Toward Microscopic Equations of State for Core-Collapse Supernovae from Chiral Effective Field Theory

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Abstract

Chiral effective field theory provides a modern framework for understanding the structure and dynamics of nuclear many-body systems. Our aim is to extend the application of chiral effective field theory to describe the nuclear equation of state required for supercomputer simulations of core-collapse supernovae. Given the large range of densities, temperatures, and proton fractions probed during stellar core collapse, microscopic calculations of the equation of state require large computational resources on the order of one million CPU hours. We investigate the use of graphics processing units (GPUs) to significantly reduce the computational cost of these calculations, which will enable a more accurate and precise description of this important input to numerical astrophysical simulations.

Core-Collapse Supernovae

Stars between $10 - 30\, M_{\odot}$ reach sufficiently high temperatures to fuse nuclei up to iron. Once the iron core reaches the Chandrasekhar mass of $1.4\, M_{\odot}$, it can no longer be supported through electron degeneracy pressure and undergoes gravitational collapse.

Short-range nuclear forces acting over femtoscale distances eventually halt the collapse, which results in a shockwave that rebounds against the inward-falling matter above. Electron capture in the core releases abundant neutrinos that deposit additional energy behind the shockwave, potentially leading to a successful supernova explosion.

The nuclear equation of state plays a critical role in simulating stellar core collapse and the hydrodynamic evolution of the resulting supernova explosion.

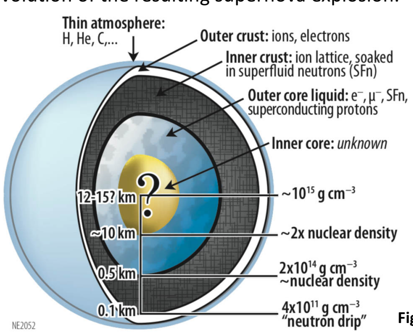

Figure 1

Hot and Dense Matter Equation of State

Realistic two- and three-body chiral nuclear forces are used to calculate the free energy of nuclear matter at varying temperature, density, and composition. The free energy per nucleon in infinite homogeneous nuclear matter can be expanded in perturbation theory as follows:

$$\bar{F}(T,\rho,\delta) = \bar{F}_o(T,\rho,\delta) + \lambda\bar{F}_1(T,\rho,\delta) + \lambda^2\bar{F}_2(T,\rho,\delta) + \lambda^3\bar{F}_3(T,\rho,\delta) + \mathcal{O}(\lambda^4)$$

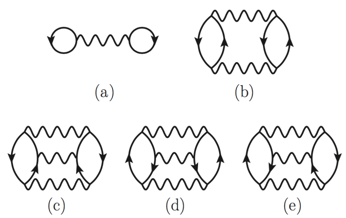

Figure 2: First-, second-, and third- order diagrammatic contributions to the ground state energy density of nuclear matter by Holt et. al.[1] The wavy lines indicate the (antisymmetrized) density-dependent NN interaction derived from chiral two- and three-body forces.

First-, second-, and third-order contributions to the energy density ρE:

$$\textbf{(1)}\ \rho E^{(1)} = \frac{1}{2}\sum_{12} n_1 n_2 \langle 12|(\bar{V}_{NN} + \bar{V}_{NN}^{med}/3)|12\rangle, \quad \textbf{(2)}\ \rho E^{(2)} = -\frac{1}{4}\sum_{1234} \left|\langle 12|\bar{V}_{eff}|34\rangle\right|^2 \frac{n_1 n_2 \bar{n}_3 \bar{n}_4}{e_3 + e_4 - e_1 - e_2}$$

$$\textbf{(3)}\ \rho E_{pp}^{(3)} = \frac{1}{8}\sum_{123456} \langle 12|\bar{V}_{eff}|34\rangle\langle 34|\bar{V}_{eff}|56\rangle\langle 56|\bar{V}_{eff}|12\rangle \times \frac{n_1 n_2 \bar{n}_3 \bar{n}_4 \bar{n}_5 \bar{n}_6}{(e_3 + e_4 - e_1 - e_2)(e_5 + e_6 - e_1 - e_2)}$$

