Shape Fluctuations in Finite Fermi System with Nonlinear Dissipativity

V. M. Kolomietz, S. V. Lukyanoy, and S. Shlomo

Institute of Nuclear Research, Kiev 03028, Ukraine

In general, the total width of a giant multipole resonance (GMR) in a cold nuclei is determined by two-body collisional damping, particle-hole fragmentation (Landau damping) and escape width. However, in hot nuclei an additional contribution to the width can appear through the thermodynamic fluctuation of the nuclear shape due to the fluctuation-dissipation theorem. In the present work, we study the fluctuation contribution to the width of a GMR caused by the nonlinear dissipativity in the collision integral.

We start from the collisional kinetic equation and then reduce it to the macroscopic equation of motion for the nuclear shape variable \( Q(t) \) assuming a quadrupole distortion of the Fermi surface in momentum space. In the case of small amplitude motion, the derived equation of motion reads

\[
B_0 \dot{Q}(t) + B \dot{Q}^2(t) + C_{LD} Q(t) + D \int_{-\infty}^{t} dt_1 e^{\frac{t-t_1}{\tau}} \dot{Q}(t_1) + \sum_{n=2}^{3} D_n \int_{-\infty}^{t} dt_1 \cdots \int_{-\infty}^{t_n} dt_n \times e^{\frac{t-t_n}{\tau}} \cdots e^{\frac{t-t_{n-1}}{\tau}} \dot{Q}(t_n) \cdots \dot{Q}(t_1) = \xi(t),
\]

where \( \tau \) is the relaxation time and \( \xi(t) \) is the random force. The transport coefficients \( B, D \), and \( C_{LD} \) have been evaluated assuming that the Fermi liquid is irrotational and incompressible. The equation of motion (1) is non-Markovian and contains the dissipative nonlinearity because of the term \( \dot{Q}^2 \) in the memory integral (last term on the left-hand side of Eq. (1)). Performing the ensemble averaging of Eq. (1) and using the correlation properties for the averaged constructions like \( \langle \dot{Q}(t_1) \dot{Q}(t_2) \dot{Q}(t_3) \rangle \sim \langle \dot{Q}(t_1) \rangle \langle \dot{Q}(t_2) \dot{Q}(t_3) \rangle + \ldots \), see Ref. [1], the equation of motion (1) was reduced to a linear equation for the average shape variable \( \langle Q(t) \rangle \). In the case of the eigenvibrations, \( \langle Q(t) \rangle \sim \exp(i\omega t) \), the problem is then reduced to the solution of the following secular equation,

\[
\omega_0^2 + \Delta \omega_0^2 - \omega^2 + i\omega(\gamma_0 + \Delta \gamma) = 0,
\]

where \( \omega_0 \) is derived from the equation

\[
\omega_0 = \sqrt{[C_{LD} + C'(\omega_0)]/B_0}
\]

with

\[
C'(\omega_0) = D/(\omega_0 \tau)^2/[1 + (\omega_0 \tau)^2].
\]

The stiffness coefficient \( C'(\omega_0) \) is caused by the Fermi distortion effect. The additional coefficient \( C'(\omega_0) \) and the corresponding eigenenergy \( h\omega_0 \) are temperature dependent because of the temperature dependency of the relaxation time [2].

\[
\tau \sim [(h\omega_0)^2 + (2\pi T)^2]^{-1}.
\]

The friction coefficient

\[
\gamma_0 = D_1 \tau / B_0[1 + (\omega_0 \tau)^2]
\]
in Eq. (2) represents the usual collisional damping. The additional contribution to the friction, \( \Delta \gamma \), and to the eigenfrequency, \( \Delta \omega_0 \), in Eq. (2) are caused by the thermodynamical fluctuations and appear due to the dissipative nonlinearity in Eq. (1).

We have performed numerical calculations of the fluctuation contribution to the friction coefficient, \( \Delta \gamma \), and the corresponding contribution to the width \( \Gamma = h\gamma = h(\gamma_0 + \Delta \gamma) \) of the isoscalar giant quadrupole resonance. In Fig. 1 we show the dependence of the ratio \( \Delta \gamma / \gamma_0 \) on the temperature \( T \).

The solid curve corresponds to the calculations with the temperature dependent eigenenergy \( \hbar \omega_0 \) (see above). For the dashed line, we used the phenomenological parameterization \( \hbar \omega_0 = 63A^{-1/3}\text{ MeV} \).

The ratio \( \Delta \gamma / \gamma_0 \) goes to zero at \( T=0 \) because the friction coefficient is caused by the thermodynamical fluctuations and it disappears at \( T=0 \).

In Fig. 2 we have plotted the temperature dependence of the width \( \Gamma \) for two sets of \( \hbar \omega_0 \) and \( \gamma \). The curves in Fig. 2 were obtained with temperature dependent \( \hbar \omega_0 \) (curve 1) and with \( \hbar \omega_0 = 63A^{-1/3}\text{ MeV} \) (curve 2). The friction coefficient \( \gamma \) was taken as \( \gamma = \gamma_0 \) (solid lines) and \( \gamma = \gamma_0 + \Delta \gamma \) (dashed lines). As seen from Fig. 2, the contribution to the width from the thermodynamical shape fluctuation does not exceed 10%.

![Figure 1](image1.png)

**Figure 1:** The ratio \( \Delta \gamma / \gamma_0 \) as a function of temperature \( T \) for the nucleus \( A=224 \) for isoscalar giant quadrupole resonance (ISGQR). For the solid curve we use the temperature dependent eigenenergy \( \hbar \omega_0 \) (see text) and for the dashed line we used the phenomenological parameterization \( \hbar \omega_0 = 63A^{-1/3}\text{ MeV} \).

![Figure 2](image2.png)

**Figure 2:** The width \( \Gamma \) of the ISGQR as a function of temperature for the nucleus \( A=224 \). The solid curves were obtained by taking into account the collision friction coefficient \( \gamma_0 \) only and the dashed curves were obtained with the additional contribution \( \Delta \gamma \) to the friction coefficient from the thermodynamical shape fluctuation. The curves 1 were obtained with the temperature dependent eigenenergy \( \hbar \omega_0 \) (see text) and for curves 2 we use the phenomenological parameterization \( \hbar \omega_0 = 63A^{-1/3}\text{ MeV} \).

**References**
