An empirical relation for studying the nuclear symmetry energy as a function of excitation energy in various mass regions


Information on the symmetry energy is crucial for understanding nuclei that can be produced in extreme conditions of density, excitation energy and $N/Z$ asymmetry. Currently, there exists no detailed understanding of how the symmetry energy evolves with the excitation energy of hot nuclei over a large range of nuclear masses. In this work, we report an empirical relation for studying the symmetry energy in various mass regions.

The symmetry energy of finite nuclei at saturation density is often extracted by fitting ground state masses with various versions of the liquid drop mass formula. However, real nuclei are cold ($T \approx 0$ MeV), nearly symmetric ($N \approx Z$) and found at equilibrium density ($\rho_o \approx 0.16$ fm$^{-3}$). Also, one needs to decompose the symmetry term of the liquid drop into bulk (volume) and surface terms along the lines of the liquid droplet model, and identify the volume symmetry energy coefficient as the symmetry energy derived from infinite nuclear matter at saturation density. In a previous work [1], we had shown how a constraint on the density dependence of the symmetry energy of infinite nuclear matter can be obtained from multifragmentation studies. Following the expression for the symmetry energy of finite nuclei at normal nuclear density by Danielewicz [2], and using the constraint obtained from our work on the symmetry energy of infinite nuclear matter, one can write the symmetry energy of a finite nucleus of mass $A$, as,

$$S_A(\rho) = \alpha(\rho/\rho_o)^\gamma/[1 + (\alpha(\rho/\rho_o)^\gamma/\beta A^{1/3})]$$

(1)

where, $\alpha = 31 - 33$ MeV, $\gamma = 0.55 - 0.69$ and $\alpha/\beta = 2.6 - 3.0$. The quantities $\alpha$ and $\gamma$ are the volume and the surface symmetry energy at normal nuclear density. At present, the values of $\alpha$, $\gamma$ and $\alpha/\beta$ remain unconstrained. The ratio of the volume symmetry energy to the surface symmetry energy ($\alpha/\beta$), is closely related to the neutron skin thickness [2]. Depending upon how the nuclear surface and the Coulomb contribution is treated, two different correlations between the volume and the surface symmetry energy have been predicted [3] from fits to nuclear masses. Experimental masses and neutron skin thickness measurements for nuclei with $N/Z > 1$ should provide further constraint on the above parameters.

Fig. 1 shows the symmetry energy as a function of excitation energy obtained from the above empirical relation, Eq. 1, for $A = 40$, 150 and 197. These are shown by the dashed curves in top three panels of Fig. 1. The calculation was done with the parameters, $\alpha = 31.6$ MeV, $\gamma = 0.69$ and $\alpha/\beta = 2.6$. The figure also shows the experimentally determined symmetry energy from our multifragmentation studies of $^{58}$Ni + $^{58}$Ni, $^{56}$Fe + $^{58}$Ni and $^{58}$Fe + $^{58}$Fe reactions. These are shown by the solid circles and inverted triangle symbols for mass $A \approx 150$. The solid squares in the figure correspond to the data measured in a previous study [4] at lower excitation energies. One observes that the result of the empirical relation is in good agreement with the experimentally deduced symmetry energy as a function of excitation energy.
In order to compare the empirical relation with a more formal calculation, we also show in the figure the results of the Thomas Fermi calculation [5] for $A = 40$, 150 and 197. These are shown by the solid blue curves in the top three panels of Fig. 1. To be consistent with the Thomas-Fermi calculation, the excitation energy dependence of the density of the expanding nucleus, assumed in Eq. 1, were the same as those obtained from the Thomas-Fermi calculation. The excitation energy dependence of the density assumed in Eq. 1 are shown by solid curves in the bottom most panel of Fig. 1 for nuclear masses of $A = 40$, 150 and 197. Also shown in this plot are the densities obtained from our multifragmentation studies (solid circles and inverted triangles) and those from Ref. [6] (star symbols). The comparison shows that the numerical values obtained from Eq. 1 agrees very well with the Thomas Fermi calculation over a wide range of nuclear mass and excitation energy. The empirical relation can be used to obtain a quick estimate of the symmetry energy as a function of excitation energy in future measurements for very light and heavy nuclei.