Kinetic Equation with Exact Charge Conservation

Z. W. Lin, C. M. Ko, V. Koch[a], K. Redlich[a,b], M. Stephanov[c,d] and Xin-Nian Wang[a]

[a]*Nuclear Science Division, Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, CA 94720*

[b]*Institute of Theoretical Physics, University of Wroclaw, PL-50204 Wroclaw, Poland*

[c]*Department of Physics, University of Illinois, Chicago, IL 60607-7059*

[d]*RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973*

Recent analyses have shown that most hadrons measured in heavy ion collisions can be described by statistical models based on the grand canonical ensemble for abundant particles [1] and the canonical ensemble for rare particles [2]. However, transport model studies indicate that the chemical equilibration time for rare particles is an order of magnitude longer than the heavy ion collision time [3], making the applicability of statistical model questionable. To gain insight to this puzzle, we have formulated a kinetic theory for the time evolution of particle production [4].

In the standard formulation, the rate equation for a binary process $a_1a_2 \leftrightarrow b_1b_2$ with $a \neq b$ is described by the following population equation:

$$\frac{d\langle N_{b_1}\rangle}{d\tau} = \frac{G}{V}\langle N_{a_1}\rangle\langle N_{a_2}\rangle - \frac{L}{V}\langle N_{b_1}\rangle\langle N_{b_2}\rangle, \quad (1)$$

where $G \equiv \langle \sigma_G v\rangle$ and $L \equiv \langle \sigma_L v\rangle$ give the momentum-averaged cross sections for the gain process $a_1a_2 \leftrightarrow b_1b_2$ and the loss process $b_1b_2 \leftrightarrow a_1a_2$, respectively. N_k represents the total number of particles k, and V is the proper volume. We consider processes in which b_1 and b_2 are constrained by $U(1)$ charge conservation, such as kaon production/annihilation via $B^+B^- \leftrightarrow K^+K^-$.

To account for the correlation between the production/annihilation of particles b_1 and b_2, the general rate equation for the average number of b_1b_2 pairs should be written as

$$\frac{d\langle N\rangle}{d\tau} = \frac{G}{V}\langle N_{a_1}\rangle\langle N_{a_2}\rangle - \frac{L}{V}\langle N^2\rangle. \quad (2)$$

For abundant production of b_1b_2 pairs, where $\langle N\rangle \gg 1$, we have $\langle N^2\rangle \approx \langle N\rangle^2$ and Eq.(2) reduces to the standard form. However, for rare production of b_1b_2 pairs, where $\langle N\rangle \ll 1$, we have instead $\langle N^2\rangle \approx \langle N\rangle$, which reduces Eq. (2) to the following form:

$$\frac{d\langle N\rangle}{d\tau} \approx \frac{G}{V}\langle N_{a_1}\rangle\langle N_{a_2}\rangle - \frac{L}{V}\langle N\rangle. \quad (3)$$

Thus, in the limit $\langle N\rangle \ll 1$, the absorption term depends on the pair number only *linearly*, instead of quadratically for the limit $\langle N\rangle \gg 1$.

In the limit when $\langle N\rangle \gg 1$, the standard Eq. (1) is valid and has the following well-known solution:

$$\langle N\rangle^{\mathrm{GC}}(\tau) = N_{\mathrm{eq}}^{\mathrm{GC}} \tanh\left(\tau/\tau_0^{\mathrm{GC}}\right), \quad (4)$$

where the equilibrium value for the number of b_1b_2 pairs N_{eq}^{GC} and the relaxation time constant τ_0^{GC} are given by

$$N_{\mathrm{eq}}^{\mathrm{GC}} = \sqrt{\epsilon}, \quad \tau_0^{\mathrm{GC}} = \frac{V}{L\sqrt{\epsilon}}, \quad (5)$$

respectively, with $\epsilon \equiv G\langle N_{a1}\rangle\langle N_{a2}\rangle / L$.

