Astrophysical Relevance of Clustering in Low Density Nuclear Matter

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Roughly, the size of an element's own niche ("I almost wrote square") is proportioned to its abundance on Earth's surface, and in addition, certain chemical similarities (e.g., Be and Al, or B and Si) are suggested by the positioning of neighbors. The chart emphasizes that in real life a chemist will probably meet O, Si, Al, ... and that he better do something about it. Periodic tables based upon elemental abundance would, of course, vary from planet to planet... W.F.S.

NOTE: TO ACCOMMODATE ALL ELEMENTS SOME DISTORTIONS WERE NECESSARY, FOR EXAMPLE SOME ELEMENTS DO NOT OCCUR NATURALLY.
Chart of the Nuclides 2009

nuclei total: 2974
in nature: 286

proton number

neutron number

STABLE NUCLIDES- BLACK SQUARES

- α - decay
- β^- decay
- β^+ decay or electron capture
- spontaneous fission
- p - emitter
- n - emitter
- isomers
The Liquid Drop Model

The Semi-Empirical Mass Formula

\[ m(Z, N) = [Z \cdot m_H + N \cdot m_n] - B(Z, N)/c^2; \quad B > 0 \quad \text{neglect e^{-} binding} \]


\[ B(Z, N) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \left( \frac{A - 2Z}{A} \right)^2 - A^{-1/2} \]

\[ \delta = \begin{cases} 
+11.2 \text{MeV for } o-o \text{ nuclei} \\
0 \text{MeV for } \text{odd-odd nuclei} \\
-11.2 \text{MeV for } e-e \text{ nuclei} 
\end{cases} \]

Wapstra, Handb. Physik, XXXVIII

INFINITE NUCLEAR MATTER \( \rightarrow \)

\[ Z = 15.835 \text{MeV} \]
\[ a_s = 18.33 \text{MeV} \]
\[ a_c = 0.714 \text{MeV} \]
\[ a_a = 23.20 \text{MeV} \]

Relative Contributions to Nuclear Mass

- \( R \propto A^{1/3} \rightarrow V_{\text{nucleus}} \approx A \) const. contribution from each nucleon \( \rightarrow \) “saturated” force
- fewer interactions on surface \( \rightarrow \) reduce contribution from each surface nucleon \( S \propto A^{2/3} \)
- different interactions between like and unlike nucleons (Fermion statistics, isospin) \( \rightarrow \) depends on \( |N-Z| \), reduces \( B \)
- Coulomb self energy becomes large for large \( Z \), heavy nuclei, makes nucleus unstable, reduces \( B \)
<table>
<thead>
<tr>
<th>Decay Type</th>
<th>Radiation Emitted</th>
<th>Generic Equation</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha decay</td>
<td>(4_2^\alpha)</td>
<td>(A^X \rightarrow A - 4 \X^* + 4_2^\alpha)</td>
<td>Parent → Daughter → Alpha Particle</td>
</tr>
<tr>
<td>Beta decay</td>
<td>(0_{-1}^\beta)</td>
<td>(A^X \rightarrow Z+1^\X^* + 0_{-1}^\beta)</td>
<td>Parent → Daughter → Beta Particle</td>
</tr>
<tr>
<td>Positron emission</td>
<td>(0_{+1}^\beta)</td>
<td>(A^X \rightarrow Z-1^\X^* + 0_{+1}^\beta)</td>
<td>Parent → Daughter → Positron</td>
</tr>
<tr>
<td>Electron capture</td>
<td>X rays</td>
<td>(A^X + 0_{-1}^e \rightarrow A^X^* + X)</td>
<td>Parent → Electron → Daughter → X ray</td>
</tr>
<tr>
<td>Gamma emission</td>
<td>(0_0^\gamma)</td>
<td>(A^X^* \rightarrow A^X^* + 0_0^\gamma)</td>
<td>Parent (excited nuclear state) → Daughter → Gamma ray</td>
</tr>
<tr>
<td>Spontaneous</td>
<td>Neutrons</td>
<td>(A\beta+C^X \rightarrow A^X^* + B^X^* + C^1_n)</td>
<td>Parent (unstable) → Daughters</td>
</tr>
</tbody>
</table>
Most Isotopes are Radioactive

Chart of the Nuclides 2009

nuclei total: 2974
in nature: 286

γ, everywhere
Reversing the Trend -- Nuclear Reactions

Experimental Chart of Nuclides 2000
2975 isotopes

Number of Protons

Number of Neutrons
ACCELERATORS

A “Livingston plot” showing the evolution of accelerator laboratory energy from 1930 until 2005. Energy of colliders is plotted in terms of the laboratory energy of particles colliding with a proton at rest to reach the same center of mass energy.

