The Use of Monte Carlo Calculations in the Determination of a Ge Detector Efficiency Curve between 50 and 1400 keV


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For our measurement of the superallowed $\beta$ decay of $^{22}\text{Mg}$ [1], we required unprecedented precision in the calibration of our 280 cm$^3$ Ge detector between 50 and 1400 keV. Our work over this energy range, combining precise source measurements with an extensive set of Monte Carlo calculations, is now complete and a manuscript has been submitted for publication [2]. We have obtained 0.15% relative and 0.2% absolute precision over the entire energy range.

The quality of measured photon emission rates and the corresponding deduced relative efficiencies depend on several factors. These include the reproducibility of the source positions, determination of precise and reproducible peak areas, proper handling of coincident and random summing, knowledge of attenuation in the source, and accurate primary decay data. The source-detector distance was carefully measured from the source to a convenient reference point on the detector cap. Our main calibration was taken at a source-detector distance of 15.1 cm, but we also took some measurements at 100.1 cm. The source-detector distance was measured with a micrometer caliper, yielding a 0.2-mm uncertainty at 15.1 cm, and 0.3 mm at 100.1 cm.

In all, we have measured thirteen individual sources of ten radionuclides – $^{48}\text{Cr}$, $^{60}\text{Co}$, $^{88}\text{Y}$, $^{108m}\text{Ag}$, $^{109}\text{Cd}$, $^{120m}\text{Sb}$, $^{133}\text{Ba}$, $^{134}\text{Cs}$, $^{137}\text{Cs}$ and $^{180m}\text{Hf}$ – with activities between 2 and 47 kBq. We prepared three of these sources ourselves, $^{180m}\text{Hf}$ ($t_{1/2} = 5.5$ h) with the Texas A&M reactor, and $^{48}\text{Cr}$ (21.6 h) and $^{120m}\text{Sb}$ (5.8 d) with the K500 Cyclotron. Two $^{60}\text{Co}$ sources were specially provided by the Physikalisch-Technische Bundesanstalt, PTB, Braunschweig, Germany and have quoted uncertainties in their activities of 0.06%. The remaining eight sources were purchased commercially and have activities quoted to 3%. Only the precisely known $^{60}\text{Co}$ sources were used to determine absolute efficiencies; all the others provided relative efficiencies that covered overlapping energy regions, which interconnected to one another and to the absolute $^{60}\text{Co}$ points.

For each source, we determined the relative efficiencies of its $\gamma$-ray lines by comparing the measured peak areas, determined with GF2, a least-squares peak-fitting program in the RADware series, with the corresponding known intensities. The sources of greatest importance to a precision calibration are those exhibiting simple $\gamma$-ray cascades uncomplicated by large conversion-electron components or by any possible $\beta$ side feeding. Except for the calculable effects of electron conversion, the intensities of such cascaded $\gamma$-ray transitions are unambiguously equal. Of particular value to this work were the following sources (and cascaded $\gamma$-ray energies in keV): $^{48}\text{Cr}$ (112.4, 308.3), $^{108m}\text{Ag}$ (433.9, 614.28, 722.9), $^{120m}\text{Sb}$ (89.8, 197.3, 1023.1, 1171.3) and $^{180m}\text{Hf}$ (215.3, 332.2).

At our source-detector distance of 15.1 cm, the detector efficiency was between 0.2 and 1.0% over the energy region of interest. Thus, coincidence summing – the simultaneous detection of two $\gamma$-rays from the same decay event – could not be neglected. This summing results in a loss of counts from the peak of any $\gamma$-ray that is in cascade with another $\gamma$-ray, and an increase in peak counts for any crossover $\gamma$-
ray. In making these corrections, we took full account of any angular correlations between the summing \( \gamma \)-rays and, where necessary, used the detector’s total efficiency (peak plus Compton), which we determined as a function of \( \gamma \)-ray energy in a separate set of measurements with sources selected for their relatively uncomplicated spectra. The sources used (and their relevant \( \gamma \)-ray energies in keV) were: \( ^{109} \)Cd (88.0), \( ^{48} \)Cr (112.4), \( ^{57} \)Co (123.7), \( ^{22} \)Na (511.0 and 1274.5), \( ^{137} \)Cs (661.7), \( ^{54} \)Mn (834.8) and \( ^{60} \)Co (average of 1253). The results appear in Fig. 1.

