Self-Consistent Hartree-Fock RPA and Energy Weighted Sum Rule

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Giant resonances of nuclei give important information about the structure of nuclei as well as bulk properties of nuclear systems. The basic theory for the microscopic description of nuclear giant resonances is the Hartree-Fock (HF) based random phase approximation (RPA). Nowadays, very accurate experimental measurement of centroid energy demands that the theoretical calculations should be highly accurate and self-consistent. Unfortunately, most of the available HF-RPA calculations are not fully self-consistent. We have shown in Ref. [1] that self-consistency violation in HF-RPA calculations may cause an error of around 1 MeV in the centroid energy for isoscalar giant monopole mode (ISGMR) which will lead to an error of 30 MeV in the estimate of nuclear matter incompressibility K. A very important and necessary condition for the self-consistency and high accuracy of HF-RPA calculation is to recover the energy weighted sum rule (EWSR) from the calculated RPA strength function $S(E)$ by,

$$ m_1 = \int_{0}^{\infty} E S(E) dE , $$

where, $S(E) = \sum_{j} < 0 | F_L | j >^2 \delta(E_j - E_0)$ and the scattering operator is given by,

$$ F_L = \sum_{i} f(r_i) Y_{L0}(i) \quad \text{for isoscalar} \quad (2) $$

$$ F_L = \frac{Z}{A} \sum_{n} f(r_n) Y_{L0}(n) - \frac{N}{A} \sum_{p} f(r_p) Y_{L0}(p) \quad \text{for isovector} \quad (3) $$

with $f(r) = r^2, r^3$ and $r^3$ for $L=0,2$ and $3$ respectively. We take for the dipole mode $f(r) = r$ for $(T=1)$ and $f(r) = r^3 - \frac{5}{3} < r^2 >$ for $(T=0)$.

The energy weighted sum rule which is obtained from the double commutator [2], can be calculated using the ground state HF density $\rho(r)$ by,

$$ EWSR(L, T = 0) = \frac{\hbar^2}{4\pi \cdot 2m} A \cdot \frac{1}{A} \int g_L(r) \rho(r) 4\pi r^2 dr $$

where,

$$ g_L(r) = \left( \frac{df(r)}{dr} \right)^2 + L(L+1)\left(\frac{L}{r}\right)^2 . $$

For isovector mode

$$ EWSR(L, T = 1) = EWSR(L, T = 0) \left[ 1 + \kappa - \kappa_{np} \right] \left( \frac{NZ}{A} \right) $$

where $\kappa$ is the enhancement factor because of the momentum dependence of the effective nucleon-nucleon interaction and is given by,
\[
\kappa = \frac{(1/2)[t_1(1+x_1/2) + t_2(1+x_2/2)]}{(h^2/2m)(4NZ/A^2)} \cdot \frac{2\int g_L(r)\rho_p(r)\rho_n(r)4\pi r^2 dr}{\int g_L(r)\rho(r)4\pi r^2 dr}
\]

where \(t_i\) and \(x_i\) are the parameters of the Skyrme interaction and the correction \(\kappa_{np}\) which arises because \((\rho_n(r) - \rho_p(r)) \neq \frac{(N-Z)}{A}\rho(r)\), is given by

\[
\kappa_{np} = \frac{(N-Z)}{A} \cdot \frac{\int g_L(r)Z\rho_n(r) - N\rho_p(r)4\pi r^2 dr}{\int g_L(r)\rho(r)4\pi r^2 dr}
\]

To see how the necessary condition of self-consistency and high accuracy is fulfilled in our calculation, we compare in Table I, the values of \(m_1\) calculated using the RPA strength functions in Eq. (1) with the corresponding EWSR obtained from Eqs. (4) and (6) for three sample nuclei \(^{40}\text{Ca}, ^{90}\text{Zr}\) and \(^{208}\text{Pb}\) from different mass regions. We have presented results for these nuclei for both isoscalar and isovector modes of various multipolarities (L=0-3). It is seen that for these nuclei and for all the modes, the deviation of \(m_1\) from the corresponding EWSR are very small (less than 0.3%). This demonstrates the high accuracy of our HF-RPA calculations.

**Table I.** Comparison of \(m_1\) calculated from RPA strength function [Eq. (1)] with those (EWSR) obtained from the double commutator, Eqs. (4) and (6). The ratio \(R=m_1/EWSR\) demonstrates the high accuracy of our calculations.

<table>
<thead>
<tr>
<th>Mode</th>
<th>(^{40}\text{Ca})</th>
<th>(^{90}\text{Zr})</th>
<th>(^{208}\text{Pb})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EWSR</td>
<td>(R)</td>
<td>EWSR</td>
</tr>
<tr>
<td>0</td>
<td>2889</td>
<td>0.9983</td>
<td>10505</td>
</tr>
<tr>
<td>1</td>
<td>896.3</td>
<td>0.9992</td>
<td>3330</td>
</tr>
<tr>
<td>1</td>
<td>57253</td>
<td>0.9990</td>
<td>289907</td>
</tr>
<tr>
<td>1</td>
<td>64.62</td>
<td>0.9999</td>
<td>148.9</td>
</tr>
<tr>
<td>2</td>
<td>7222</td>
<td>1.0001</td>
<td>26262</td>
</tr>
<tr>
<td>1</td>
<td>2241</td>
<td>1.0000</td>
<td>8326</td>
</tr>
<tr>
<td>3</td>
<td>238240</td>
<td>0.9996</td>
<td>1300645</td>
</tr>
<tr>
<td>1</td>
<td>69328</td>
<td>0.9994</td>
<td>389266</td>
</tr>
</tbody>
</table>

We note that for the SGII interaction used in this calculation, the enhancement factor \(\kappa=0.314, 0.381, 0.314\) and 0.253 and the factor \(\kappa_{np}=0.010, 0.000, 0.010\) and 0.024 for the isovector \(L=0, 1, 2\) and 3 in \(^{208}\text{Pb}\), respectively. We point out that the correction term \(\kappa_{np}\) in Eq.(8) for isovector modes, usually missing in the literature, is not negligible for asymmetric nuclei. As for example, for \(^{208}\text{Pb}\), \(L=3\), \(\kappa_{np}\) has an effect of 2% in the calculation of EWSR. The effect of \(\kappa_{np}\) will be more significant for nuclei near the drip lines because of the large difference between the neutron and proton density distributions and also for the large asymmetry \((N-Z)/A\).