Isobaric yield ratios and the symmetry energy in Fermi energy heavy ion reactions


According to the Modified Fisher model[1,2], the fragment yield of A nucleons with I = N-Z, Y(A,I) is given by,

\[ Y(A,I) = CA^{-\tau} \exp\{[(W(A,I) + \mu_n N + \mu_p Z)/T] + N \ln(N/A) + Z \ln(Z/A)\}. \]  (1)

C is a constant. The A^{-\tau} term originates from the entropy of the fragment and the last two terms are from the entropy contributions for the mixing of two substances in the Fisher Droplet Model [3]. \( \mu_n \) is the neutron chemical potential and \( \mu_p \) is the proton chemical potential. \( W(A,I) \) is the free energy of the cluster at temperature T. In the model \( W(A,I) \) is given by the following generalized Weiszacker-Beth semi-classical mass formula at a given temperature T and density \( \rho \),

\[ W(A,I) = a_v(\rho,T)A - a_v(\rho,T)A^{2/3} - a_s(\rho,T)Z(Z-1)/A^{1/3} - a_{sym}(\rho,T)I^2/A - \delta(N,Z). \]  (2)

The indexes, v, s, c and sym represent volume, surface, Coulomb, and symmetry energy, respectively. I= N-Z.

The isotope yield ratio between isobars differing by 2 units in I, \( R(I,I+2,A) \) can be deduced from Eq. (1) and Eq. (2),

\[ R(I+2,I,A) = \exp\{[(\mu_n - \mu_p) + 2a_v(Z-1)/A^{1/3} - 4a_{sym}(I+1)/A \]

\[ - \delta(N+1,Z-1) - \delta(N,Z)/T + \Delta(I+2,I,A)\}. \]  (3)

Hereafter, in order to simplify the description, the density and temperature dependence of the coefficients in Eq.(2) is omitted as \( a_i = a_i(\rho,T) \) (i=v,s,c,sym,p).

\(^{64}\)Zn, \(^{70}\)Zn and \(^{64}\)Ni projectiles were incident on targets of \(^{58}\)Ni, \(^{64}\)Ni, \(^{112}\)Sn, \(^{124}\)Sn, \(^{197}\)Au and \(^{232}\)Th at 40A MeV. Isotopes were measured inclusively at \( \theta = 20^\circ \) using quad-Si detector telescope. Isotopes are clearly identified up to Z<= 18. The measured energy spectrum of each isotope was integrated using a moving source fit to evaluate the multiplicity.

Initially we focus on the isobars with I= -1 and 1. For these isobars the symmetry term in Eq. (3) drops out and, since these isobars are even-odd nuclei, the pairing term also drops out. Taking the logarithm of the resultant equation, one can get

\[ \ln R(I,-1,A) = [(\mu_n - \mu_p) + 2a_v(Z-1)/A^{1/3}] / T. \]  (4)
In Fig. 1 the experimental values of ln[R(1,-1,A)] plotted for $^{64}$Zn+$^{112}$Sn reactions as a function of A. Fitting these values using $(\mu_n - \mu_p)/T$ and $a_c/T$ as fitting parameters, Eq.(3) leads to $(\mu_n - \mu_p)/T=0.71$ and $a_c/T=0.35$.

![Figure 1](image1.png)

**FIG. 1.** The experimental values (solid circles) of ln[R(1,-1,A)] for $^{64}$Zn+$^{112}$Sn reactions is plotted as a function of A for the isobars of I=-1. The dotted line shows the result of fitting with Eq.(4).

We next compare isobars with I=1 and 3, the symmetry energy coefficient term in Eq.(3) is given as a function of A by

$$a_{sym}/T = -A/8\{ln[R(3,1,A)] - [(\mu_n - \mu_p) + 2a_c(Z - 1)/A^{1/3}] / T - \Delta(3,1, A)\}$$  \hspace{1cm} (5)

![Figure 2](image2.png)

**FIG. 2.** Extracted values of the symmetry energy coefficient from $^{64}$Zn+$^{112}$Sn reactions (solid circles) and results of calculations for the secondary fragments (circles). Triangles show results obtained for primary fragments.
In Fig. 2 results for $^{64}$Zn$+^{112}$Sn reactions (solid circles) of $a_{\text{sym}}/T$ calculated from Eq. (5), using the values $(\mu_n - \mu_p)/T$ and $a_e/T$ determined in Eq. (4), are plotted as a function of A. The extracted values from the experiments are in good agreement with those calculated for the secondary fragments. In general the values increase from ~5 to ~16 as $cA$ increases from 9 to 37. These values are generally much larger than those extracted for the primary fragments (triangles) observed in the AMD calculations. Over the same mass interval the primary fragment values range from 4 to 5. The comparisons between the experimentally extracted results and those of the calculations indicate that the experimental determination of symmetry energy coefficients, $a_{\text{sym}}/T$, are significantly affected by these secondary decay processes of the primary fragments.