Semi-Classical Investigation of the Efimov Potential in Small Nuclei

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Introduction

Various theories exist to explain the disagreement between the energy predicted by classical models of nuclei and established experimental values of the energy. One such theory that has had some verification in a quantum model when extended to atomic systems [3] was introduced by Vitaly Efimov and is known as the Efimov potential. However, each of the successes of the Efimov potential in predicting small systems of particles comes with pairing the Efimov potential to a complex quantum mechanical model. A semi-classical model and understanding of the Efimov potential has not been reached.

Purpose

The primary purpose of this research is to attempt to model small nuclei by pairing the Efimov potential with a semi-classical system. The secondary purpose is to explore the causes and ramifications of the pairing and to attempt to locate the Efimov states.

Methods

We used the Yukawa potential for the majority of our investigation with a kinetic energy factor

$$ KE = \frac{p^2}{2m} $$

such that $\rho=\hbar$. Our Efimov potential was derived from the work of Braaten and Hammer [4] to satisfy

$$ E_{\text{yuk}}(r) = \frac{4}{3} \frac{\mu}{r} + \frac{1}{3} \frac{\mu}{r^2} + \frac{1}{2} \frac{\mu}{r^3} - \frac{1}{2} \frac{\mu}{r^4} $$

We used two different geometric models each with a slightly different Efimov potential to investigate the tritium model system of identical particles in both geometrical configurations discussed earlier. The two-body conditions of the system were $E(r) = 0$ and $E'(r) = 0$, where

$$ E(r) = KE + V_{\text{yuk}}(r) $$

or

$$ E(r) = c_2 r^{-2} + c_3 r^{-3} + \frac{R^2}{3} - \frac{4}{3} \frac{\mu}{r} - \frac{1}{3} \frac{\mu}{r^2} $$

In this equation $r_0$ is the two body radius, $\mu$ is the pion attractive parameter, and $\mu$ is the pion repulsive parameter.

After choosing a radius $r_0$ and solving for the parameters $c_2$ and $c_3$ the results were used to calculate the energy of the two models

$$ E_2(r) = \frac{3}{4} \left( r_0^{-2} a_0 + r_0^{-3} a_1 + \frac{R^2}{3} \right) $$

and for Model B this yielded

$$ E_3(r) = \frac{3}{4} \left( r_0^{-2} b_0 + r_0^{-3} b_1 + \frac{R^2}{3} \right) $$

With reasonable results from the ideal model, both geometric models A and B were then applied to three different cases of triton with the modification of a pion replacing the particle at position $a$ (See Fig. 1). (The pion was placed at the apex of the triangle because it minimized distortion of the deuteron-like bond.) In Case I, identical to the idealized case, parameters $c_2$ and $c_3$ were calculated for a two nucleon interaction where $E(r_0) = 0$ and $E'(r_0) = 0$. In Case II, the scattering length of the two nucleon interaction was set at 23.71 fm and the parameters were recalculated. In Case III, $E(r_0) = 0$ and $E'(r_0) = 0$ and once again the parameters were recalculated [See Fig. 2]. A separate set of parameters, $c_0$, and $c_0'$, were calculated in all cases for the deuteron-like nuclear interactions in triton, such that $E_2(r_0) = -2.2$ and $E_3(r_0) = 0$, when $r_0 = 2.14$.

Results

Results were obtained by setting the energy of the two nucleon interaction to -8.48 MeV and varying the radius of the two nucleon interaction (non-deuteron) to obtain a value close to 1.01 fm and a radius close to 1.94 fm [See Table 1].

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<th>Case</th>
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<th>$E_2(r_0)$</th>
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<th>$r_3$</th>
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<th>$\sigma r$</th>
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Table 1. Results of triton Model A and B case I, II, and III. Energies are in MeV and radii are in fm.

References


Acknowledgements

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