ANCs and Radiation Widths of the First Two Resonances in $^{17}$F
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Asymptotic normalization coefficients (ANC), defining the normalization of the tail of nuclear bound-state wave functions projected onto the two-body channels, play an important role in nuclear reaction and structure theory. In our previous work, we have demonstrated that ANCs can be successfully used as normalization coefficients for the nonresonant capture amplitudes in the R matrix approach [1]. The purpose of this work is to demonstrate in the R matrix approach that ANCs can be used to put limits on the value of the radiative width of a resonance decaying to a bound state with known ANC.

Let us consider proton radiative capture to a bound state through an isolated single-channel resonance with the proton partial width $\Gamma_p$. In the R matrix method, the radiative width of the resonance decaying to the bound state is given by

$$\Gamma_\gamma(E) = |\Gamma_{\gamma(ch)}^{1/2}(E) - \Gamma_{\gamma(int)}^{1/2}(E)|^2. \quad (1)$$

Here,

$$\Gamma_{\gamma(ch)}^{1/2}(E) = \lambda \eta^{-1/2} e^{-i\phi} C \Gamma^{1/2}$$

$$\times \int_{r_0}^{\infty} drr^{L+1} W_{-\eta, j_{1}+1/2}^{1/2}(r) O_{j_{1}}(r), \quad (2)$$

and

$$\Gamma_{\gamma(int)}^{1/2}(E) = \lambda \int_{0}^{\infty} drr^{L+2} \phi(r) \times \psi_{l}(r), \quad (3)$$

$\lambda$ is the kinematical factor, $\psi_{l}(r)$ is the proton-target standing wave describing the resonance in the nuclear interior; $O_{j_{1}}$ is the outgoing spherical wave, $\nu$ is the proton-target relative velocity in the initial channel, $\phi$ is the solid-sphere scattering phase shift, $\phi(r)$ is the bound-state radial wave function which at $r > r_0$ behaves as

$$\phi(r) = C W_{-\eta, j_{1}+1/2}(r)/r, \quad (4)$$

$C$ is the ANC of the bound state, $W_{-\eta, j_{1}+1/2}(r)$ is the Whittaker function, $\eta$ is the bound-state Coulomb parameter $l_{1}$ and $l$ are the resonance and bound-state orbital angular momenta, $L$ is the multipolarity of the transition from the resonance to the bound state, and $r_0$ is the channel radius in the R matrix method. $\Gamma_{\gamma(int)}^{1/2}(E)$ and $\Gamma_{\gamma(ch)}^{1/2}(E)$ are the internal and external contributions to the radiative width. In the R matrix, for $r < r_0$ the resonant scattering wave function $\psi_{l}(r)$ is real, while for $r > r_0$ it is complex because it is given by the outgoing spherical wave. Hence, $\Gamma_{\gamma(int)}^{1/2}(E)$ is real and $\Gamma_{\gamma(ch)}^{1/2}(E)$ is complex. Then, $(\Im \Gamma_{\gamma(ch)}^{1/2}(E))^2$ gives a lower limit for the radiative width. If the bound state is loosely bound, the external contribution to the resonance width dominates. Then, if the signs of $\Re \Gamma_{\gamma(ch)}^{1/2}(E)$ and $\Gamma_{\gamma(ch)}^{1/2}(E)$ are the same, $|\Gamma_{\gamma(ch)}^{1/2}(E)|^2$ provides an upper limit of the radiative width. Thus, the knowledge of the ANC and proton partial width would be enough to estimate the limits of the radiative widths.
We demonstrate this for the radiative widths of the first two resonances in $^{17}$F. The parameters of the second resonance are $J^\pi = 5/2^-$, energy $E_R = 3.257$ MeV, proton partial width $\Gamma = 1.5 \pm 0.2$ keV. This resonance decays entirely to the ground state $1d_{5/2}, l = 2$ with binding energy $\varepsilon = 0.605$ MeV. The measured radiative width is $\Gamma \leq 0.11 \pm 0.02$ eV [2]. The ANC for the ground state of $^{17}$F has been determined in our work [2] from $^{16}$O($^3$He, d)$^{17}$F: $C^2 = 1.1 \pm 0.1$ fm$^{-1}$. Using this ANC, we were able to reproduce the experimental low-energy astrophysical factor for $^{16}$O(p, $^\gamma$)$^{17}$F [2]. Using the experimental proton partial width and our ANC we get for the radiative width, $\Gamma_{\gamma}(E_R) \geq 0.037 \pm 0.004$ meV for $r_0 = 5$ fm. This result does not contradict the experimental radiative width. Note that, due to the low binding energy of the bound state, the dependence on the lower limit of the radiative width obtained here is quite weak.

Now we consider the first resonance $J^\pi = 1/2^-$, $l = 1$ with resonance energy $E_R = 2.500$ MeV, proton partial width $\Gamma = 19 \pm 1.0$ keV. This resonance decays entirely to the first excited state $2s_{1/2}, l = 0$ with binding energy $\varepsilon = 0.105$ MeV. The measured radiative width is $\Gamma_{\gamma} = 0.012 \pm 0.002$ eV [1]. The ANC for the excited state of $^{17}$F has been determined in our work [2] from $^{16}$O($^3$He, d)$^{17}$F: $C^2 = 6490 \pm 680$ fm$^{-1}$. We get for the radiative width $\Gamma_{\gamma}(E_R) \geq 0.030 \pm 0.004$ eV for $r_0 = 5.0$ fm, and $\Gamma_{\gamma}(E_R) \geq 0.027 \pm 0.003$ eV for $r_0 = 5.5$ fm. Thus, our low limit exceeds by more than $3\sigma$ the upper limit of the experimental radiative width given in [3], but agrees with the upper limit $\Gamma_{\gamma}(E_R) \leq 0.03$ eV determined in [4]. It calls for a new measurement of the radiative width of the first resonance in $^{17}$F.

References