Isoscalar giant monopole resonance in $^{48}$Ca


The location of the isoscalar giant monopole resonance (ISGMR) is important because it can be directly related to the incompressibility coefficient of nuclear matter (NM), an important ingredient in equation of state (EOS) of NM. In recent years, studies of the isotope dependence and the extraction of the symmetry term $K_\tau$ are mostly concentrated in heavy nuclei, especially in Sn isotopes where the neutron excess ratio $(N-Z)/A$ value changes from 0.107 in $^{112}$Sn to 0.194 in $^{124}$Sn. This gives a relative large deviation in the isotope dependence. However, in the calcium isotopes, $(N-Z)/A$ is 0 in $^{40}$Ca and 0.167 in $^{48}$Ca, a much larger variation than in the Sn isotopes, even though the neutron excess in $^{48}$Ca is not as large as in $^{124}$Sn. Thus a study of $^{40-48}$Ca might provide a more precise determination of the symmetry coefficient $K_\tau$.

We have carried out fully self-consistent Hartree-Fock based RPA calculations for the $^{40}$Ca and the $^{48}$Ca isotopes, with commonly used Skyrme type interactions, and compare the results with the recent experimental data obtained at our Institute [1]. We emphasize, in particular, the importance of self-consistency.

The microscopic mean-field based RPA provides a good description of collective states in nuclei. It is common to calculate the RPA states $|n>$ with the corresponding energies $E_n$, and obtain the strength function

$$S(E) = \sum_n |\langle 0|F|n\rangle|^2 \delta(E - E_n),$$

for a certain single particle scattering operator $F = \sum f(i)$, and then determine the energy moments

$$m_k = \int E^k S(E) \, dE.$$

The constrained energy, $E_{con}$, centroid energy, $E_{cen}$, and the scaling energy, $E_s$, of the resonance are then obtained from

$$E_{con} = (m_1/m_t)^{1/2}, \quad E_{cen} = m_1/m_0, \quad E_s = (m_3/m_t)^{1/2}.$$

The energy moment $m_t$ can also be calculated using the Hartree-Fock (HF) ground state wave function, leading to an energy weighted sum rule (EWSR). In a fully self-consistent mean-field calculation of the response function, one adopts an effective two-nucleon interaction $V$, usually fitted to ground states properties of nuclei, and determines the mean-field. Then, the random-phase approximation

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(RPA) calculation is carried out with all the components of the two-body interaction using a large configuration space. In this sense, the calculations are fully self-consistent.

In Figure 1 we compare the measured centroid energies in $^{40,48}\text{Ca}$ to our results from HF-based RPA calculations using the Skyrme type, SGII, SKM*, KDE0 and SK255 interactions. The selected Skyrme interactions are associated with a wide range of nuclear matter (NM) incompressibility coefficients $K_{\text{NM}} = 215 - 255$ MeV and a wide range of NM symmetry energy coefficients $J = 27 - 37$ MeV. In Figure 1 we show the centroid energies as a function of $K_{\text{NM}}$. As can be seen in Fig. 1b, for $^{48}\text{Ca}$, the centroid obtained with SKM* is in agreement with the data, while that for KDE0 is slightly outside the errors while those for the other two interactions are a few hundred keV outside the errors. For $^{40}\text{Ca}$ (Fig. 1a) the centroid obtained with SkM* is high and ~600 keV outside the errors, while those for the other interactions are yet higher and over an MeV outside the errors.

Whereas in the Sn isotopes the ISGMR energy decreases with increasing mass, the measured $^{48}\text{Ca}$ centroid energy is higher than that for $^{40}\text{Ca}$. The measured centroid energy for $^{40}\text{Ca}$ is 0.7 MeV below that of $^{48}\text{Ca}$. The energies of the ISGMR in $^{48}\text{Ca}$ obtained in our fully self-consistent calculations using various Skyrme type interactions are all 0.7 to 1.2 MeV below those of $^{40}\text{Ca}$ (Fig. 1c). It is well known that the effects of self-consistency violation associated with neglecting the particle-hole spin-orbit and Coulomb interactions in HF-based RPA calculations can shift giant resonance energies by hundreds of keV. Neglecting the particle-hole Spin-orbit and Columb interactions in the RPA calculations give $^{48}\text{Ca}$...
energies higher relative to $^{40}\text{Ca}$ than those that include these interactions by 0.4 to 1.2 MeV. Leaving out these interactions, the predicted ISGMR centroid energies (Fig. 1d) in $^{48}\text{Ca}$ are higher than those in $^{40}\text{Ca}$ by $\Delta E_{\text{cen}} = 0.5, 0.3$ and 1.0 MeV for the SGII, KDE0 and SkM* interactions, and SK255 gives a $^{48}\text{Ca}$ energy below $^{40}\text{Ca}$ by 0.4 MeV.

In summary, the experimental value for the centroid energy in ISGMR in $^{48}\text{Ca}$ is larger than that in $^{40}\text{Ca}$ by about 0.7 MeV, whereas self consistent HF-RPA calculations predict a lower centroid energy in this neutron rich Ca isotope. Moreover, the HF based RPA calculations do not reproduce the strength distributions, and it would be interesting to extend them beyond the RPA to include coupling to more complex configurations.