Particle Energy

Proton Storage Ring Colliders
Proton Synchrotrons
Proton Synchrotrons
Electron Linac
Electron Linac
Sector-Focused Cyclotrons
Electrostatic Generators
Rectifier Generators

Year of Commissioning

1930
1950
1970
1990
2005

1,000,000 TeV
100,000 TeV
10,000 TeV
1,000 TeV
100 TeV
10 TeV
1 TeV
100 GeV
10 GeV
1 GeV
100 MeV
10 MeV
1 MeV

LHC
RHIC
TAMU

With such reactions new isotopes and new elements can be made.
Synthesis of superheavy elements (cold and hot fusion)

**Cold synthesis:**
\[ ^{208}\text{Pb} + ^{64}\text{Ni}, ^{70}\text{Zn}, \ldots \rightarrow ^{272}\text{110}, ^{278}\text{112}, \ldots \]

**Hot synthesis:**
\[ ^{238}\text{U}, \ldots, ^{249}\text{Cf} + ^{48}\text{Ca} \rightarrow ^{286}\text{112}, \ldots, ^{297}\text{118} \]
Theoretical Limits to the Existence of Nuclei

- Only a fraction of the theoretically possible isotopes have been produced and studied.
- A new generation of accelerators being constructed will accelerate radioactive ions and probe the region of unknown isotopes.
Limits of Stability

- Neutron Drip Line?
- Proton Drip Line?
- Known Nuclei
- Heavy Elements?
- Fission Limit?
Time (fm/c) = 300

32 MeV/nucleon $^{48}\text{Ca} + ^{124}\text{Sn}$

Higher Energy Reactions

200 GeV/nucleon

A Gold-on-Gold Collision at RHIC

The Little Bang
The Big Bang
First Atoms, H and He

Heavier Elements
Our Sun

Start | Step 2 | End

10,000,000 Degrees
NEUTRINO ASTROPHYSICS
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Figure 1: The Homestake Mine’s chlorine detector, which Ray Davis Jr. and colleagues operated for over three decades.

Figure 3: The left panel shows the Super-Kamiokande detector during filling, with scientists cleaning PMT surfaces as the water rises. The right panel is a fish-eye photo of the SNO detector and cavity, showing the PMTs and support structure prior to cavity and detector filling.
Origin of the Lighter Elements
Floor of Binding Energy Valley is Not Flat

Energetics of Transmutation

- Energy released in fusion
- Energy released by fission

Radioactive nuclei decay (lose energy) toward the valley of stable isotopes
Energetics of Transmutation

A massive star near the end of its lifetime has “onion ring” structure

Periodic Table of the Elements

How are the heavier elements produced?
One Important Source for Generation of THE ELEMENTS BEYOND IRON is EXPLOSIONS of SUCH STARS

DID 1987A EXPLODE IN 1987?

NO!

It was $9.87 \times 10^{17}$ MILES Away! The light took 168,000 years to reach us.
One Important Source for Generation of
THE ELEMENTS BEYOND IRON
is EXPLOSIONS of SUCH STARS

Nucleosynthesis in the r-process

JINA
Joint Institute for Nuclear Astrophysics 2002

Movie : H. Schatz, National Superconducting Cyclotron Laboratory
Calculation : K. Vaughan, J.L. Galache, and A. Apriahmanian, University of Notre Dame
Model : B. Meyer, Clemson University and R. Surman, North Carolina State

Temperature: 1.50 GK
Time: 2.7e-14 s
Mostly protons

Mostly Neutrons

R \sim 10^5 - 10^6 \text{ km}

R \sim 10 \text{ km}

STARS

Giant Nuclei

And Sites of Nucleosynthesis

Large Changes in Temperature, Density, Proton/Neutron content

Big Bang

Creation of the Elements

Stars

Supernovae, Binary Mergers

R \sim 10 \text{ km}

Mostly Neutrons
How does the physics of nuclei impact the physical universe?

- What is the origin of elements heavier than iron?
- How do stars burn and explode?
- What is the nucleonic structure of neutron stars?

Nuclear Input (experiment and theory)
- Masses and drip lines
- Nuclear reaction rates
- Weak decay rates
- Electron capture rates
- Neutrino interactions
- Equation of State
- Fission processes

Supernova

Stellar burning

rp process

Crust processes

neutron-Star

protons

events

neutrons
• Relevance of heavy ion collisions to core collapse supernovae
  – Allow probing different densities in the lab
  – Comparisons of heavy ion data to supernovae calculations may help discriminate between different models.

