Self-Consistent Approximations to a Model with Spontaneously Broken O(N) Symmetry

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Self-consistent Φ-derivable Schwinger-Dyson resummation schemes for quantum field theories (QFT's) are preferable to other schemes since only they obey the conservation laws for the expectation values of energy, momentum and charge and at the same time guarantee the thermodynamic consistency for thermal-equilibrium systems [1]. However, these kind of approximations violate the Ward-Takahashi identities of symmetries underlying the QFT's for two-point and higher vertex functions, leading especially to the violation of the Nambu-Goldstone (NG) theorem in the phase of spontaneously broken symmetry (NG phase) [2].

Earlier we have shown that Φ-derivable approximations to renormalizable QFT's can be renormalized with temperature-independent counter terms [3]. Recently a "gapless Φ-derivable" approximation scheme to a linear O(N)-σ model has been proposed which obeys the NG theorem in the NG phase [4]. Here we apply the previously developed renormalization procedure [3] to this model and discuss remaining problems in the NG phase at finite temperature [5].

As has been demonstrated in [4], for the linear O(N)-σ model a modification to the two-loop approximation of the Φ functional can be uniquely defined by the assumption that (i) it restores the NG theorem in the NG phase, (ii) it leaves the approximation in the phase with restored symmetry (Wigner-Weyl phase) unchanged to the two-loop approximation (i.e., the Hartree-Fock (HF) approximation), (iii) it does not change the HF equation of motion for the mean field. Thus, according to [3], this model can be renormalized with temperature-independent counter terms. In contrast to the usual HF equation, the renormalized gapless HF (gHF) approximation for T=0 resulting from the modified Φ functional is O(N) symmetric in the mass-independent renormalization scheme, and thus the physical mass and coupling can be chosen independently from the renormalization-scale, μ.

As shown in the figure, at finite temperature the gHF equations lead to a rather complex phase structure: There exist two critical temperatures, T_1 and T_2. For $T>T_1$ the equations have a metastable solution in addition to the stable one which shows a phase transition at the temperature $T_2>T_1$. While the latter critical temperature is approximately renormalization-scale independent, the former depends logarithmically on the renormalization scale, μ. For $T>T_2$, in the stable branch the mean field vanishes, but the pion and the σ masses are different up to a temperature $T_{\text{cross},}$ where chiral symmetry is finally completely restored. Above a certain temperature, no physically meaningful solutions of the renormalized gHF equations exist. This can be traced back to large renormalization-scale dependent logarithms which become the driving terms of the equations for higher temperatures. For the same reason, another metastable symmetric solution with much higher meson masses exists. This solution corresponds to the usual HF approximation in the Wigner Weyl phase of the model and ceases to exist at the same limiting temperature as the other branches, described above. The existence of this artificial limiting temperature has been already found in the paper by Baym and Grinstein [2].

This renormalization-scale dependence of Φ-derivable approximations has also been analyzed from the point of view of the renormalization-group equation [6]: The β function, evaluated from a Φ-derivable approximation, deviates from its perturbative expansion, beginning at orders in the expansion parameter (like the coupling or number of loops in the diagrams taken into account) higher than that explicitly used to define the approximate Φ functional. The reason is the violation of "crossing symmetry" in the sense of [3]: Solving the self-consistent equations of motion leads to a partial resummation of perturbative diagrams to any order in the expansion parameter which is necessarily incomplete for any truncation of the Φ functional.

Within the here applied renormalization scheme for the gHF approximation, the renormalization-scale dependence at finite temperature originates from the subtraction of the "hidden subdivergence" of the four-point function inside the self-consistent tadpole loop. As shown in [3], the corresponding four-point function consists in the resummation of diagrams of arbitrarily high orders in the coupling, λ, but only in one of three channels, and thus the β function of this resummed four-point function deviates from the correct one at orders $O(\lambda^2)$.

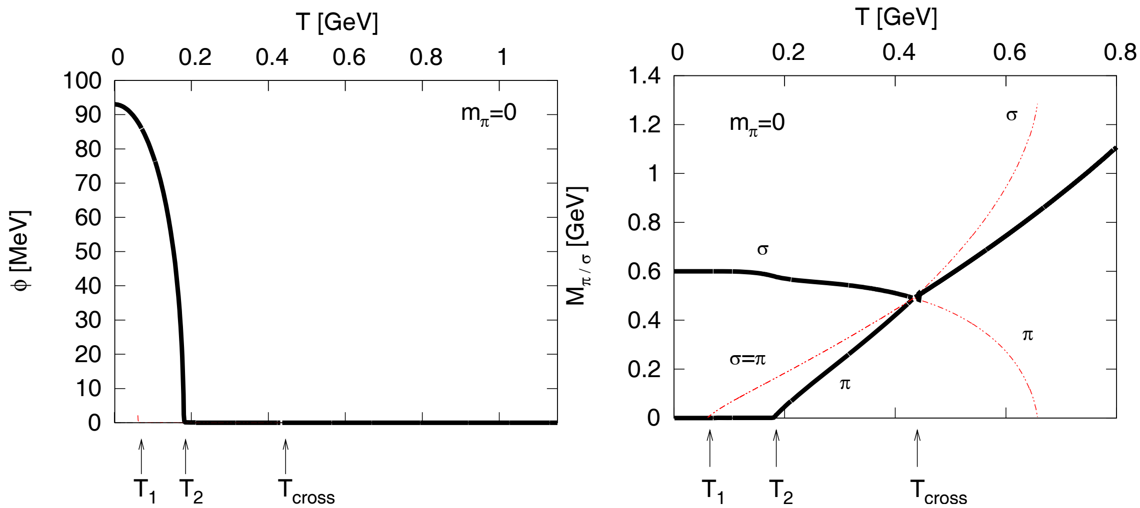

Figure 1. The solution of the gHF equation for the mean field, ϕ (left panel), and the meson masses (right panel) in the chiral limit. The solid (dashed) lines represent the stable (metastable) solutions. The renormalization scale has been chosen to μ=0.6 GeV.

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