Nuclear Incompressibility Coefficient within Fermi Liquid Drop Model

O. Pochivalov, S. Shlomo, and V. M. Kolomietz

Institute for Nuclear Research, Kiev 03680, Ukraine

New experimental data for the isoscalar giant monopole (ISGMR) and dipole (ISGDR) resonances and inability of the self-consistent Hartree-Fock (HF) random phase approximation (RPA) calculations to simultaneously reproduce the experimental values of the centroid energies $E_0$ and $E_1$ and widths $\Gamma_0$, and $\Gamma_1$, of the ISGMR and ISGDR, respectively, and the experimental value $E_1/E_0$, have renewed interest in research of theoretical description of multiple particle-hole excitation. We have previously shown that experimental value of ratio $E_1/E_0$ can be reproduced by the introduction of collisional damping into the Fermi Liquid Drop Model (FLDM). However, the absolute values of the centroid energies $E_0$, and $E_1$, and the widths $\Gamma_0$, and $\Gamma_1$, obtained from FLDM with collisional FSD, are overestimated with respect to experimental data.

To address this issue we study the dependence of the energies for ISGMR and ISGDR in $^{208}$Pb as functions of the nucleus incompressibility coefficient $K_A$. We start from the fluid dynamic equations of motion for the viscous Fermi liquid [1]. Within the FLDM, the eigenfrequency $\omega$ of the compression modes satisfies the dispersion equation

$$\omega^2 - c_0^2 q^2 + i \omega \gamma q^2 = 0, \quad c_0^2 = \left( K_A + 12 \mu / \rho_0 \right) / 9 m, \quad \gamma = 4 \nu / 3 \rho_0 m, \quad (1)$$

where $\rho_0 = 0.17$ fm$^{-3}$ is the bulk particle density corresponding to the sharp radius $R_s = 1.12 A^{1/3}$ fm, and $m$ is the nucleon mass. The transport coefficients $\mu$ and viscosity $\nu$ in Eq. (1) are due to the Fermi surface distortion (see Ref. [1]) and are given by

$$\mu = \text{Im} \left( \omega \tau / (1 - i \omega \tau) \right) P_{eq}, \quad \nu = \text{Re} \left( \tau / (1 - i \omega \tau) \right) P_{eq}, \quad (2)$$

where $P_{eq}$ is the equilibrium pressure in the nuclear interior. The memory effect is introduced into the model by the dependence of transport coefficients on the excitation eigenfrequency through the relaxation time $\tau$

$$\tau = 4 \pi^2 \beta \hbar / (\hbar \text{Re}(\omega))^2. \quad (3)$$

The wave number $q$ in Eq. (1) is derived by the boundary conditions on the free surface of the nucleus. Solving appropriate secular equations for boundary conditions of isoscalar monopole and dipole excitations we obtain centroid energies and widths of compression excitations. We have found that the damping coefficient $\beta = 0.65$ provides a good fit of calculated value of the ratio $E_1/E_0$ to the experimental value. Then, by varying the nucleus incompressibility $K_A$ in Eq. (1) as a parameter we studied behavior of $E_1$, and $E_0$, appropriate widths $\Gamma_0$, and $\Gamma_1$, and compared the nucleus
incompressibility coefficient \( K_A \) with the scaling approximation value \( K_S \), obtained with ISGMR energy taken as described in [2]:

\[
K_S = \frac{m\langle r^2 \rangle}{\hbar^2} \left( E_0^2 + 3(\Gamma_0/2.35)^2 \right).
\]

(4)

Assuming a Fermi density distribution, the rms-radius was calculated using

\[
\langle r^2 \rangle = \frac{3}{5} R^2 \left[ 1 + \frac{10}{9} \left( \frac{\pi a}{R} \right)^2 + \frac{4}{3} \left( \frac{\pi a}{R} \right)^4 \right] \left/ \left[ 1 + \left( \frac{\pi a}{R} \right)^2 \right] \right.,
\]

(5)

where \( a = 0.53 \text{ fm} \) and \( R \) are the diffuseness and half radius of the charge distribution, corresponding to \( R_S \). Results of the calculation for \(^{208}\text{Pb}\) with \( \beta = 0.65 \) are presented in Table I. Results of our calculations show that within the current model, the experimental values of \( E_0 \), \( E_1 \), and \( E_1/E_0 \) for \(^{208}\text{Pb}\) are very well reproduced.

<table>
<thead>
<tr>
<th>( K_A )</th>
<th>( E_0 )</th>
<th>( E_1 )</th>
<th>( \Gamma_0 )</th>
<th>( \Gamma_1 )</th>
<th>( K_S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>130.0</td>
<td>12.61</td>
<td>20.52</td>
<td>2.76</td>
<td>7.06</td>
<td>120.78</td>
</tr>
<tr>
<td>135.0</td>
<td>12.83</td>
<td>20.76</td>
<td>2.81</td>
<td>7.09</td>
<td>125.00</td>
</tr>
<tr>
<td>140.0</td>
<td>13.04</td>
<td>20.98</td>
<td>2.86</td>
<td>7.10</td>
<td>129.30</td>
</tr>
<tr>
<td>145.0</td>
<td>13.26</td>
<td>21.19</td>
<td>2.91</td>
<td>7.10</td>
<td>133.67</td>
</tr>
<tr>
<td>150.0</td>
<td>13.46</td>
<td>21.43</td>
<td>2.96</td>
<td>7.12</td>
<td>137.67</td>
</tr>
<tr>
<td>155.0</td>
<td>13.67</td>
<td>21.65</td>
<td>3.01</td>
<td>7.12</td>
<td>142.17</td>
</tr>
<tr>
<td>Exp.*</td>
<td>13.97 ± 0.20</td>
<td>22.20 ± 0.30</td>
<td>2.88 ± 0.20</td>
<td>9.39 ± 0.35</td>
<td>145.96</td>
</tr>
</tbody>
</table>

* Ref. [4].