• Clusters appear in shock heated nuclear matter
  – Clusters Role on the explosion dynamics and the subsequent cooling and compression of the proto-neutron star is not yet fully understood
  – Valid treatment of the correlations and clusterization in low density matter is a vital ingredient of astrophysical models

• Equation of state (EOS)
  – Many fundamental connections between the equation of state and neutrino interactions
  – Crucial input for understanding proto-neutron star evolution
Light Charged Particle Emission Studies

- \( p + 112\text{Sn} \) and \( 124\text{Sn} \)
- \( d + 112\text{Sn} \) and \( 124\text{Sn} \)
- \( 3\text{He} + 112\text{Sn} \) and \( 124\text{Sn} \)
- \( 4\text{He} + 112\text{Sn} \) and \( 124\text{Sn} \)
- \( 10\text{B} + 112\text{Sn} \) and \( 124\text{Sn} \)
- \( 20\text{Ne} + 112\text{Sn} \) and \( 124\text{Sn} \)
- \( 40\text{Ar} + 112\text{Sn} \) and \( 124\text{Sn} \)
- \( 64\text{Zn} + 112\text{Sn} \) and \( 124\text{Sn} \)
- **Projectile Energy** - 47A MeV

**Thesis – L. Qin**
**TAMU- 2008**
Higher Energy Reactions

Time (fm/c) = 1

32 MeV/nucleon $^{48}$Ca + $^{124}$Sn

b = 11 10 9 8 7 6 5 4 3 2 1 0

Velocity Plots
Light Charged Particles - Most Violent Collisions

**Velocity Plot Protons**

$^{40}\text{Ar}+^{124}\text{Sn}$

- **Experiment**
- **From Fitting**
  - **PLF**
  - **Evaporation-like**
  - **NN**
  - **Coalescence-like**
  - **Sampling the GAS-early emission**
  - **Sampling the Liquid – late emission**
- **Sum of Source Fits**

**V** parallel

**V** perpendicular
Crab Nebula, HST Image

Supernova

Mass: $4.6 \pm 1.8 M_{\odot}$ ($\sim 9.2 \times 10^{30}$ kg)

- **47 MeV/u Ar + $^{112,124}$Sn**
- **Select the most violent collisions**
- **Identify the femtonova**
  - Intermediate velocity source
    - nucleon-nucleon collisions early in the reaction
  - Observe light nuclei emitted from that source.
- **Temperature from relative yields of particles**
- **Density from Coalescence analysis**
- **Evolution time scale from velocity of products from intermediate velocity source**
CLUSTER FORMATION
Modifies Nuclear EOS

Astrophysical Implications, e.g.,
Core-collapse Supernovae

K. Sumiyoshi et al.,

Density, electron fraction, and
temperature profile of a
15 solar mass
supernova at 150 ms
after core bounce -- as
function of the radius.

K. Sumiyoshi, G. Roepke
PRC 77, 055804 (2008)

Cluster formation
Influences neutrino
flux

Figure 1. Schematic view of the phase diagram of nuclear matter. The phase diagram
is empirical accessible by heavy ion collisions, excited nuclei, observation of neutron
stars and the early universe as indicated in the diagram. New plans at GSI aim at
exploring the color superconducting phase as well.
Temperatures and Densities Are Correlated

- System starts hot
- As it cools, it expands

\[ 47 \text{ MeV/u } ^{40}\text{Ar} + ^{112}\text{Sn} \]
Temperatures and Densities

**47 MeV/u $^{40}$Ar + $^{112}$Sn**

- SN are “infinite”, but HIC are finite
- The “infinite” matter in SN is charge neutral, but HIC has a net charge
- Proton fraction, $Y_p$ can differ
- Composition of nuclear matter in calculations
  - Different calculations include different species

<table>
<thead>
<tr>
<th></th>
<th>Supernova</th>
<th>Heavy Ion Nuclear reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (nuc/fm$^3$)</td>
<td>$10^{-10} &lt; \rho &lt; 2$</td>
<td>$2 \times 10^{-3} &lt; \rho &lt; 3 \times 10^{-2}$</td>
</tr>
<tr>
<td>Temperature (MeV)</td>
<td>$\sim 0 &lt; T &lt; 100$</td>
<td>$5 &lt; T &lt; 11$</td>
</tr>
<tr>
<td>Electron fraction</td>
<td>$0 &lt; Y_p &lt; 0.6$</td>
<td>$Y_p \sim 0.41$</td>
</tr>
</tbody>
</table>
The equilibrium constant of a chemical reaction

\[ \alpha A + \beta B \rightleftharpoons \rho R + \sigma S \]

is the value of the reaction quotient when the reaction has reached equilibrium.

