“Whatever Nature has in store for mankind, unpleasant as it may be, men must accept, for ignorance is never better than knowledge”

- Enrico Fermi
The atomic nucleus is made of \( N \) neutrons and \( Z \) protons.

The number of nucleons, \( A = N + Z \).

The general notation is, \( {}^{A}_{Z}X_{N} \).

<table>
<thead>
<tr>
<th>particle</th>
<th>( m ) (kg)</th>
<th>( m ) (amu)</th>
<th>( mc^2 ) (MeV)</th>
<th>charge</th>
<th>spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>proton</td>
<td>( 1.6727 \times 10^{-27} )</td>
<td>1.007276</td>
<td>938.27</td>
<td>+e</td>
<td>1/2</td>
</tr>
<tr>
<td>neutron</td>
<td>( 1.6749 \times 10^{-27} )</td>
<td>1.008665</td>
<td>939.57</td>
<td>0</td>
<td>1/2</td>
</tr>
</tbody>
</table>
Radius of a typical nucleus is about $10 \text{ fm} = 10^{-14} \text{ m}$

Neutron scattering from nuclei can determine the nuclear radius.

$$R = (1.07 \pm 0.02) A^{1/3} \text{ fm}$$

$1 \text{ fm} = 10^{-15} \text{ m}$
The atomic nucleus is positively charged.

In the interior of heavier nuclei (Au, Bi, …), charge is uniformly distributed.

For lighter nuclei (He, C, Mg ..) there is a steady decrease of density.

Elastic scattering of electrons from nuclei can accurately determine the nuclear charge distribution.
Nuclear Masses and Binding Energies

Binding Energy determines the stability of a nucleus

Binding Energy = sum of all proton and neutron mass-energies minus nuclear mass-energy

\[ B = Zm_{\text{proton}}c^2 + Nm_{\text{neutron}}c^2 - M_{\text{nucleus}}c^2 > 0 \]

For all but the lightest nuclei the average binding energy per nucleon is about 8 MeV.
Nuclear Shapes

Nuclei with quadrupole $Q = 0$ are **spherical**.

Electric quadrupole moment $Q$ is a measure of the shape of a nucleus.
A nucleus can rotate with very high spin and deform itself.

Super-deformation has been found in several regions of the nuclear chart, in nuclei around $A=60$, $A=80$, $A=130$, $A=150$ and $A=190$.

Theory also predicts some exotic shapes for the spinning nucleus.
A nucleus can vibrate or oscillate in different modes, just like the string of a violin can vibrate with different notes.

- Protons & neutrons behave as two interpenetrating but separate rigid distributions.
- Rigid distributions undergo harmonic displacement w. r. t. each other.
- $E_{\text{GDR}} \propto A^{-1/6}$

**Giant Resonances**

- **Monopole**
  - GMR
  - $A^{1/6}$

- **Dipole**
  - Spurious state

- **Quadrupole**
  - GQR

- **Octupole**
  - HEOR
Classification of Nuclei

- Classification of nuclei
  - Unstable nuclei found in nature
    - Give rise to *natural radioactivity*
  - Nuclei produced in the laboratory through nuclear reactions
    - Exhibit *artificial radioactivity*
- Three series of natural radioactivity exist
  - Uranium
  - Actinium
  - Thorium
Decay Series of $^{232}$Th

- Series starts with $^{232}$Th
- Processes through a series of alpha and beta decays
- Ends with a stable isotope of lead, $^{208}$Pb
A Nucleus can decay by emitting three types of radiation:

- **Alpha particle**
  - The particle is $^4\text{He}$ nucleus

- **Beta particle**
  - The particle is either electron or positron
    - the positron is the *antiparticle* of the electron
    - It is similar to the electron except its charge is $+e$

- **Gamma ray**
  - They are high energy photons
• The number of nuclei that decay in given time follows a decay curve given as

\[ N = N_0 e^{-\lambda t} \]

\( \lambda \) – decay constant

• The half-life \( T_{1/2} \) is also a useful parameter

• The half-life is defined as the time it takes for half of any given number of radioactive nuclei to decay

\[ T_{1/2} = \frac{\ln 2}{\lambda} = 0.693 \]
When a nucleus emits an alpha particle it loses two protons and two neutrons
- N decreases by 2
- Z decreases by 2
- A decreases by 4

Symbolically,

\[ {A \atop Z} X \rightarrow {A-4 \atop Z-2} Y + {4 \atop 2} \text{He} \]

- X is called the parent nucleus
- Y is called the daughter nucleus

A typical α emitter
Alpha Decay Paradox

Consider,

\[ \frac{238}{92} U \rightarrow \frac{234}{90} Th + \alpha \]

\[ KE(\alpha) = 4.275 \times \left( \frac{234}{238} \right) = 4.2 \text{ MeV} \]

A 4.2 MeV \( \alpha \) particle is able to come out of the Uranium nucleus

However, \( \alpha \) particle with \( KE(\alpha) = 9 \) MeV (from \( ^{212}\text{Po} \)) is unable to penetrate \( ^{238}\text{U}_{92} \)!

