Mean Field Approximations of $^{11}$Be

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Introduction

The nucleus $^{11}$Be is an interesting one because the neutron-proton asymmetry (N-Z)/A=3-11 is relatively large and experimental data suggests that in its ground state the valence neutron is in the 1S$_{1/2}$ orbit rather than the 0P$_{1/2}$ orbit as the independent particle model predicts. We have carried out Hartree-Fock calculations for $^{11}$Be, using six Skyrme type nucleon-nucleon interactions, with varying valence nucleon orbit. For each interaction, Skyrme parameters $t_0$ or $t_1$ were adjusted such that 1S$_{1/2}$ orbit single particle energy would match the experimental values of -0.50 MeV.

Mean Field

The wave equation of a many-body system is described by a non-linear system of differential equations that normally cannot be solved analytically

$$\hat{H}_{\text{total}} = T + V$$

$$T = \sum \frac{p_i^2}{2m_i}$$

$$V = \sum_{i<j} V(r_{ij})$$

The Hartree wavefunction of a single particle of spin $\frac{1}{2}$ in the mean field approximation approaches the solution

$$\phi_i(r) \rightarrow \phi_i(r) \exp(-\frac{1}{2} \int \nabla \cdot V)$$

In the mean field approximation the antisymmetric wavefunction of the many nucleon system can be written as a Slater determinant, the determinant of a matrix with single particle wavefunctions and the Hartree-Fock equation once all substitutions are done and all operators.

$$\phi_{\text{SD}} = \frac{1}{\sqrt{N!}} \phi_{\text{SD}}(r_1, r_2, \ldots, r_N)$$

Each single-particle wave function is determined by a mean field that approximates its interactions with the other particles in the system. Through spherical symmetry of the nucleus, the solution becomes separable between its radial component and angular components

$$\phi_i(r_1, \sigma_1, \tau_1) = R_{\text{SD}}(r_1) Y_{\text{JM}}(r_1, \sigma_1, \tau_1(r_1))$$

Hartree-Fock Method

The hamiltonian operator is the sum of both the kinetic and potential energy operators.

$$\hat{H}_{\text{total}} = T + V$$

$$T = \sum \frac{p_i^2}{2m_i}$$

$$V = \sum_{i<j} V(r_{ij})$$

total energy of the system is the expectation of the hamiltonian operator

$$E = \langle \psi_{\text{SD}} | \hat{H}_{\text{SD}} | \psi_{\text{SD}} \rangle$$

to find the wave functions, we desire the minimum energy possible with the condition that the sum of the density integral over all space equals A so as to conserve the number of nucleons in the system

$$\sum_{i=1}^{N} |\phi_i(r)|^2 d^3r = \sum_{i<j} \rho_{\text{SD}}(r) \delta^3 = A$$

and thus the Hartree-Fock Equation is derived

$$\frac{-\hbar^2}{2m} \Delta \phi_i(r) + \sum_{j \neq i} \phi_j^*(r) V(r_i, r_j) \phi_j(r) - \mu_i \delta_i(r) = 0$$

Hartree-Fock with Skyrme Interaction

Skyrme nucleon-nucleon two-body potential

$$V_{ij}^{\text{Skyrme}} = t_0 (1 + x_1 P_{ij}^{0p}) \delta(r_i - r_j) + t_1 (1 + x_2 P_{ij}^{pp}) \frac{1}{2} \delta(r_i - r_j) + \delta(r_i - r_j) \delta(r_j)$$

For the adjusted interactions, the calculated total binding energies are much larger in magnitude than experimental values by factors of 2-3 when $t_0$ and $t_1$ values are calibrated.

Conclusions

Hartree-Fock calculations for $^{11}$Be, using six Skyrme type nucleon-nucleon interactions, fail to appropriately reproduce the single particle structure of the $^{11}$Be nucleus. For all original interactions 1S$_{1/2}$ orbit is unbound. For all interactions the 0P$_{1/2}$ single particle energy is lower than that of 1S$_{1/2}$ predictions fail whether the valence neutron is placed in the 1S$_{1/2}$ orbit or the 0P$_{1/2}$ orbit and for both original Skyrme parameters and calibrated parameters for proper 1S$_{1/2}$ orbit single particle energy. Further research is needed to carry out deformed HF calculations, improve the predictability of the Skyrme interactions, and go beyond the mean-field approximation (i.e. including correlations).