In the special case where particle momentum distributions are thermal, the gain

(G) and loss (L) terms just represent the thermal averages of the production and absorption cross sections, and their ratio is

$$\frac{G}{L} = \frac{d_{b_1}\alpha_{b_1}^2 K_2(\alpha_{b_1}) d_{b_2}\alpha_{b_2}^2 K_2(\alpha_{b_2})}{d_{a_1}\alpha_{a_1}^2 K_2(\alpha_{a_1}) d_{a_2}\alpha_{a_2}^2 K_2(\alpha_{a_2})}, \qquad (6)$$

where d_k's denote the degeneracy factors, "$_k \equiv m_k/T$, and K_2 represents the modified Bessel function. The equilibrium value for the number of b_1b_2 pairs in Eq.(5) now reads as

$$N_{\rm eq}^{\rm GC} = \frac{d_{b_1}}{2\pi^2} VT^3 \alpha_{b_1}^2 K_2(\alpha_{b_1}). \qquad (7)$$

Thus it is described by the Grand Canonical (GC) result with vanishing chemical potential due to our requirement of the $U(1)$ charge neutrality of the system.

In the opposite limit where $\langle N\rangle \ll 1$, the time evolution is described by Eq.(3), which has the following solution:

$$\langle N\rangle^{\rm C}(\tau) = N_{\rm eq}^{\rm C}\left(1 - e^{-\tau/\tau_0^{\rm C}}\right), \qquad (8)$$

with the equilibrium value and relaxation time given by

$$N_{\rm eq}^{\rm C} = \epsilon, \quad \tau_0^{\rm C} = \frac{V}{L}. \qquad (9)$$

With a thermal momentum distribution the equilibrium value of b_1b_2 pair multiplicity has the following form:

$$N_{\rm eq}^{\rm C} = \left[\frac{d_{b_1}}{2\pi^2} VT^3 \alpha_{b_1}^2 K_2(\alpha_{b_1})\right] \times \left[\frac{d_{b_2}}{2\pi^2} VT^3 \alpha_{b_2}^2 K_2(\alpha_{b_2})\right]. \qquad (10)$$

This equation thus demonstrates the locality of the U(1) charge conservation. With each particle b_1, a particle b_2 with the opposite charge is produced in the same event in order to conserve charge locally. This is the result expected from the Canonical (C) formulation of conservation laws.

Eqs.(7,10) show that the equilibrium multiplicity in the canonical formulation for rare particles is much lower than what is expected from the grand canonical result,

$$N_{\rm eq}^{\rm C} = (N_{\rm eq}^{\rm GC})^2 \ll N_{\rm eq}^{\rm GC}. \qquad (11)$$

We also note that the volume dependence in the two cases is different. The particle density in the GC (abundant) limit is independent of V, whereas in the opposite canonical (rare) limit the density scales linearly with V.

Secondly, the relaxation time for a canonical system is far shorter than what is expected from the grand canonical result,

$$\tau_0^{\rm C} = \tau_0^{\rm GC} N_{\rm eq}^{\rm GC} \ll \tau_0^{\rm GC}, \qquad (12)$$

due to small number of particles ($N_{eq}^{GC} \ll 1$). For example, the total number of produced kaons in Au+Au collisions at 1 GeV/A is of the order of 0.02. Thus the canonical relaxation time is a factor of 7 shorter than what is expected from the grand canonical formulation.

References

[1] P. Braun-Munzinger, I. Heppe and J. Stachel, Phys. Lett. **B465**, 15 (1999).

[2] J. S. Hamieh, K. Redlich and A. Tounsi, Phys. Lett. **B486**, 61 (2000).

[3] J. E. L. Bratkovskaya, W. Cassing, C. Greiner, M. Effenberger, U. Mosel and A. Sibirtsev, Nucl. Phys. **A675**, 661 (2000).

[4] C. M. Ko, V. Koch, Z. W. Lin, K. Redlich, M. Stephanov, and X. N. Wang, Phys. Rev. Lett., in press; nucl-th/0010004.