Self-Consistent Hartree-Fock RPA and Energy Weighted Sum Rule

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Giant resonances of nuclei give important information about the structure of nuclei as well as bulk properties of nuclear systems. The basic theory for the microscopic description of nuclear giant resonances is the Hartree-Fock (HF) based random phase approximation (RPA). Nowadays, very accurate experimental measurement of centroid energy demands that the theoretical calculations should be highly accurate and self-consistent. Unfortunately, most of the available HF-RPA calculations are not fully self-consistent. We have shown in Ref. [1] that self-consistency violation in HF-RPA calculations may cause an error of around 1 MeV in the centroid energy for isoscalar giant monopole mode (ISGMR) which will lead to an error of 30 MeV in the estimate of nuclear matter incompressibility K. A very important and necessary condition for the self-consistency and high accuracy of HF-RPA calculation is to recover the energy weighted sum rule (EWSR) from the calculated RPA strength function S(E) by,

$$m_1 = \int_0^\infty E S(E) dE \,, \tag{1}$$

where, $S(E) = \sum_j |< 0 | F_L | j >|^2 \; \delta(E_j - E_0)$ and the scattering operator is given by,

$$F_L = \sum_i f(r_i) Y_{L0}(i) \qquad \text{for isoscalar} \tag{2}$$

$$F_L = \frac{Z}{A}\sum_n f(r_n) Y_{L0}(n) - \frac{N}{A}\sum_p f(r_p) Y_{L0}(p) \qquad \text{for isovector} \tag{3}$$

with $f(r) = r^2, r^2$ and r^3 for L=0,2 and 3 respectively. We take for the dipole mode $f(r) = r$ for (T=1) and $f(r) = r^3 - \frac{5}{3} < r^2 > r$ for (T=0).

The energy weighted sum rule which is obtained from the double commutator [2], can be calculated using the ground state HF density ρ(r) by,

$$EWSR(L, T = 0) = \frac{1}{4\pi}\frac{\hbar^2}{2m} A . \frac{1}{A}\int g_L(r)\rho(r) 4\pi r^2 dr \tag{4}$$

where,

$$g_L(r) = \left(\frac{df}{dr}\right)^2 + L(L+1)\left(\frac{f}{r}\right)^2 . \tag{5}$$

For isovector mode

$$EWSR(L, T = 1) = EWSR(L, T = 0)\left[1 + \kappa - \kappa_{np}\right]\left(\frac{NZ}{A}\right) \tag{6}$$

where κ is the enhancement factor because of the momentum dependence of the effective nucleon-nucleon interaction and is given by,

$$\kappa = \frac{(1/2)[t_1(1+x_1/2)+t_2(1+x_2/2)]}{(\hbar^2/2m)(4NZ/A^2)} \cdot \frac{2\int g_L(r)\rho_p(r)\rho_n(r)4\pi r^2 dr}{\int g_L(r)\rho(r)4\pi r^2 dr} \tag{7}$$

where t_i and x_i are the parameters of the Skyrme interaction and the correction κ_{np} which arises because $\left(\rho_n(r) - \rho_p(r)\right) \neq \frac{(N-Z)}{A}\rho(r)$, is given by

$$\kappa_{np} = \frac{(N-Z)}{A}\frac{A}{NZ}\frac{\int g_L(r)[Z\rho_n(r) - N\rho_p(r)]4\pi r^2 dr}{\int g_L(r)\rho(r)4\pi r^2 dr}. \tag{8}$$

To see how the necessary condition of self-consistency and high accuracy is fulfilled in our calculation, we compare in Table I, the values of m_1 calculated using the RPA strength functions in Eq. (1) with the corresponding EWSR obtained from Eqs. (4) and (6) for three sample nuclei (^{40}Ca, ^{90}Zr and ^{208}Pb) from different mass regions. We have presented results for these nuclei for both isoscalar and isovector modes of various multipolarities (L=0-3). It is seen that for these nuclei and for all the modes, the deviation of m_1 from the corresponding EWSR are very small (less than 0.3%). This demonstrates the high accuracy of our HF-RPA calculations.

Table I. Comparison of m_1 calculated from RPA strength function [Eq. (1)] with those (EWSR) obtained from the double commutator, Eqs. (4) and (6). The ratio R=m_1/EWSR demonstrates the high accuracy of our calculations.

Mode		^{40}Ca		^{90}Zr		^{208}Pb	
L	T	EWSR	R	EWSR	R	EWSR	R
0	0	2889	0.9983	10505	0.9994	41872	0.9971
	1	896.3	0.9992	3330	0.9994	13041	0.9998
1	0	57253	0.9990	289907	0.9995	1823110	0.9999
	1	64.62	0.9999	148.9	1.0001	337.8	0.9999
2	0	7222	1.0001	26262	1.0007	104681	1.0015
	1	2241	1.0000	8326	0.9998	32604	1.0002
3	0	238240	0.9996	1300645	0.9999	8584813	1.0003
	1	69328	0.9994	389266	0.9996	2519881	1.0001

We note that for the SGII interaction used in this calculation, the enhancement factor κ=0.314, 0.381, 0.314 and 0.253 and the factor κ_{np}=0.010, 0.000, 0.010 and 0.024 for the isovector L=0, 1, 2 and 3 in ^{208}Pb, respectively. We point out that the correction term κ_{np} in Eq.(8) for isovector modes, usually missing in the literature, is not negligible for asymmetric nuclei. As for example , for ^{208}Pb, L=3, κ_{np} has an effect of 2% in the calculation of EWSR. The effect of κ_{np} will be more significant for nuclei near the drip lines because of the large difference between the neutron and proton density distributions and also for the large asymmetry (N-Z)/A.

[1] T. Sil, S. Shlomo, B. K. Agrawal and P.-G. Reinhard, Phys. Rev C **73**, 034316 (2006).
[2] A. Bohr and B. R. Mottelson, Nuclear Structure (Benjamin London 1975), Vol. II