For a general chemical equilibrium the thermodynamic equilibrium constant can be defined such that, at equilibrium,

\[ K^\Theta = \frac{\{R\}^\rho \{S\}^\sigma \cdots}{\{A\}^\alpha \{B\}^\beta \cdots} \]

where curly brackets denote the thermodynamic activities of the chemical species. The right-hand side of this equation corresponds to the reaction quotient \( Q \) for arbitrary values of the activities. The reaction coefficient becomes the equilibrium constant as shown when the reaction reaches equilibrium.

An equilibrium constant value is independent of the analytical concentrations of the reactant and product species in a mixture, but depends on temperature and on ionic strength. Known equilibrium constant values can be used to determine the composition of a system at equilibrium.

The equilibrium constant is related to the standard Gibbs free energy change for the reaction.

\[ \Delta G^\Theta = -RT \ln K^\Theta \]

If deviations from ideal behavior are neglected, the activities of solutes may be replaced by concentrations, \([A]\), and the activity quotient becomes a concentration quotient, \( K_c \).

\[ K_c = \frac{[R]^\rho [S]^\sigma \cdots}{[A]^\alpha [B]^\beta \cdots} \]

\( K_c \) is defined in an equivalent way to the thermodynamic equilibrium constant but with concentrations of reactants and products instead of activities. \( (K_c \) appears here to have units of concentration raised to some power while \( K \) is dimensionless; however the concentration factors in \( K_c \) are properly divided by a standard concentration so that \( K_c \) is dimensionless also.

Assuming ideal behavior, the activity of a solvent may be replaced by its mole fraction, \( \text{mole fraction} \), (approximately by 1 in dilute solution). The activity of a pure liquid or solid phase is exactly 1. The activity of a species in an ideal gas phase may be replaced by its partial pressure.

** In chemical thermodynamics, activity is a measure of the “effective concentration” of a species in a mixture. The species' chemical potential depends on the activity. Activity depends on temperature, pressure and composition of the mixture, among other things. The difference between activity and other measures of composition arises because molecules in non-ideal gases or solutions interact with each other, either to attract or to repel each other.
Constraining supernova equations of state with equilibrium constants from heavy-ion collisions

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Joseph Natowitz, Kris Hagel, Stefan Typel, and Gerd Röpke
(Dated: January 29, 2015)

• Dependence of Equilibrium constants on various quantities
  – Asymmetry of system
  – Coulomb effects
  – Particle degrees of freedom
• Include comparison where possible to other particle types observed in experiment (d, t, $^3$He)
• Other EOS models
Equilibrium constants for $\alpha$-particles

$$K_c(A, Z) = \frac{\rho(A, Z)}{\rho_p \rho_n} \frac{Z}{(A-Z)}$$

- Many tests of EOS are done using mass fractions and various calculations include various different competing species.
- If any relevant species are not included, mass fractions are not accurate.
- Equilibrium constants should be more robust with respect to the choice of competing species assumed in a particular model if interactions are the same.
- Differences in the equilibrium constants may offer the possibility to study the interactions.
- Models converge at lowest densities, but are significantly below data.
$K_{eq}(T)$

- $K_{eq}(T)$
- Uncertainty in temperature measurement including at low density
- Ideal gas $K_{eq}$ is function of $T$ only.
Mott Points and Lines

O. L. Caballero et al.
arXiv:1410.7663

Surface of Last Neutrino Interaction

\[ T, \text{ MeV} \]
\[ \rho, \text{nuc/fm}^3 \]
• Core-collapse supernovae (SN)
  – Explosions of massive stars that radiate 99% of their energy in neutrinos
  – Birth places of neutron stars
  – Wide range of densities ranging from much lower than normal nuclear density to much higher are sampled

• Core Collapse Supernovae dynamics and the observed neutrino signals are sensitive to the details of neutrino interactions with low density nuclear matter at the **Neutrinosphere**
  – Last scattering site of neutrinos in proto-neutron star: $\sim 10^{12} \text{ g/cm}^3$ ($\sim 6 \times 10^{-4} \text{ fm}^{-3}$), $T \sim 5 \text{ MeV}$
  – A thermal surface from which around $10^{53} \text{ ergs (}10^{37} \text{ MeV)}$ are emitted in all neutrino species during the explosion

  – The neutrino interactions determine the nucleosynthesis conditions in the so-called neutrino-driven wind

  – Detailed information on the composition and other thermodynamic properties of matter in the neutrinosphere region is important to evaluate role of neutrino scattering.
THANK YOU!