In this work we address the Trojan Horse (TH) formalism for the direct binary processes \( x + A \rightarrow b + B \). In the TH method the cross section for the binary process \( x + A \rightarrow b + B \) is determined from the TH reaction \( a + A \rightarrow y + b + B \), where \( a = (yx) \). The half-off-energy-shell (HOES) post-form amplitude of the direct binary reaction, \( x + A \rightarrow b + B \), which is extracted from the TH reaction, is given by

\[
M_{\text{HOES}} = \langle \psi_{bb} | \Delta V_{bb} | \varphi_x \varphi_A e^{ip_{rel}} \rangle
\]  

Here, \( \varphi_i \) is the bound state wave function of nucleus \( i \), \( \psi_{bb}^{(+)} \) is the scattering wave function of nuclei \( b \) and \( B \) in the final state, \( \Delta V_{bb} = V_{bb} - U_{bb} \), \( V_{bb} \) and \( U_{bb} \), are the interaction potential of nuclei \( b \) and \( B \) and their optical potential, respectively; \( r_{rel} \) and \( p_{rel} \) are the radius-vector connecting nuclei \( x \) and \( A \) and their relative momentum.

The HOES amplitude contains the off-shell plane wave which describes the relative motion of the virtual \( x \) and \( A \) nuclei in the initial channel of the binary reaction rather than the distorted wave describing the initial state in the OES amplitude. Hence the HOES amplitude does not contain a Coulomb barrier factor. The on-energy-shell (OES) amplitude to be compared with the HOES one is

\[
M_{\text{OES}} = \langle \psi_{bb}^{(+)} | \Delta V_{bb} | \varphi_x \varphi_A \chi_{xA}^{(+)} \rangle
\]  

Here, \( \chi_{xA}^{(+)} \) is the distorted wave describing the scattering of particles \( x \) and \( A \). The only difference in the energy dependence between the HOES and OES astrophysical factors comes from the use of the HOES plane wave in the initial state in Eq. (1) and the OES distorted wave in Eq. (2). The distorted wave at low energies can be approximated by the pure Coulomb scattering wave function which contains the Gamow penetration factor, hypergeometric wave function and the OES plane wave. When calculating the astrophysical \( S \) factor the Gamow penetration factor is dropped. The energy dependence of the hypergeometric function is very weak. Hence one can expect that HOES and OES astrophysical factors should have similar energy behavior. From Fig. 1 it is clear that the results we calculated justify the TH method. The calculations have been done for two reactions, \(^6\text{Li}(d,a)^4\text{He}\) and \(^7\text{Li}(p,a)^4\text{He}\). The corresponding TH reactions are \(^6\text{Li}(d,\alpha)^4\text{He}\) and \(^7\text{Li}(d,n)^4\text{He}\), respectively. The energy dependence of the HOES and OES astrophysical factors at low energies are practically identical. Since only the energy dependence is of interest, the HOES results in Fig. 1 have been normalized to the OES ones at an relative \( x-A \) kinetic energy \( E_{rel} = 1 \) keV for ease of comparison.
Figure 1. Energy dependence ($E \equiv E_{\text{ast}}$) of the HOES (red dashed line) and OES (black solid line) astrophysical factor for (a) $^6\text{Li}(d,\alpha)^4\text{He}$ and (b) $^7\text{Li}(p,\alpha)^4\text{He}$ reactions.