$$\textbf{(4)}\ \rho E_{hh}^{(3)} = \frac{1}{8}\sum_{123456} \langle 12|\bar{V}_{eff}|34\rangle\langle 34|\bar{V}_{eff}|56\rangle\langle 56|\bar{V}_{eff}|12\rangle \times \frac{\bar{n}_1 \bar{n}_2 n_3 n_4 n_5 n_6}{(e_1 + e_2 - e_3 - e_4)(e_1 + e_2 - e_5 - e_6)}$$

$$\textbf{(5)}\ \rho E_{ph}^{(3)} = -\sum_{123456} \langle 12|\bar{V}_{eff}|34\rangle\langle 54|\bar{V}_{eff}|16\rangle\langle 36|\bar{V}_{eff}|52\rangle \times \frac{n_1 n_2 \bar{n}_3 \bar{n}_4 n_5 \bar{n}_6}{(e_3 + e_4 - e_1 - e_2)(e_3 + e_6 - e_2 - e_5)}$$

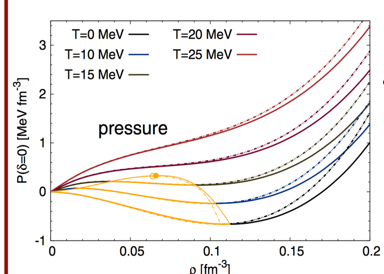

Figure 3: Pressure in isospin symmetric nuclear matter for temperatures in the range $T = 0 - 25 MeV$ by Wellenhofer et. al.[2]

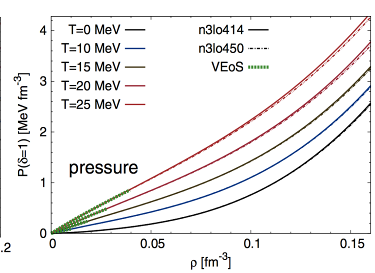

Figure 4: Pressure in pure neutron matter by Wellenhofer et. al.[2]

Numerical Optimization on GPUs

We explore the possibility to calculate the nuclear equation of state using massive parallel computing on GPU accelerators. The GPU architecture allows for thousands of independent threads to execute simultaneously, which may lead to significantly improved computational efficiency of current codes.

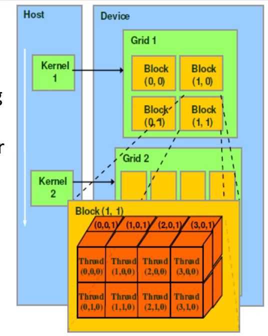

Figure 5: CUDA Memory Model

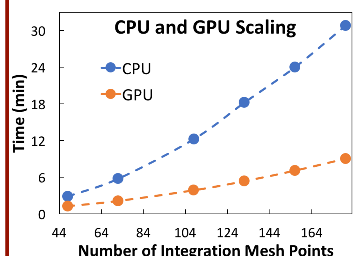

Figure 6: The plot to the left compares GPU and CPU runtime as a function of desired numerical precision (encoded in the number of integration mesh points). The GPU code exhibits a much more favorable scaling compared to the optimized CPU version.

Conclusions and Future Work

The Current GPU implementation offers a more efficient algorithm than the original CPU program, especially the highest numerical precisions achieved. This provides a first step toward faster calculations needed for constructing equation of state tables for astrophysical simulations.

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References

1. J. W. Holt, PRC **95** 034326 (2017)
2. C. Wellenhofer, PRC **92** 015801 (2015)

Figures:

1. Figure 1: https://sites.google.com/site/cudapros/website-builder
2. Figure 5: https://heasarc.gsfc.nasa.gov/docs/nicer/nicer_about.htm