Quark Condensate in Neutron Matter from Chiral Effective Field Theory

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Motivation

- We want to know what goes on inside highly dense astrophysical objects, specifically neutron stars.
- Neutron stars – the final stage of a massive star (~10-30 $M_\odot$) fit inside the width of Rhode Island three times over.
- Not much is known about what happens within the inner core of such dense neutron material.
Background

- Looking at quark properties shines light on the properties of highly dense neutron matter
- QCD asymptotic freedom suggests chiral symmetry restoration at high densities
  - Asymptotic freedom says that quarks interact weakly at high energies, not allowing them to form a condensate
- Chiral symmetry – the application of symmetry to the “handedness” of a gauge theory (in this case, the field theory QCD)
  - An approximate theory of the strong interactions based on vanishing quark masses (up and down quarks are so light next to hadronic scales)
- Chiral Condensate ($\langle \bar{q}q \rangle$) – a scalar quark density value which, when vanishing, means chiral symmetry is being restored
  - Analyzing the behavior of this condensate sheds light on the symmetry restoration phase transition of neutron matter
Chiral Symmetry Breaking Analogy

• Analogy – spontaneous magnetization in ferromagnetic material
  ▫ Spontaneous magnetization is when ferromagnetic material (below Curie temperature) with no magnetic field applied gain an ordered spin state
• Dependence on Curie temperature, where the symmetry breaking happens
  ▫ Above which ferromagnetic material becomes paramagnetic and the spins (spin waves or magnons) within the material have spherical symmetry
  ▫ Below which the direction of the spins follow a preferred axis, or the magnetization direction
• As temperature decreases, ferromagnetic material’s spherical symmetry vanishes
  ▫ Looking at these temperature limits \((T \to 0 \text{ and } T \to T_C)\) shows the behavior of this symmetry breaking and restoration
  ▫ Similar to looking at chiral symmetry breaking and restoration
Chiral Effective Theory & Perturbation Theory

ChEFT - putting together nuclear force order-by-order
• Top figure: potential (force inferred) relation to distance
MBPT – solving for the quantum mechanical ground state of a many-body system
• First-order: expectation value of the chiral potential in the non-interacting neutron matter ground state
• Second-order: matrix elements of the potential connecting the non-interacting ground state and the excited states of the non-interacting system
• Bottom figure: perturbation theory diagrams included in present work


Pure Neutron Matter EOS

- Pure neutron matter equation of state at finite temperatures
  - Looking at the free energy and pressure of pure neutron matter (with isotherms)
  - Benchmark with virial EOS at low densities
  - This project is similar with in-medium neutron matter

Deriving the Chiral Condensate

\[ \rho F = 2 \int_0^\infty dpK_1 n(p) + \int_0^\infty dp_1 \int_0^\infty dp_2 K_2 n(p_1)n(p_2) + \int_0^\infty dp_1 \int_0^\infty dp_2 \int_0^\infty dp_3 K_3 n(p_1)n(p_2)n(p_3) \]

\[ \frac{\partial K_1}{\partial m_\pi^2} = \frac{\sigma_N}{m_\pi^2} \left( 1 + \frac{3\rho}{2m_N \Omega_0} + \frac{p^2}{3m_N^2} + \frac{3p_4}{8m_N^4} \right) \]


- Start with free energy equation based on the different many-body interaction contributions
  - \( K_1 \) is the leading contribution to the free energy density above
  - \( K_1 \) dependence on the light quark mass, which is equivalently the pion mass, is what matters

- **Hellmann-Feynman theorem**
  - This expectation value is the chiral condensate and the parameter is the pion mass squared

\[ \langle \psi(\lambda) | \frac{d}{d\lambda} H(\lambda) | \psi(\lambda) \rangle = \frac{d}{d\lambda} E(\lambda) \]

Zero Temperature Chiral Condensate

Krüger et al. analysis with free energy and interaction energy derivatives

Analysis with just free energy derivative

Thermodynamic Properties

- Chiral condensate vanishing with increasing density
- Approach to zero is quickened with higher temperature systems
- Higher temperatures favor chiral symmetry
- Increasing densities seem to delay symmetry restoration
Contribution Analysis

Omega0 Contribution – first order noninteracting term found to be the dominant term
Omega1 Contribution – change in condensate with respect to the one-body term
Omega2 Contribution – change in condensate with respect to the two-body interaction term

Although the dominant term is the noninteracting term, this analysis reinforces the idea that nuclear interactions are important at high densities.
Conclusions

• With the discovery of more massive neutron stars (~2 $M_\odot$), the neutron matter at the core is becoming more constrained to known models and less likely to be exotic
  ▫ This works supports that claim showing that the existence of exotic matter in neutron stars is less likely

Future Work

• Repeating analysis
  ▫ Chiral potentials with different cutoffs (different resolution scales)
  ▫ Chiral potentials at lower orders in the expansion which would help with uncertainties
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