Spinodal instability in baryon-rich quark matter

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The baryon-rich quark matter is expected to undergo a first-order phase transition at finite baryon density. Using the Polyakov Nambu-Jona-Lasinio (PNJL) model [1], which shows both the deconfinement and chiral phase transitions of quark matter at high temperature and density, we have studied the spinodal instability in a baryon-rich quark matter [2]. Shown in Fig. 1 is the spinodal boundary in the temperature and density plane for different values of the wave number $k$ of the unstable mode with (right window) and without (left window) the repulsive quark interaction. It is seen that the boundary of the spinodal region shrinks with increasing wave number, and for the same wave number, it is reduced by the repulsive quark vector interaction.

Using the linear response theory, we have also calculated the growth rate of the unstable mode, which is given by the imaginary part of its dispersion relation, as a function of the wave number. Fig. 2 shows that for quark matter at temperature $T = 70$ MeV and net quark density $n_q = 0.7 \text{ fm}^3$, the growth rate of the unstable mode in the absence of vector interaction first increases and then decreases with the wave number after reaching a peak value of about $0.01 \text{ fm}^{-1}$ at $k \sim 0.15 \text{ fm}^{-1}$. The growth rate is, however, significantly reduced after including the quark vector interaction, resulting in a suppression of the spinodal instability that is greater for unstable modes of shorter wavelength.

We have further studied how density fluctuations develop in a baryon-rich quark matter inside the spinodal region by numerically solving the Vlasov equation derived from the PNJL model for the quark distribution function inside a box of periodic boundary conditions. With an initial net quark density of $n_q = 0.6 \text{ fm}^3$ and temperature of $T_0 = 70$ MeV, the quark density distributions in the x-y plane at later times...
of 20 fm/c and 70 fm/c are shown in the left and right windows of Fig. 3, respectively. Density fluctuations are seen to appear already at 20 fm/c and become very large at 70 fm/c.

**FIG. 2.** Growth rate of unstable modes in quark matter of net quark density $n_q = 0.7 \text{ fm}^{-3}$ and at temperature $T = 70 \text{ MeV}$ for quark vector interaction $G_V = 0$ and $G_V = 0.2 G_S$.

**FIG. 3.** Density distributions at $t = 20 \text{ fm/c}$ (left window) and $70 \text{ fm/c}$ (right window) on the plane $z = 0$ of a quark matter with initial uniform density of $n_q = 0.6 \text{ fm}^{-3}$ and temperature $T_0 = 70 \text{ MeV}$.
To quantify the density fluctuations, we have calculated the density moments $\langle \rho^N \rangle$ of all quarks in the box [3]. Shown in Fig. 4 is the time evolution of scaled density moments $\langle \rho^N \rangle / \langle \rho \rangle^N$ for $N = 2$, 4, and 6. All are seen to increase with time, and the higher the moment, the larger its magnitude is.

![Graph showing time evolution of density moments](image)

**FIG. 4.** Time evolution of density moments of a quark matter with initial density $n_0 = 0.6$ fm$^{-3}$ temperature $T_0 = 70$ MeV.

Work is in progress to study the density fluctuations of an expanding fireball to investigate if they can still be developed before the system disintegrate due to expansion. Also, physical observables that are sensitive to density fluctuations are being studied.