Semi-classical approximation description of static properties of nuclei

S. Shlomo and A.I. Sanzhur[1]
[1]*Institute for Nuclear Research, 03680 Kyiv, Ukraine*

Investigating many-body nuclear system of strongly interacting neutrons and protons is very challenging. The mean-field approximation, a zero order approximation in perturbation theory, ignoring the interparticle correlations, is widely employed in variable approaches, such as the Hartree-Fock (HF) and Thomas-Fermi (TF) approximations. The mean field approach has been shown to provide good description of a wide range of phenomena of many body-systems. We point out that an important feature of strongly interacting system is that the free two-body nucleon-nucleon interaction is strongly renormalized inside the system, leading to a parametrized effective nucleon-nucleon interaction. The adopted effective interaction provides the saturation properties of the nuclear system which exhibits the properties of gas (shell model) and a Fermi liquid simultaneously. The Wigner transform (WT) provides a reformulation of quantum mechanics in terms of classical concepts. For many-body Schrodinger equation, coupled hierarchy equations for Wigner distribution functions (WDF) of the reduced density matrices can be derived. Simple semi-classical equations for static and dynamic properties of a many body system are then obtained by truncating the set of the coupled equations.

As an example, we present below results of calculations of the deformation energy E_{def} of ^{240}Pu carried out within the semi-classical extended Thomas Fermi approximation (ETFA) and compare with the results obtained within the liquid drop model (LDM). In the ETFA calculation we: (i) Employed the direct variational method with trial proton and neutron densities $\rho_p(\mathbf{r})$ and $\rho_n(\mathbf{r})$ that are generated by diffuse-layer profile functions, and determine the stationary energy with respect to variations of these profiles, and; (ii) Adopted the modified well-known two-parameters “Funny-Hills" form {c,h}, where c and h are the elongation and neck parameters, respectively, to describe the shape of the deformed nucleus. The LDM deformation energy $E_{\mathrm{def}}^{(\mathrm{LDM})}$ is determined by the Coulomb energy and the shape factor. In Fig. 1 we present the results [1] of numerical calculations of the ETFA deformation energy for ^{240}Pu, using the Skyrme interactions SkM*, SLy230b, T6, and KDE0. The different lines in Fig. 1 show the dependence of the optimal deformation energy $E_{\mathrm{def}}^{(\mathrm{ETFA})}$ (fission trajectory) on the elongation parameter c. The cross at the end of a line in Fig. 1 is the location of the scission point. Note that the deformation energy is quite sensitive to the Skyrme interaction parametrization, but the scission point depends only slightly on the specific Skyrme interaction. We also present in Fig. 1 the results of the LDM calculations. It is seen in Fig. 1 that the LDM results significantly underestimate the fission barrier energy B_f (below 1.0 MeV), compared to the ETFA results (above 4.0 MeV). Also, the sensitivity of the neck parameters on the Skyrme-interaction parametrization is stronger for the LDM case than for the ETFA. These significant differences between the LDM and ETFA results are due to the finite diffuse-layer effects in the ETFA approach, which are sensitive to the gradient terms (surface terms) in the Skyrme interaction and depend on the deformation parameters. The finite diffuse layer in the ETFA approach, which is missing in the LDM approach, produces the curvature correction to the surface energy of liquid drop that leads to a significant effect of diffuse layer on the fission barrier.

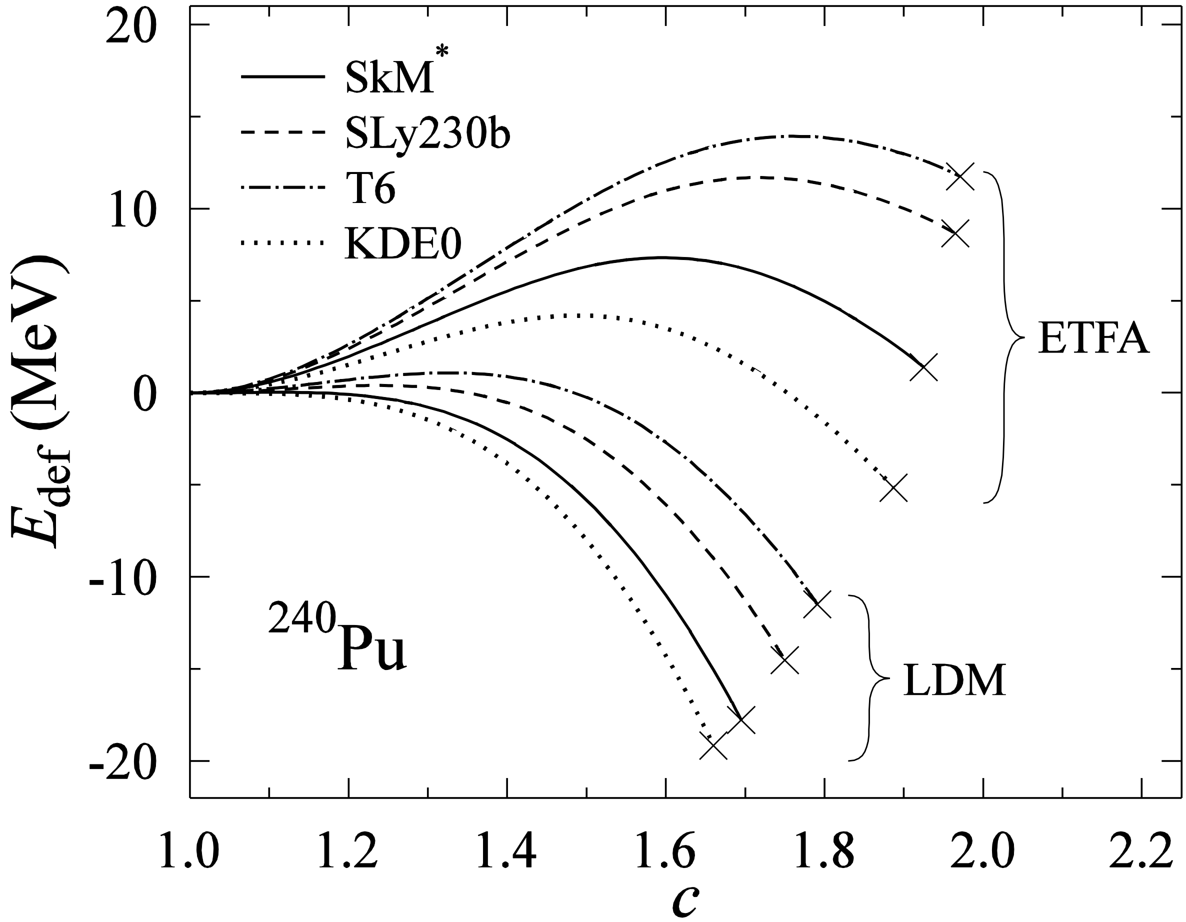

Fig. 1. The dependence of the deformation energies E_{def} of ^{240}Pu for different Skyrme interactions (see text) on the fission trajectory. The four upper lines are for the ETFA and lowest lines are for the LDM approach. The crosses on the lines indicate the scission points

[1] S. Shlomo and A.I. Sanzhur, Mod. Phys. Lett. A **36**, 2130008 (2021).