The status of $V_{ud}$

J. C. Hardy

The up-down quark mixing element, $V_{ud}$, of the Cabibbo Kobayashi Maskawa (CKM) matrix can be determined via three $\beta$-decay routes: superallowed $0^+ \rightarrow 0^+$ nuclear decays, neutron decay and pion decay. Historically, the nuclear decays have yielded the most precise measurements, but the resultant value for $V_{ud}$ was often thought to be severely limited in accuracy by the calculated nuclear-structure-dependent corrections that were required to extract it from the data. The neutron measurements, though free of nuclear corrections, are experimentally challenging and have therefore been less precise, not to mention occasionally inconsistent with one another. Pion $\beta$ decay, being a $10^{-8}$ decay branch, is even more challenging experimentally and has produced larger uncertainties for $V_{ud}$ than either of the other two approaches.

All three decay modes require that small radiative corrections be applied to the primary experimental data in the process of obtaining a value for $V_{ud}$, and naturally there are uncertainties associated with these calculated corrections. In fact, in the case of the superallowed nuclear decays, the experimental uncertainties have become so well controlled that it is these theoretical uncertainties that dominate the uncertainty quoted on $V_{ud}$. However, it may be a surprise to some readers that the nuclear-structure-dependent correction is not the main contributor. Instead, the dominant theoretical uncertainty originates from the so-called inner radiative correction, which is a correction that is common to all three decay modes and, unless it is improved, will ultimately limit the precision with which $V_{ud}$ can be determined by any route.

As more and more superallowed $0^+ \rightarrow 0^+$ nuclear transitions are measured with high precision, the nuclear-structure-dependent corrections continue to prove their validity. The calculations themselves are based on well-established nuclear structure information derived from nuclear measurements that are totally independent of the superallowed decay experiments. The magnitude of the calculated correction for each transition, though always less than 1.5%, differs considerably from transition to transition. The measured superallowed transition strengths, which are an order of magnitude more precise than that, actually reproduce these predicted differences and, as a result, lead to completely consistent values of $V_{ud}$. All evidence points to the nuclear-structure-dependent corrections being completely reliable within their quoted uncertainties.

The current status of $V_{ud}$ as determined by the three different experimental routes, has been assessed and summarized [1,2]. The up-to-date values are:

$$V_{ud} = \begin{array}{ll}
0.97378(27) & \text{[superallowed nuclear decays]} \\
0.9766(20) & \text{[neutron decay]} \\
0.9751(27) & \text{[pion beta decay]} \\
\end{array}$$

(I-46)
The corresponding error budgets for these three methods are given in Figure 1, from which it can be seen that uncertainties in the theoretical corrections currently limit the nuclear result, while the neutron and pion decays are limited to a much larger uncertainty by experimental factors.

As of now, superallowed $0^+ \rightarrow 0^+$ nuclear $\beta$ decay clearly dominates in the determination of $V_{ud}$. A weighted average of the three results quoted in Eq. 1, yields a result that differs only in the fifth place of decimals from the nuclear result alone; and, considering the experimental ambiguities present in the other decays, especially in that of the neutron, it seems best for the time being to rely simply upon the un-averaged nuclear result. Although this result is now limited by uncertainties originating in the theoretical corrections applied to the data, the largest theoretical uncertainty is not due to the nuclear-structure-dependent corrections, $\delta_C$ and $\delta_{NS}$, but rather to the “inner” radiative correction, $\Delta_R$, which actually is common to all three $\beta$ decays: nuclear, neutron and pion. Clearly it would be beneficial to have confirmation of the nuclear $V_{ud}$ by results of comparable precision from the other two $\beta$-decay modes, but one should not anticipate that either of the latter will actually surpass the nuclear result in the near future.

![Figure 1](image_url)

**Figure 1.** Error budgets for the three different methods to determine $V_{ud}$, illustrating the relative importance of experimental uncertainties (exp) and theoretical ones ($\delta_C$, $\delta_{NS}$, $\delta_R$ and $\Delta_R$).