In addition to acquiring calibration spectra, we also made a number of measurements designed to reveal the physical dimensions and location of the detector's Ge crystal in its housing. These measurements included a scan of the side of the detector with a tightly collimated \( ^{133} \)Ba source, to determine the crystal length; a pair of \( ^{57} \)Co spectra recorded at 4- and 20-cm source-detector distances, to locate the front surface of the crystal; and an overall x-ray picture of the crystal in its housing, to establish its exact orientation. This information was then used as input to Monte Carlo calculations performed with the electron and photon transport code CYLTRAN.

One further complication arose when we compared the Monte Carlo calculations with measurements at energies below \( \sim 80 \) keV: the former did not exactly reproduce the strengths of the measured Ge X-ray escape peaks, presumably because we had, of necessity, assumed that the detector has a uniform front deadlayer. In reality, this region of the detector is probably not uniform and, furthermore, it likely has only partial charge collection. Therefore, it is quite reasonable that the simple representation in the Monte Carlo calculations does not completely represent the measured data. Accordingly we undertook another separate study, of X-ray escape from our detector, with the following sources (and their relevant \( \gamma \)-, or X-ray energies in keV): \( ^{109} \)Cd (22.1, 24.9), \( ^{120m} \)Sb (25.2, 28.6), \( ^{137} \)Cs (32.0), \( ^{152} \)Eu (39.9), \( ^{180m} \)Hf (93.3) and \( ^{48} \)Cr (112.4). The results appear in Fig. 2. In subsequent comparisons with efficiency data, we added the Monte-Carlo calculated full-energy and X-ray-escape peaks together and used the experiment-based escape-to-full-energy function (Fig. 2) to derive the full-energy peak area from that sum. It is this result that we refer to as the “Monte-Carlo calculated” efficiency for peaks below 120 keV.

![Figure 1](image-url)
Now, with only the detector’s two dead-layers as adjustable parameters, we achieved excellent agreement ($\chi^2/N = 0.8$) between the Monte Carlo efficiency results and our 40 measured data points between 22 and 1836 keV. The results are shown in Fig. 3, where they are plotted as differences between the measured efficiencies and the Monte Carlo calculated values, expressed in percent. Each source is separately identified to make clear how the energy ranges overlap one another. We also compared calculation and experiment at a source-detector distance of 100.1 cm with no further parameter adjustment. They differed by a mere (energy independent) 0.8%.

Our data show that a well-determined set of Monte Carlo calculations can be used to interpolate with high precision between measured relative (or absolute) Ge-detector efficiencies. In our case, with 10 sources that included one whose activity was known to 0.06%, we have obtained an efficiency curve with relative uncertainties of 0.15% and absolute uncertainties of 0.2% from 50 to 1400 keV. Although the curves only strictly apply to a source distance of 15.1 cm, the distance for which we actually require the calibration, it is evident from our results at 100.1 cm that one could expect similar uncertainties if the Monte Carlo calculations were used to obtain efficiencies at distances substantially different from 15 cm.

**Figure 2.** The ratio of the summed x-ray escape-peak areas to the full-energy peak area, plotted as a function of incident photon energy. The open circles with uncertainties are our measured data; the three points marked with an X are Monte Carlo calculated. The dashed curve is calculated with an expression due to Hansen et al [3]; the solid curve is the same calculation scaled upwards by 16%.

**Figure 3.** Differences between the measured efficiencies (points) and Monte Carlo calculated values (line at 0) plotted for the individual sources.
References

