Comprehensive Parameterization of the ρ-Meson Spectral Function in Hot and Dense Matter

Thomas Onyango
Ralf Rapp

Supported by NSF under grant nos. PHY-1614484 & PHY - 1659847 and DOE under grant DE-FG02-93ER40773

ABSTRACT

The goal of this research is to study how hadronic matter transitions into quark-gluon plasma. This transition is believed to have occurred in the early universe about 10 microseconds after the big bang. It created more than 95% of the visible mass in the universe and confined quarks and gluons into hadrons. Hot nuclear matter can be recreated in the laboratory by colliding heavy atomic nuclei at very high energies. The transition into the quark-gluon plasma can be probed by analyzing the invariant-mass distributions of ρ-mesons. The ρ-meson was chosen because it decays into dilepton pairs, e.g. e^+e^- or $\mu^+\mu^-$. Dilepton pairs are an excellent observable because they do not interact through the strong nuclear force inside the strongly interacting fireball, therefore ρ-mesons which decay into dileptons in the medium can be measured during heavy ion collisions and used to study its properties in hot and dense matter [1]. In this project, we developed a parameterization of this process which will help to describe quark-gluon plasma which filled the early universe. The original theoretical calculations [2] involve rather complex integrals. The purpose of the parametrization is to make the ρ-meson spectral function readily available applications in a compact form over a wide range of conditions.

BACKGROUND

Quantum mechanics describes the composition of the world around us at atomic and sub-atomic scales. It has helped to form the periodic table of atomic nuclei and the standard model of elementary particles, see Fig. 1. The particles described in this research are the ρ-meson which is composed of a quark-antiquark pair ($d\bar{d}$ or $u\bar{u}$) as shown in Fig. 2, charged pions which are also composed of a quark-antiquark pair ($u\bar{d}$ or $\bar{u}d$), electrons , muons, and gluons. Our preferred decay of the ρ-meson is into a dilepton pair, but a pion pair is usually the outcome as a result of the strong nuclear force, see Fig. 3.

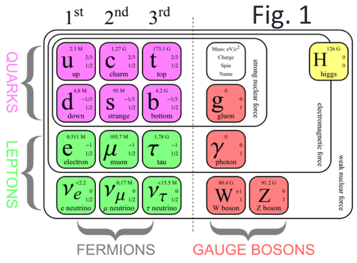

Fig. 1

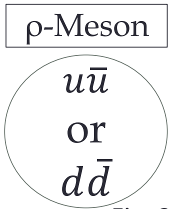

Fig. 2

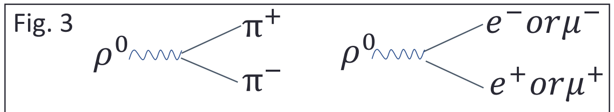

Fig. 3

99% Occurrence 1% Occurrence

DILEPTON PRODUCTION RATE

$$(1)\ \frac{dN_{ee}}{d4_x d4_q} = constant * \frac{1}{M^2} * e^{-q_0/T} * Im\left[D_\rho\left(M, q, T, \frac{\rho}{\rho_0}\right)\right]$$

Eq. (1) is the rate of the dilepton pairs produced by the heavy ion collisions that produce hot dense matter. The above relation highlights how the dilepton production rate depends on the ρ-meson spectral function.

SPECTRAL FUNCTION

$$(2)\ D_\rho(M,q,T) = \frac{1}{M^2 - \left(m_\rho^{(0)}\right)^2 - \Sigma_{\rho\pi\pi}(M,q,T) - \Sigma_{\rho M}(M,q,T)}$$

$$(3)\ \Sigma_{\rho\pi\pi}(M,q,T) = -aM - iM\Gamma_\rho(M) \qquad (4)\ a = 0.162 GeV \qquad (5)\ \Gamma_\rho(M) = \left\{M \le 0.4, \frac{g_\rho^2}{6\pi} * \frac{M}{8} * \left(\frac{M}{0.4}\right)^4; M > 0.4, \frac{g_\rho^2}{6\pi} * \frac{M}{8}\right\}$$

$$(6)\ \Sigma_{\rho M}(M,q,T) = -im_\rho\left(\frac{T}{0.15}\right)^{\alpha_I}(\Gamma_1 + \Gamma_2) + Re(\Sigma_{\rho M}) \qquad (7)\ \Gamma_1 = \Gamma_0 * \left(\frac{M+0.1}{1.3}\right)^5\left(\frac{\Lambda_1^2 + m_\rho^2}{\Lambda_1^2 + M^2}\right)^8 \qquad (8)\ g_\rho = 5.86$$

$$(9)\ \Gamma_2 = \left[b(M) * \left(\frac{q}{0.4}\right)^3 + (1 - b(M))\left(\frac{q}{0.15}\right)^2\right]\left(\frac{\Lambda_2^2 + m_\rho^2}{\Lambda_2^2 + q^2}\right)^2 * \frac{0.001}{1 + 25 * M^6} \qquad (10)\ b(M) = \frac{1}{e^{\left(\frac{M-0.2}{0.1}\right)} + 1} \qquad (11)\ m_\rho = 0.77 GeV$$

$$(12)\ \mathrm{Re}(\Sigma_{\rho M}) = \left\{M < 0.75,\ (-0.01)c_m m_\rho\left(\frac{T}{0.15}\right)^{\alpha_R};\ 0.75 < M < 1.2,\ c_m m_\rho\left(\frac{T}{0.15}\right)^{\alpha_R}\left(\frac{M-0.85}{10}\right);\ 1.2 < M,\ (0.035)c_m m_\rho\left(\frac{T}{0.15}\right)^{\alpha_R}\right\}$$

$\Sigma_{\rho\pi\pi}$ is the vacuum self − energy of the ρ−Meson describing its propagation through the vacuum including the pion pion interaction that the ρ−Meson eventually decays into. The dilepton spectra that the ρ−Meson decays into are calculated using $\mathrm{Im}(D_\rho)$, see Eq. (1). Once the ρ−Meson starts to travel through a medium the spectral function activates the in-medium self-energy $\Sigma_{\rho M}$ which encodes the changes in the spectral properties through its interactions with a hot medium of mesons.

