Figure 1: Measured (solid circles) and calculated (solid lines) 2 MeV/u projectile charge fractions as a function of nitrogen target thickness for incident projectile charge of 18+.

Charge Equilibrium of 2 MeV/u Xe Ions in Nitrogen

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A complete set of cross sections for 2 MeV/u Xe projectile charge change in single collisions with nitrogen gas molecules was derived from a representative sample by studying the underlying systematics and expressing the findings in terms of analytical functions [1]. Here we use these results to predict the target thickness dependence of the projectile charge distribution, as well as the parameters of its charge equilibrium state.

Projectiles repeatedly undergo charge-changing collisions as they travel through the target and the distribution of their charge (charge fractions $F_i$) as a function of target thickness $x$ can be calculated from the rate equations

$$\frac{dF_i}{dx} = \sum_{j=0}^{Z-1} F_{ij} (F_j - F_i)$$

in which indices $i$ and $j$ run from zero to $Z$ (the atomic number of the projectile). The rate equations can be solved numerically in a straightforward way. However, the accuracy of the solutions will depend on the accuracy of the charge-changing cross section values $F_{ij}$. In this work, these values were estimated using eqs. (1) through (4) given in the preceding report [1].

Calculated projectile charge fractions as a function of target thickness are compared with the results of measurements in Fig. 1 for the incident projectile charge of 18+. Generally, the agreement is satisfactory, which implies that the estimated values of relevant cross sections were reasonably accurate. To a good approximation, each calculated curve is linear, as long as the product of target atom areal density and the corresponding cross section for direct charge change from the incident projectile charge is much smaller than 1. Multiple collisions result in the addition of a quadratic term to the charge fraction dependence on target thickness, which is manifested by the initial "bending" of the curves in Fig. 1. Contributions from multiple collisions can be properly taken into account only if the charge-changing cross sections are known for initial charges other than that of the incident projectile.

A large number of collisions between the projectiles and target atoms will result in the equilibration of the projectile charge distribution. The equilibrium charge distribution calculated from the rate equations will be accurate if the relevant charge-changing cross sections are estimated with good accuracy and if they predominantly depend on the initial and final
Figure 2: Calculated equilibrium charge distribution of 2 MeV/u Xe ions traveling in nitrogen gas (solid circles). The solid line represents a Gaussian fit to the calculated data.

It was noted that the equilibrium charge distribution calculated from the rate equations applies to projectiles that reach charge equilibrium with energy equal to that corresponding to the charge-changing cross section values used in the calculations. Equilibrium-state charge fractions can be obtained from the rate equations as the asymptotic values of its solutions when the target thickness tends to infinity. The charge-equilibration target thickness can be estimated based on these results.

The equilibrium charge distribution of 2 MeV/u Xe ions traveling in nitrogen gas was calculated from the rate equations using the charge-changing cross section values from eqs. (1) through (4) given in the preceding report [1]. The results are shown in Fig. 2. It is apparent that the calculated distribution is rather symmetric. A Gaussian fit to the calculated points yields a most probable charge of 23.8 and a standard deviation of 2.1 charge units. This best-fit value of the most probable charge agrees with the value of 23.8 predicted by energy loss calculations [2].

\[ S(Z) = \text{stopping power of the ion with atomic number } Z. \]

The standard deviation of the calculated distribution agrees well with the value of 2.1 calculated using the formula of Nikolaev and Dimitriev [3].

Equation (3) in which \( S(Z) \) denotes the stopping power of the ion with atomic number \( Z \).

Fig. 3 shows the average projectile charge calculated from the charge fractions shown in Fig. 1. The average charge approaches its equilibrium value within 0.1 charge units at a target thickness greater than about 0.28 atoms/Mb (6.5 \( \text{g/cm}^2 \)). This value is expected to be even larger at higher projectile energies. Therefore, a direct measurement of the projectile equilibrium charge distribution would typically require at least a 50 cm long differentially pumped gas cell at a pressure of up to 100 mTorr. The combination of a longer gas cell and a lower maximum pressure would be preferred in order to increase the average time the projectile spends between consecutive collisions. However, the longer the gas cell, the more difficult it becomes to align and focus the beam.

It was discovered that the equilibrium charge of the projectile can be determined with good accuracy from thin target measurements of the charge-changing cross sections for just a few selected projectile incident charge states without using any extrapolation techniques and without...
The average charge change in a single collision between a projectile having incident charge $q$ and a target atom/molecule can be determined using the expression

$$\Delta q = q - q_0$$

in which

$$q_0 = q_0(q)$$

is the charge-changing probability and

$$\sigma_{cc} = \sigma_{cc}(q)$$

is the total charge-changing cross section.

The calculated average charge change for 2 MeV/u Xe projectiles colliding with nitrogen atoms is plotted in Fig. 4 as a function of projectile charge. It was found that can be represented to good accuracy by the function shown by the solid curve in Fig. 4. The best-fit parameter values are $A = 0.5212$, $B = -1.853$, $q_c = 22.04$, and $q_w = 5.560$. The expected charge change is equal to zero when $q = 23.7$. Therefore, this value of $q$ can be interpreted as the equilibrium charge. Indeed, its value is within less than 0.2 charge units from that determined using the rate equations. The same result can be obtained by plotting the "weighted" capture and loss probability as a function of the projectile incident charge, as shown in Fig. 5. The weighted probabilities are defined here as
Figure 5: Weighted capture (solid circles) and loss (open circles) probability as a function of the 2 MeV/u Xe projectile incident charge, calculated using eqs. (8, 9). The solid line represents the fitting function defined by eq. (10).

The two weighted probabilities are well represented by the function

\[ P = q_a \frac{e^{-q/a}}{q_a} - q_c \frac{e^{-q/c}}{q_c} \]

as shown in Fig. 5. The best-fit parameter values are \( q_a = 23.6 \) and \( q_c = 4.5 \). The + sign in eq. (10) is for electron capture, while the - sign is for electron loss. The equilibrium charge \( q \) should be equal to the charge \( q_a \) at which the two weighted probabilities are equal to each other and to 0.5. This is in good agreement with the calculated average charge values presented above. An interesting feature here is that the capture and loss fitting functions [eq. (10)] are mirror images of each other reflected about the vertical line \( q = q_a \) and also about the horizontal line \( P = 0.5 \).

According to this analysis, the average equilibrium charge of a projectile in a gas target may be determined by finding the incident charge for which the weighted capture probability is the closest to the weighted loss probability (or 0.5). This procedure should be valid when the equilibrium charge distribution is symmetric and shell effects are absent. It is particularly applicable to heavy MeV/u projectiles for which the equilibrium charge is not close to \( Z \).

References

