Spectra of $L$ x rays emitted from Ho targets bombarded by 6 MeV/amu C, Ne, Ar, Kr, and Xe ions were measured in high resolution using a curved crystal spectrometer, employing second-order diffraction from a LiF crystal. The spectra were analyzed in order to examine the systematic evolution of the $L$ x-ray satellite structure as a function of projectile atomic number. A relatively simple interpretation of the structure in the measured spectra, based on the Independent Electron Approximation and Dirac-Fock calculations, was used to develop an analysis procedure that reproduced the shape of the spectra with reasonably good accuracy. It was found that transitions from atoms with single $L$- and multiple $M^+$ outer-shell vacancies in the initial state dominate the spectra. In addition, contributions from $L$ x rays emitted by atoms with two or three $L$ vacancies in the initial state were clearly identified [1]. These contributions were found to account for up to 21% of the total $L$ x-ray intensity.

In this reporting period, the results obtained in the analysis of the spectra were used to establish scaling rules for the apparent average double to single $L$-vacancy population ratio at the time of $L$ x-ray emission. The results were then compared with those obtained for $K$-vacancy populations at the time of $K$ x-ray emission for Co, Ni, and Cu atoms, since their $K$ binding energies are comparable to Ho $L$ binding energies.

In the Independent Electron Approximation [2], the cross section ($\sigma_m$) for the removal of exactly $m$ electrons from a shell containing $N$ electrons ($m \leq N$) is expressed in terms of the single-electron ionization probability $p(b)$ as

$$\sigma_m = 2\pi \int_0^\infty \left( \begin{array}{c} N \\ m \end{array} \right) p(b)^m [1 - p(b)]^{N-m} b db , \quad (1)$$

where $b$ is the impact parameter. If $p(b)$ is approximated by an exponential decay function [3],

$$p(b) = p_o \exp(-b/r) , \quad (2)$$

in which $p_o$ and $r$ are parameters that do not depend on $b$, then

$$\sigma_m = 2\pi \left( \begin{array}{c} N \\ m \end{array} \right) p_o^m \sum_{i=0}^{N-m} (-1)^i \left( \begin{array}{c} N-m \\ i \end{array} \right) p_o^i /(m+i)^2 . \quad (3)$$

In this approximation, the ratio $\sigma_2/\sigma_1$ depends only on $p_o$ and not on $r$. The parameter $p_o = p(0)$ is the ionization probability per electron at impact parameter equal to zero. The solid lines in Fig. 1(a) represent the values of $\sigma_2/\sigma_1$ for $L$–vacancy production in Ho and $K$–vacancy production in Ni, calculated using eq.(3) and plotted as a function of $p_o$. 

IV-1
According to the Geometrical Model [4], ionization probabilities per electron \( p_n \) for near-central collisions should lie on a universal curve when plotted as a function of the Universal Variable \( X_n \), given by

\[
X_n = 4V[G(V)]^{1/2}Z_1/(nv_1) .
\]  

(4)

In eq.(4), \( Z_1 \) is the projectile atomic number, \( v_1 \) is its speed in atomic units, \( n \) is the principal quantum number of the ionized target electron, \( V = v_1/v_2 \) is the scaled projectile speed (\( v_2 \) is the average target electron speed prior to ionization), and \( G(V) \) is the Binary Encounter Approximation (BEA) scaling function [5], originally formulated to describe universal scaling of the cross sections for inner-shell ionization.

Based on the results obtained in the analysis of Ho L and Co, Ni, and Cu K x-ray spectra, the scaling predicted by the Geometrical model was found to apply to \( p_n^x \), the apparent average \( n \)-vacancy fraction at the time of x-ray emission, provided that the x-ray transition results in filling of a vacancy in the shell specified by a quantum number \( n' \), such that \( n' < n \). Specifically, it was found experimentally [6] that

\[
p_n^x = a/[1 + (b / X_n)^c] ,
\]  

(5)

where \( a = 0.579 \pm 0.016 \), \( b = 1.86 \pm 0.11 \), and \( c = 1.95 \pm 0.09 \).

The values of \( p_n^x \) obtained using eq.(5) can be compared to the \( p_o \) value obtained using eq.(2) under the assumption that the net effect of post-collision vacancy rearrangement and fluorescence yield enhancement is small. This assumption was found to be reasonably valid (within 10 to 20\%) based on a previous analysis of these effects on the \( p_L^x \) values for K-shell ionizing collisions involving a Cu target and 10 MeV/amu projectiles ranging in atomic number from 10 (Ne) to 83 (Bi) [7].

The apparent average double to single K-vacancy population ratio at the time of K x-ray emission in Co, Ni, and Cu \( (R_K^{2/1}) \), and the apparent average double to single L-vacancy population ratio at the time of L x-ray emission in Ho \( (R_L^{2/1}) \) were determined in analyses of the Co, Ni, and Cu K x-ray spectra and the Ho L x-ray spectra, respectively. Under the assumption that the net effects of post-collision vacancy rearrangement and fluorescence yields on the double and single vacancy decays are nearly the same, this ratio is approximately equal to the ratio of the double and single vacancy production cross sections.

The values of \( \sigma_2/\sigma_1 \) obtained using eq.(3) (as shown in Fig. 1a) are compared with the measured values of \( R_L^{2/1} \) and \( R_K^{2/1} \), for which the corresponding values of \( p_o \) (i.e. \( p_n^x \) for \( n = 2 \) and \( n = 1 \), respectively) were determined using eq.(5). The calculations agree with both sets of measured data reasonably well. Predicted values of \( \sigma_2/\sigma_1 \) can also be presented as universal functions of \( X_n \) [eq.(4)] by expressing \( p_o \) in eq.(3) in terms of the Universal Variable using either the empirical curve [eq.(5)] or the curve predicted by the Geometrical Model, given by
\[ p_n = \frac{X_n^2}{\{4.2524 + \frac{X_n^2}{[1 + 0.5 \exp(-X_n^2/16)]}\}}. \]  

Both results are shown in Fig. 1(b) and compared with the experimental data. Predictions of this model agree reasonably well with both sets of measured data. The general validity of these results remains to be tested experimentally for different collision parameters.

![Graph showing double-vacancy production to single-vacancy production cross section ratios](image)

**Figure 1.** Double-vacancy production to single-vacancy production cross section ratios (\(\sigma_2/\sigma_1\)), calculated using eq.(3), plotted (a) as a function of the single-electron ionization probability at small impact parameters (\(p_o\)) and (b) as a function of the corresponding Universal Variable (\(X_n\)) determined using eq.(5). The solid lines represent the results for the L-shell ionization of Ho (red) and K-shell ionization of Ni (blue). Also shown are the measured values of \(R_{2/1}^L\) (open circles) and \(R_{2/1}^K\) (solid circles), for which \(X_n\) was determined using eq.(4) and the corresponding values of \(p_o\) were determined as \(p_n^1\) (for \(n = 2\) and \(n = 1\), respectively) using eq.(5). The data point shown by the solid diamond (measured using a 6 MeV/amu N beam on a Ni target) was taken from Ref.[8]. The dashed lines represent the universal function predicted by the Geometrical Model, as given by eq.(6).