Mirror superallowed beta transitions

I. S. Towner and J. C. Hardy

Of the 20 superallowed $0^+ \rightarrow 0^+$ $\beta$ decays surveyed in 2009 [1] there are four pairs that are isospin mirrors. These transitions deserve a renewed scrutiny because they have the potential to distinguish clearly between competing calculations of the isospin-symmetry breaking correction. To illustrate this, we start with a few definitions. The measured quantity in a beta transition is the $f_t$ value. To this is applied two sets of theoretical corrections: a radiative correction, $\delta_R$, and an isospin-symmetry-breaking correction, $\delta_C$, defining in the process a corrected $F_t$ value

$$F_t = f_t (1 + \delta_R)(1 - \delta_C).$$

The corrected $F_t$ values have one very important property: for superallowed transitions between isospin $T = 1$ states, the $F_t$ values are constant, independent of the nucleus under study. This follows from the conserved vector current (CVC) hypothesis. Indeed one of the strongest vindications of the CVC hypothesis is the fact that the superallowed $F_t$ values in many different nuclei are all consistent with one another. Here, however, we will turn the argument around. We assume the validity of the CVC hypothesis and use the experimental results to test the theoretical isospin-symmetry-breaking corrections. This strategy has already been proposed and implemented by Towner and Hardy [2]. However by focusing on a pair of mirror transitions, a number of systematic uncertainties in the theoretical corrections drop out, sharpening even further the test of the theory.

In discussing a pair of mirror transitions, we will use the superscript $a$ for a property of the decay of the proton-rich member of the isospin triplet (the $T_z = -1$ member), and the superscript $b$ for its mirror ($T_z = 0$) transition. Then the experimental property we wish to examine is the ratio of $F_t$ values, $f_t^a/f_t^b$. From the CVC hypothesis, this ratio can be written

$$\frac{f_t^a}{f_t^b} = 1 + (\delta_R^b - \delta_R^a) - (\delta_C^b - \delta_C^a).$$

The test therefore is to measure $f_t^a/f_t^b$ and compare with the theoretical value calculated from the right-hand side of Eq. (2). The expected values are given in Table I. There are many model calculations of the isospin-symmetry breaking correction, but for our illustration here we concentrate on just two: one based on phenomenological Woods-Saxon (WS) eigenfunctions the other based on Hartree-Fock (HF) mean-field eigenfunctions. These two models give quite different predictions for $f_t^a/f_t^b$: The Woods–Saxon calculation predicts a larger deviation from one than does the Hartree-Fock calculation. It is anticipated that an experimental measurement of $f_t^a/f_t^b$ will have sufficient precision to distinguish between these two models. Indeed a first result has just been published [3] for the $A = 38$ doublet where a result of 1.0036(22) favours the Woods-Saxon calculation.
Table I. Calculated $ff^a/ff^b$ ratios for four doublets with Woods-Saxon(WS) and Hartree-Fock(HF) radial wave functions used to calculate $\delta_C$. The uncertainties due to differences in $\delta_R$ and $\delta_C$ are combined in quadrature.

<table>
<thead>
<tr>
<th>Decay pairs, $a; b$</th>
<th>$(\delta^b_R - \delta^a_R)$ (%)</th>
<th>$\delta^b_C - \delta^a_C$ (%)</th>
<th>$f t^a / f t^b$</th>
</tr>
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<tbody>
<tr>
<td>$^{26}\text{Si} \rightarrow ^{26m}\text{Al} ; ^{26m}\text{Al} \rightarrow ^{26}\text{Mg}$</td>
<td>0.264(20)</td>
<td>-0.125(16)</td>
<td>0.075(16)</td>
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<tr>
<td>$^{34}\text{Ar} \rightarrow ^{34}\text{Cl} ; ^{34}\text{Cl} \rightarrow ^{34}\text{S}$</td>
<td>0.126(15)</td>
<td>-0.045(21)</td>
<td>0.155(40)</td>
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<td>$^{38}\text{Ca} \rightarrow ^{38m}\text{K} ; ^{38m}\text{K} \rightarrow ^{38}\text{Ar}$</td>
<td>0.101(20)</td>
<td>-0.095(34)</td>
<td>0.125(38)</td>
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<td>$^{42}\text{Ti} \rightarrow ^{42}\text{Sc} ; ^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$</td>
<td>0.296(30)</td>
<td>-0.270(57)</td>
<td>0.000(29)</td>
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