonstock.com/directory/g/grand_unified_theory.asp

Derivative


$T^{00}_{(4)}$ with Derivative Solution
Energy Density
4000000
2000000
0
−2000000
−4000000
−8.4
−8.3
−8.2
−8.1
−8.0
−7.9
−7.8
x [fm]

Copying



$T^{00}_{(8)}$ with Copying Solution
4000000
2000000
0
−2000000
−4000000
−8.4
−8.3
−8.2
−8.1
−8.0
−7.9
−7.8
x [fm]

Full