Superallowed Fermi beta decay: the isospin-symmetry-breaking correction

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For weak vector interactions in hadron states, the conserved vector current (CVC) hypothesis protects the decay amplitudes from strong-interaction corrections. However, there is a caveat. The CVC hypothesis requires the hadron state to be an exact eigenstate of *SU*(2) symmetry (isospin) in the case of nucleons and pions, and an exact eigenstate of *SU*(3) in the case of kaons.

In nuclei, *SU*(2) symmetry is always broken, albeit weakly, by Coulomb interactions between protons. There may be other charge-dependent effects as well. These influences shift the value of the hadron matrix element from its exact symmetry limit to a new value and this shift has to be evaluated before weak-interaction physics can be probed with hadrons. In the case of superallowed nuclear β decay, the hadron matrix element, M_F, is written

$$|M_F|^2 = |M_0|^2 (1-\delta_C), \tag{1}$$

where M_0 is the exact-symmetry value, and δ_C is the correction we seek to evaluate. In our approach to these calculations [1] we have identified two contributions,

$$\delta_C = \delta_{C1} + \delta_{C2}, \tag{2}$$

where δ_{C1} is the correction caused by adding charge-dependent terms to the shell-model Hamiltonian, and δ_{C2} accounts for the difference in radial forms between the proton in the parent β-decaying nucleus and the neutron in the daughter nucleus. These radial forms are integrated together and, if there were no difference between them, the integral would just be the normalization integral of value one. Any departure of the square of this overlap integral from one represents the correction, δ_{C2}. There is a strong constraint on the calculation: the asymptotic forms of the radial functions are matched to the separation energies, S_p and S_n , where S_p is the proton separation energy in the decaying nucleus and S_n the neutron separation energy in the daughter nucleus.

In the past year, we have revisited the calculation of the δ_{C2} correction. In the shell model, the *A*-particle wave functions can be expanded into products of (*A*-1)-particle wave functions $|\pi\rangle$ and single-particle functions $|j\rangle$. In terms of this expansion, the Fermi matrix element is given by:

$$M_F = \sum_{j,\pi} S_j^\pi \Omega_j^\pi ,$$

$$\Omega_j^\pi = \int_0^\infty R_{\pi\, j}^p(r) R_{\pi\, j}^n(r) r^2 dr. \tag{3}$$

The expansion coefficients S_j^π are generalized fractional parentage coefficients and represent the spectroscopic overlap of the *A*- and (*A*-1)-particle wave functions; and $R_{\pi\, j}^{p}(r)$ and $R_{\pi\, j}^{n}(r)$ are proton and neutron radial functions, whose asymptotic forms are matched to the corresponding separation energies between the *A*- and (*A* -1)-particle states, π. The correction δ_{C2} is then

$$\delta_{C2} \approx \frac{2}{M_0} \sum_{j,\pi} S_j^\pi (1 - \Omega_j^\pi), \tag{4}$$

which shows that a large contribution to δ_{C2} requires both a large spectroscopic amplitude and a significant departure of the radial integral from unity.

We have focused our attention on the four superallowed cases in the *fp* shell: the decays of ^{42}Sc, ^{46}V, ^{50}Mn and ^{54}Co. The CVC hypothesis has the consequence that the corrected *ft* values for superallowed β decay should all be the same for all nuclei of the same isospin, after radiative and isospin-symmetry-breaking corrections have been applied: *i.e.*

$$\mathcal{F}t = ft(1+\delta_R)(1-\delta_C) = \text{constant} \quad . \tag{5}$$

In the current data set, the $\mathcal{F}t$ values for both ^{42}Sc and ^{46}V are high and could be indicating that their calculated δ_{C2} values should be larger. Up to now, we have restricted the summation over orbitals *j* in Eq. (4) to the shell-model space of $d_{3/2}$, $f_{7/2}$, $p_{3/2}$ orbitals for ^{42}Sc, and $f_{7/2}$, $p_{3/2}$ orbitals for ^{46}V, ^{50}Mn and ^{54}Co. This past year, we were able to include one additional *sd*-shell core orbital into the calculation. For ^{42}Sc, this increased δ_{C2} from 0.44% to 0.52% and decreased its $\mathcal{F}t$ from 3076.5 to 3074.0*s*. Similarly in ^{46}V, δ_{C2} increased from 0.34% to 0.49% such that its $\mathcal{F}t$ value reduced from 3079.4 to 3074.8*s*. Both these are highly desirable results as the $\mathcal{F}t$ values are then much closer to the average, $\mathcal{F}t$ = 3073.8*s*. However, similar increments of 0.08% and 0.06% in δ_{C2} for the ^{50}Mn and ^{54}Co transitions was detrimental, reducing the $\mathcal{F}t$ value of ^{50}Mn from 3071.6 to 3069.1*s*, and for ^{54}Co from 3071.9 to 3070.0*s*.

We conclude that enlarging the model space of the shell-model calculations for δ_{C2} to include the next available core orbital will give an increment to δ_{C2} that on the whole improves the CVC test. However, this raises the question: why just include the next available core orbital? Why not include all core orbitals? We have evaluated the effect of doing so in the extreme *jj* model and the result for the heavier nuclei, such as ^{62}Ga and ^{74}Rb, is a correction to the δ_{C2} value that is far too large. In this extreme limit, the CVC test would be severely compromised. More work needs to be done to evaluate the effect of core orbitals in a more realistic model.

[1] I. S. Towner and J. C. Hardy, Phys. Rev. C **66**, 035501 (2002).