Correlation functions in constrained molecular dynamics (CoMD)

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The interactions between nucleons (proton-proton, neutron-neutron, proton-neutron) are not particularly well constrained, and continue to be an area of study [1-3]. One method for characterizing these interactions is through correlation functions, which can graphically and numerically show the effects from quantum, Coulomb, and other interactions between nucleons. The shape of correlation functions is strongly influenced by these effects, and correlation functions have long been used to extract spatial and temporal information about the excited emitting sources [1,2,4,5]. Theoretical predictions have suggested that the contribution of the symmetry potential to the behavior of nucleon-nucleon interactions may be large enough that neutron-neutron, proton-proton, and neutron-proton correlation functions are sensitive to the size of the source and the timescale of particle emission [3]. The shape of the correlation functions may therefore be sensitive to the symmetry energy in the nuclear equation-of-state. Constrained Molecular Dynamics (CoMD) simulations of neutron-rich and neutron-poor calcium on nickel systems have been run using two different formulations of the aforementioned nucleon interactions to investigate the possibility of experimentally probing the density-dependence of the symmetry energy in these correlation functions using detectors with high angular resolution.

The correlation functions are defined and plotted in terms of the relative angular momentum \( \vec{q}_{\text{Rel}} \) of any two protons \( \vec{p}_1, \vec{p}_2 \). Where

\[
|\vec{q}_{\text{Rel}}| = \frac{1}{2} |\vec{p}_1 - \vec{p}_2|.
\]  

The correlation function can be written as [1]

\[
C(p_1, p_2) = N \frac{A(p_1, p_2)}{B(p_1, p_2)},
\]  

and plotted as a function of \( q_{\text{Rel}} \). The correlation function is \( C(p_1, p_2) \), \( N \) is a normalization coefficient defined to make \( C(p_1, p_2) = 1 \) at large values \( q_{\text{Rel}} \) (the protons are assumed to have very little correlation when separated by large momentum, and, therefore, space), \( A(p_1, p_2) \) is the yield of pairs of protons at each \( q_{\text{Rel}} \) from the same event (which are defined to be correlated) and \( B(p_1, p_2) \) is the yield of pairs of protons from different events (which are by definition uncorrelated) at each relative momentum point \( (q_{\text{Rel}}) \). This method of calculating the uncorrelated portion of the ratio is known as “event mixing” [1].

In this work, CoMD [6] simulations were used to simulate nucleus-nucleus reactions, and the resulting data were used to construct proton-proton correlation functions for 31 MeV/nucleon \(^{40}\text{Ca}+^{40}\text{Ca}\) and 30 MeV/nucleon \(^{48}\text{Ca}+^{64}\text{Ni}\) reactions. Fig. 1 demonstrates that CoMD is able to create a correlation function of an expected shape for charged fermions. A ratio \([C(q)]\) of greater than one means that there was a higher yield of correlated proton pairs than uncorrelated with that relative momentum. The \( C(q) \) values at the lowest \( q_{\text{Rel}} \) in Fig 1 shows the Coulomb effects on the protons, they are less likely to have the
same momentum if they are close in space and time. Fig. 2 shows the difference between a "stiff" and "soft" density-dependence of the symmetry energy. The bump towards more correlations at a relative momentum of about 20 MeV/c is caused by the Coulombic repulsion of proton-on-proton interactions. Protons that are near in space in the same event tend to repel each other to a relative momentum near 20

![Graph showing correlation function for protons.](image)

**FIG. 1.** Correlation function for protons, central impact parameter events of 31 MeV/u $^{40}$Ca+$^{40}$Ca events.

![Graph showing correlation function for all impact parameter events.](image)

**FIG. 2.** Correlation function for protons for all impact parameter events of 30 MeV/u $^{48}$Ca+$^{64}$Ni events. The green circles denote the "soft" implementation of the symmetry energy, while the blue triangles denote the "stiff".
More events are being run now in CoMD, in order to reduce the effects of statistical uncertainties. Transport models such as isospin-dependent Boltzmann-Uehling-Uhlenbeck (iBUU) have been used in the past to construct these correlation functions, which shows the relative correlation of pairs of protons at any relative momentum [3]. A transport model code will also be run, in order to compare with previously published simulations [3].

Position-sensitive dual-axis dual-lateral (DADL) silicon detectors will be incorporated in the Faust array to experimentally test these calculations [7]. The Faust array in its current configuration will be tested in-beam at the end of this summer, and tests with the DADL detectors are underway with a couple of different types of electronics.