If 9 MeV \( \alpha \) particle is not able to penetrate the Coulomb barrier from outside, then how is the 4.2 MeV \( \alpha \) particle able to penetrate from inside?
Experimental observation in $\alpha$ decay study

Geiger-Nuttall relation

$$\log t_{1/2} + n \cdot \log K_\alpha = \text{Const.}$$

![Graph showing the Geiger-Nuttall relation with a logarithmic scale for $t_{1/2}$ and $K_\alpha$. The graph includes a linear trend line with data points.](image-url)
Gamow, Gurney & Condon applied quantum mechanics of particle tunneling through the barrier to the problem of $\alpha$ decay.

\[ \phi_1(x) = Ae^{jk_1x} + Be^{-ik_1x} \]

\[ \hbar k_1 = \sqrt{2mE} \]

\[ \phi_2(x) = Ce^{k_2x} + De^{-k_2x} \]

\[ \hbar k_2 = \sqrt{2m(V_0 - E)} \]

\[ \phi_3(x) = Fe^{jk_1x} \]

\[ T = \frac{F^*F}{A^*A} = \left[ 1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2(k_2R) \right]^{-1} \]

\[ T \sim \exp \left\{ -a \frac{Z}{\sqrt{E}} + b\sqrt{ZR} \right\} \]

\[ a = \frac{e^2\sqrt{2m}}{2\epsilon_0\hbar} = 3.97 \ (MeV)^{\frac{1}{2}} \]

\[ b = \frac{8}{\hbar} \sqrt{\frac{me^2}{4\pi\epsilon_0}} = 2.98 \ (fm)^{\frac{1}{2}} \]

$E = \alpha$ energy in MeV

$R = \text{radius of ‘daughter’ in fm}$

$Z = \text{atomic number of parent}$
The α particle can tunnel through the potential barrier attempting to confine it to the nuclear interior. The greater the energy the shorter the half-life.

The half-life is in years, the energy is in MeV, and Z refers to the daughter nucleus.
Calculating half-life from the penetration probability $T$

\[ ^{238}_{92}U \rightarrow ^{234}_{90}Th + \alpha \]

\[ E = 4.2 \text{ MeV}, \quad Z_D = 90 \quad \& \quad R \sim 9.3 \text{ fm} \]

\[ T \sim \exp \left\{ -3.97 \frac{90}{\sqrt{4.2}} + 2.98 \sqrt{90 \times 9.3} \right\} \]

\[ = \exp(-88) = 6 \times 10^{-39} \]

Time to ‘cross’ the nucleus is
\[ t = \frac{2R}{v_\alpha} \]

Attempt frequency (‘knocking rate’) is
\[ f = \frac{1}{t} = \frac{v_\alpha}{2R} \]

Alpha particle speed = ?
\[ E_\alpha \sim 4.2 \text{ MeV} \quad (m = 3727.4 \text{ MeV}) \]

\[ \therefore v_\alpha = \sqrt{\frac{2E}{m}} \sim 1.4 \times 10^7 \text{ m/s} \]

\[ \therefore f = \frac{1.4 \times 10^7}{2 \times 9.3 \times 10^{-15}} \sim 7.5 \times 10^{20} \text{ s}^{-1} \]

\[ 6 \times 10^{-39} \times 7.5 \times 10^{20} = 4.5 \times 10^{-18} \text{ s}^{-1} \]

\[ = \lambda = \frac{\ln 2}{t_{1/2}} \]

\[ \therefore t_{1/2} = 1.54 \times 10^{17} \text{ s} = 4.9 \times 10^9 \text{ yr} \]

(expt = 4.46 \times 10^9 \text{ yr})
The $\alpha$ decay theory is able to account for the Geiger-Nuttall relation

$$\log t_{1/2} + n \cdot \log K_\alpha = \text{Const.}$$

![Graph showing the relation between $\log(t_{1/2})$ and $\log(K_\alpha)$](image)
Beta Decay

- Symbolically
  \[
  \begin{align*}
  ^{A}_{Z}X & \rightarrow ^{A}_{Z+1}Y + e^- + \bar{\nu} \\
  ^{A}_{Z}X & \rightarrow ^{A}_{Z-1}Y + e^+ + \nu
  \end{align*}
  \]
  - $\nu$ is the symbol for the neutrino
  - $\bar{\nu}$ is the symbol for the antineutrino

- In beta decay, the following pairs of particles are emitted
  - An electron and an antineutrino
  - A positron and a neutrino
Just like the $\alpha$ decay, $\beta$ decay also is an energy transition between two definite energy.

