Equation of state for strongly coupled systems with emerging bound states

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The interplay of bound and continuum states is a key feature in a wide variety of quantum many-body systems, e.g., in cold atomic gases, electromagnetic and electron-hole plasmas, nuclear matter, and its transition to the quark-gluon plasma (QGP). A microscopic description of their equation of state (EoS) becomes particularly challenging when a strong coupling between the constituents mandates methods beyond the quasi-particle approximation. One such method is the Luttinger-Ward-Baym (LWB) formalism [1-3],

$$\Omega(G) = \mp \text{Tr} \{ \ln(-G^{-1}) + (G_0^{-1} - G^{-1})G \} \pm \Phi(G)$$

where the grand potential is evaluated in terms of Feynman diagrams with fully dressed in-medium single-particle propagators, $G$. The predictive power of this formalism hinges on including the relevant diagrams in the calculation of the Luttinger-Ward functional (LWF), $\Phi(G)$. Typically, $\Phi(G)$ is constructed to finite order in the \``skeleton diagram\” expansion, from which the integral equation for $G$ should be solved self-consistently. However, for strongly coupled systems non-perturbative resummation for $\Phi(G)$ is required. In our recent work [4-5], we developed a resummation method to evaluate the $\Phi(G)$ non-perturbatively via a generalized $T$-matrix formalism [5],

$$\Phi(G) = -\frac{1}{2} \int d^4\bar{p} \text{Tr} \left\{ \text{Log} \left( 1 - VGG(\bar{p}) \right) \right\}$$

Here, Log denotes a matrix-logarithm operation, and $V$ and $G$ are matrices in energy and momentum space after discretization of the interaction kernel $V$ and propagator $G$. By self-consistently solving for the propagators $G$, we can calculate the EoS non-perturbatively including both in-medium bound and 1-particle states systematically.

**FIG. 1.** Evolution of the pressure in the self-consistent iteration procedure (left panel), and temperature dependence of quark and gluon masses (middle panel) needed to fit lQCD data [7] for the scaled pressure, $P/T^4$ (right panel).
We apply this method to evaluate the EoS of QGP. The starting point is an effective Hamiltonian where the interaction $V$ is constrained by the static quark-antiquark free energy as computed in lattice QCD (lQCD) [6]. Tuning two fit parameters (the light parton masses), the EoS obtained through our approach can describe the lQCD data as shown in Fig.1; in the right panel, the LWF $\Phi$ contribution to the EoS, which encodes dynamical bound/resonance states, is found to dominate the pressure when the temperature approaches the pseudo-critical one. The resonance interactions, in turn, strongly distort the single-parton spectral functions, see Fig.2, which is a prediction of the approach.

[Graphs showing spectral functions for quarks and gluons at different temperatures]

**FIG. 2.** In-medium spectral functions for quarks (left two panels) and gluons (right two panels) at $T = 194$ MeV and $T = 400$ MeV for 3-momenta $p = 0, 1, 2, 3$ GeV.

The spectral functions are broad and non-quasiparticle like at low momenta and low temperatures while they recover quasi-particle structures at high momenta and/or temperatures. This is a direct reflection of the remnants of the strong confining force in QCD at low energy scales and asymptotic freedom at high energy scales, as encoded in the potential $V$. The pertinent 2-particle T-matrices are shown in Fig. 3 in attractive channels, illustrating strong resonances at low temperature and their dissolution at higher temperatures.

In summary, utilizing a newly developed many-body method, we have unraveled a strongly coupled picture of QGP near the transition region where quantum effects play a key role: as the pseudocritical temperature is approached from above, broad single-parton spectral functions give way to dynamically formed bound states, driven by the confining force as constrained by lattice QCD.
FIG. 3. Imaginary part of the in-medium $T$-matrix for $P = 0$ in the color-singlet $q\bar{q}$ (left) and gg (right) channels.