The use of Geant4 simulate the response function of a plastic $\beta$-detector to standard $\beta$-sources of $^{133}$Ba and $^{137}$Cs

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Knowledge of the response function of our plastic $\beta$-detector is crucial in precision measurements of branching ratios, which is one component of our experimental program aimed at testing the Standard Model. In particular, the $\beta$ values from superallowed $0^+ \to 0^+$ nuclear transitions yield the value of $V_{ud}$, the up-down quark-mixing element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. This requires that the half-lives, branching ratios and decay energies be measured to high precision. (The most recent review of this work can be found in Ref. [1]). Since branching ratios are typically determined from the intensities of $\beta$-delayed $\gamma$ rays that are observed in $\beta$-$\gamma$ coincidence measurements, the $\beta$-detector response function – and thus its efficiency as a function of energy – is crucially important to our achieving high precision. Here we report studies of our $\beta$-detector's response function based on Monte Carlo (MC) calculations performed with the Geant4 code [2]. The Geant4 results will be compared with experimental measurements.

The $\beta$-detector assembly is illustrated in Fig. 1a, with a detailed description of materials given in Ref [3]. For our measurements of the detector response function, each radioactive source was placed at a distance of 13 mm from the havar window of the detector assembly and axially aligned with it. Both were placed on a lab table (in air at atmospheric pressure) as far away as possible from other objects. The three radioactive sources we used – $^{133}$Ba, $^{137}$Cs and $^{207}$Bi – were 37-kBq conversion-electron sources purchased from Isotope Products Laboratories. Previous work [4] has reported the response function of $\beta$-particles from the $^{207}$Bi source. Here, we concentrate on the comparison of MC simulated data with the measurements from $^{133}$Ba and $^{137}$Cs. All three sources, being specially prepared to minimize scattering or degradation of the emitted electrons, were deposited as 5-mm-diameter spots on a thin foil – stainless steel in the case of $^{207}$Bi, aluminized Mylar for the other two – and covered only by a 100-$\mu$gm/cm$^2$ acrylic film. The source-holder geometry is shown in Fig. 1b.

In defining the laboratory geometry in Geant4 we included all components of the detector assembly and source housing (see Fig. 1a and Fig. 1b), with all objects placed in air. Special care was taken to include all elements present in the various materials, and the natural abundance of isotopes for each chemical element was properly accounted for. In this work, we concentrated on comparisons between Geant4 calculations and experiment for the energy deposited by electrons in a thin plastic scintillator. We have used the simplest possible laboratory geometry so that the Monte Carlo geometry could reproduce it exactly.

The Geant4 code software used in treating the transportation of $\beta$ and $\gamma$ particles through matter is both modular and flexible, especially in the description of low-energy electromagnetic processes down to 250 eV. In addition, it is also possible to simulate a rather complicated 3-D geometry, select a variety of materials and decay products (including radioactive ions), and choose how to handle the physical processes governing particle interactions. Moreover, it provides output of the simulated data at different
stages in the calculation and under various selection criteria. An overview of recent developments in diverse areas of this toolkit is presented in Ref. [4].

Figure 1. (a): Incident $\beta$-particles from a radioactive source pass through a thin havar-foil window (3) into the plastic-scintillator disc (2). The disc is recessed into a lucite light guide (4), which is coupled to a phototube (not shown in the figure) at surface 5. The scintillator and light guide are enclosed in a cylindrical cover (1) made from PVC. (b): Radioactive-source geometry: the support ring is made from aluminum.

All three radioactive sources emit $\gamma$ rays as well as $\beta$ particles and, although our thin scintillator is relatively insensitive to the former, we nonetheless took extra precautions to ensure that we were only studying the detectors’ response to the latter. In addition to recording a spectrum from each source as already described, we also recorded a second spectrum with a 2-mm-thick aluminum plate inserted between the source and the detector. This plate was thick enough to remove all the $\beta$ particles without significantly attenuating the $\gamma$ rays. We then subtracted this second spectrum from the first, and considered the resultant spectrum to be a “pure” $\beta$ spectrum. This method has one flaw, however: the spectrum obtained with the aluminum plate includes some contribution from the bremsstrahlung created by the $\beta$ particles as they stop. Thus, we took the same approach with the calculated Monte Carlo spectra: we made two calculations for each source, one with an aluminum plate and one without, took the difference between them and then compared that difference spectrum with the “pure” experimental $\beta$ spectrum.

To simulate the decay of a radioactive nuclide with Geant4, it is possible to define each $\gamma$ transition, internal-conversion line and $\beta$-decay spectrum individually and require Geant4 to transport all particles through the specified materials and determine the spectrum in the scintillator. However, the code also offers a radioactive decay module, which generates all the decay components radiated from a specified source using information from the Evaluated Nuclear Structure Data File (ENSDF) [5]. We used this module to generate the emission spectra from $^{207}$Bi and $^{137}$Cs. An example – the primary electron spectrum emitted from $^{207}$Bi – is shown in Fig. 1 of Ref. [3].
To our surprise, in the case of $^{133}$Ba we found that the initial decay spectrum produced by the radioactive module of Geant4 was simply not correct, yielding relative conversion-electron intensities in significant disagreement with ENSDF data. Therefore, for this decay only, we inserted each decay mode and transition individually.

Based on a primary emission spectrum thus generated, the Monte Carlo code then determines the total energy deposited into the scintillator for each source. However, before this result can be compared with the experimental spectrum, it is necessary to add in the effects of statistical fluctuations introduced by the processes of light production and transmission, as well as photomultiplication and electronic pulse analysis. For this purpose, we looked to a published study of the response of a plastic scintillator to mono-energetic beams of positrons and electrons [6], which graphed the width of the full-energy Gaussian peak as a function of energy between 0.8 and 3.8 MeV. Since we also needed to deal with energies lower than that, we took the width to be linearly dependent on energy below 0.8 MeV.

Our procedure was to take the scintillator spectrum produced by Geant4 and process it by a randomization algorithm written in C++ in the ROOT [7] analysis framework. In essence, this process spread the number of counts in each energy bin into a Gaussian distribution centered at the original energy and with a width, $\sigma$, taken or extrapolated from Ref. [6]. The results could then be compared directly with the measured spectra. The comparisons for two sources, $^{133}$Ba and $^{137}$Cs, appear in Figs. 2a and 2b respectively.

![Figure 2](image1.png)

**Figure 2.** (a) Net spectrum of the decay of $^{133}$Ba as recorded from our plastic $\beta$ detector. (b) Net spectrum of the decay of $^{137}$Cs similarly recorded. Each is compared with a Geant4-simulated result.

The results obtained in this work from Monte Carlo simulations are in quite good agreement with the data. All features that appear in the experimental spectra are also seen in the simulated ones. Additional final turning of the Monte Carlo model is required for each case however. For example, in calculating the $\beta$ continuum from the decay of $^{137}$Cs, we used an “allowed” shape. Since the decay is actually of “forbidden” character with a known shape-correction factor, it is likely that the observed small difference between the simulated and measured spectra (see Fig. 2b) can be removed by our using the
correct shape-correction factor. We plan to incorporate this improvement. In addition, some adjustment of the low-energy behavior of the peak resolution is probably warranted.