Thus, mono-energetic (single energy) $\beta$ ray is expected.

However, the kinetic energy spectrum of $\beta$ particle (electron) is continuous, i.e., the electrons emitted in $\beta$ decay have range of kinetic energy.

Also, the beta particle emission violates the conservation of energy and angular momentum.
Pauli’s Neutrino Hypothesis

- To account for the continuous energy spectrum and the violation of energy and momentum conservation, Pauli proposed the existence of another particle – the neutrino.

- Pauli postulated that the neutrino must have
  - Zero electrical charge
  - Mass much smaller than the electron, probably not zero
  - Spin of $\frac{1}{2}$
  - And interact very weakly with matter

$$^A_ZX \rightarrow ^{A}_{Z+1}Y + e^- + \bar{\nu}$$

$$^A_ZX \rightarrow ^A_{Z-1}Y + e^+ + \nu$$

Diagram showing the sharing of total disintegration energy between the beta particle and the neutrino
Fermi’s theory of Beta Decay

Using Pauli’s neutrino Fermi proposed a simple theory of $\beta$ decay using his golden rule

The transition probability is given by

$$\lambda_{fi} = \frac{2\pi}{\hbar} |V_{fi}|^2 \rho_f$$

“matrix element” $V_{fi} = \int \psi_f^* V \psi_i \, dv$

The density of states $\rho \propto p_e^2 (E - E_e)^2$

The transition rate is therefore:

$$\lambda \propto p_e^2 (E - E_e)^2$$

$$\therefore \frac{\sqrt{\lambda}}{p_e} \propto (E - E_e)$$

(Fermi-) Kurie plot
Gamma Decay

• Gamma rays are given off when an excited nucleus “falls” to a lower energy state
  – Similar to the process of electron “jumps” to lower energy states and giving off photons
• The excited nuclear states result from “jumps” made by a proton or neutron

\[ ^{110m}_{\text{Ag}} \rightarrow ^{110}_{\text{Ag}} + \gamma \]

• No change in Z, N or A
• Nucleus can also de-excite by ‘Internal Conversion’ (excess energy given to an ATOMIC electron)
Multipolarities in Gamma transition

- Multipole Radiation: Electric and Magnetic
- Opposite parities
  \[ \pi(EL) = (-1)^L \quad \& \quad \pi(ML) = (-1)^{L+1} \]
- \( L = 1 \rightarrow \) Dipole
- \( L = 2 \rightarrow \) Quadrupole
- \( L = 3 \rightarrow \) Octupole
- \( L = 4 \rightarrow \) Hexadecapole etc

- Transition between nuclear states:
  \[ I_i \overset{\gamma}{\rightarrow} I_f \]
- A multipole of order \( L \) transfers \( L\hbar \) angular momentum per photon
  \[ \vec{l}_i = \vec{L} + \vec{l}_f \]
  \[ e.g. \quad (I_i, I_f) = \left(\frac{3}{2}, \frac{5}{2}\right) \rightarrow L = 1, 2, 3, 4 \]
  \[ i.e. \quad |I_i - I_f| \leq L \leq (I_i + I_f) \]
- ‘Electric’ or ‘Magnetic’ depends on parities of nuclear states
There are 266 stable nuclear isotopes. There are about 3000 radioactive (unstable) nuclides with lifetimes greater than about 1 millisecond and need to be studied.

The line of stability lies above the line N=Z because of the Coulomb repulsion between protons.
Neutron-rich Nuclei (exotic property of nucleus)

A nucleus can have excess neutrons than those found in stable nucleus and have exotic structures.
The **skin thickness**, $t$, is defined to be the distance from 90% to 10% of the central nuclear density.
The halo nucleus $^{11}\text{Li}$ is almost as large as a $^{208}\text{Pb}$ nucleus yet it is a bound system.
How does one study the properties of nucleus?

- Nuclear reactions