PARAMETERIZATION

Starting from the parameterizations given in Eqs. (3)-(12), we fit momentum dependent parameters to the spectral function data at finite temperature, as listed below. The process we used to create the fit functions of the parameters was to first leave them as free parameters during initial fit at each momentum q. Then all of the parameters at individual momenta were fit using Mathematica's nonlinear regression model package. We applied constraints so that the results were physically significant. Then we plot the values of the parameters at the individual momenta as data points on a plot. After that, we interpolate them to acquire a fit function. Finally we would replace the free parameters with the fit functions and test the spectral function to make sure that we receive the same values for the self energy as when using original parameters, see Fig. 4 and Fig. 5. For the temperature dependence a T- and q-dependent power law was found to be adequate, see Fig. 6 and Fig. 7 for comparison.

$$c_m = 2.0166 - 2.179 * q \qquad \alpha_R = \left\{q < 0.5, 5.02569\left(1 - \frac{q}{0.5}\right); q \ge 0.5, 0\right\} \qquad \alpha_I = \left\{q \ge 0.5,\ 3.93722;\ q < 0.5,\ 10.3502 - \frac{10.3502 - 3.9372}{0.5} * q\right\}$$

$$\Lambda_1 = 1.235 + e^{1.3907 - 2.8 * q} \qquad \Lambda_2 = 0.68997 + 78498 * \frac{q^{0.000015} - 18.584 * 0.0538}{(q^{-1.4774} - 24.2505 * 0.0538)^2 + 0.17} \qquad \Gamma_0 = 0.19 + 218309 * \frac{1 - q^{8.121 * 10^{-6}}}{\left(1 - q^{-3.086}\right)^2 + 0.4533^2} + 0.2005 * q^2$$

SELF-ENERGY

Self-Energy (GeV^{-2}) vs Mass (GeV)

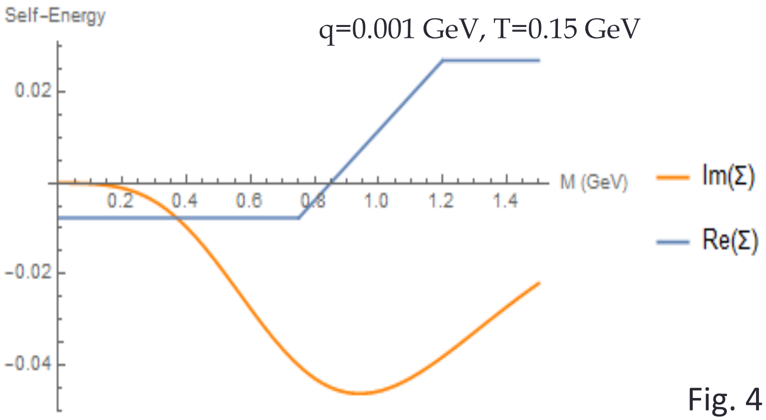

Fig. 4

IN-MEDIUM PROPAGATION

Imaginary Part of the Spectral Function (GeV^{-2}) vs Mass (GeV)

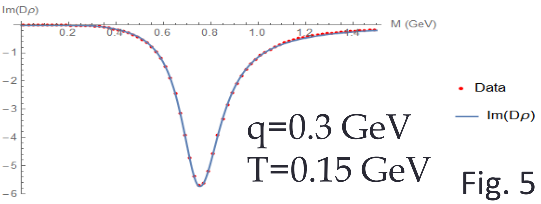

Fig. 5

RATE OF THE SPECTRAL FUNCTION VS. MESON INTERACTION DATA

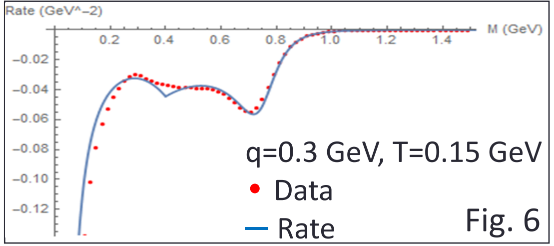

Fig. 6

RATE OF THE SPECTRAL FUNCTION VS. MESON INTERACTION DATA

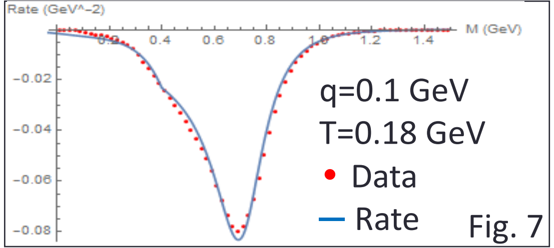

Fig. 7

FUTURE PLANS

Complete the nuclear density dependency in baryon interaction term of the spectral function

Use the spectral function to probe quark-gluon plasma to analyze the plasma's properties

REFERENCES

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[2] R.Rapp and J. Wambach, Eur. Phys. J. A6 (1999) 415
[3] http://www.physik.uzh.ch/groups/serra/images/SM1.png