# Equation of State and Phase Transitions in Asymmetric Nuclear Matter

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We have studied the structure of 3-dimensional pressure-temperature-asymmetry surface of equilibrium and the caloric curves for the asymmetric nuclear matter. The thermal Thomas-Fermi approximation with the effective SkM force was used. We have evaluated the free energy *F*, the pressure *P* (equation of state) and the chemical potential $:_q$ (*q* = *n* for neutron and *q* = *p* for proton) as functions of the particle density $\Delta = \Delta_n + \Delta_p$, temperature *T* and asymmetry parameter $X = (\Delta_n - \Delta_p)/(\Delta_n + \Delta_p)$. A numerical calculation of the pressure *P* shows a van der Waals-like behavior for the isotherms $P = P(T, X, \Delta)$ describing both the liquid and the vapor phases. To get the shape of the (*P*, *T*, *X*)-surface of equilibrium we have used the Gibbs equilibrium conditions:

$$P^{\text{liq}}(T,X,\rho) = P^{\text{vap}}(T,X,\rho),$$

$$\mu_q^{\text{liq}}(T,X,\rho) = \mu_q^{\text{vap}}(T,X,\rho), \qquad (1)$$

where indices "liq" and "vap" denote the liquid and vapor phases, respectively.

The fragment of the equilibrium surface obtained in our calculation is shown in Fig. 1. This figure represents the equilibrium states having $X > 0$, i.e. states of a neutron-rich nuclear matter. We point out that there are two sheets to the surface of equilibrium. The upper sheet is the surface of boiling and the lower one is the surface of condensation. The interior space between the sheets is the phase separation region where the two phases co-exist. The crossing points of both sheets with a straight line *P* = const, *T* = const give the equilibrium asymmetry parameters $X^{\text{liq}}$ and $X^{\text{vap}}$ for liquid and vapor phases, respectively. In general the vapor asymmetry $X^{\text{vap}}$ exceeds the corresponding liquid one $X^{\text{liq}}$ (at $X^{\text{liq}} > 0$). This is a feature of the nuclear matter. The density dependence of the isospin symmetry energy provides the condition $|\mu_n| < |\mu_p|$ and thus induces a preferable emission of neutrons. The above mentioned sheets of the surface of equilibrium coincide along the aseotropic (*P, T*)-line in *X* = 0 plane denoted by the letter A in Fig. 1. The aseotropic line is cut off at the critical temperature $T_c^{(\text{sym})}$ (point E in Fig. 1) of the symmetric nuclear matter derived by the condition

$$(\partial P/\partial\rho)_{T,X=0} = (\partial^2 P/\partial\rho^2)_{T,X=0} = 0. \qquad (2)$$

Both sheets of the surface of equilibrium also coincide along the critical line marked by the letter C in Fig. 1. The critical line of an asymmetric nuclear matter was derived by the condition

$$(\partial\mu_q/\partial X)_{T,P} = (\partial^2\mu_q/\partial X^2)_{T,P} = 0. \qquad (3)$$

We point out that the critical temperature $T_c$ derived by Eq. (3) at $X \neq 0$ is different than the one, $T_c^{(\text{sym})}$, obtained from Eq. (2). If one goes along the critical line C, the critical temperature

$T_c$ decreases and the corresponding pressure increases as the value of $X$ increases.

The crossing point of the critical line C with the drip line B of $:_n = 0$ (point D in Fig. 1) provides the maximum possible asymmetry for the bound ($\mu_q^{liq}$<0) liquid phase. This point is located at $X = 0.68$, $T = 10.4$ MeV and $P = 0.26$ MeV/fm$^3$ for the SkM interaction. Note that the cold nuclear matter at $T = 0$ is bound at $X$ <0.31. Thus, the hot nuclear matter can exist (in a bound state) at higher asymmetry than the cold one. This feature of hot nuclear matter appears

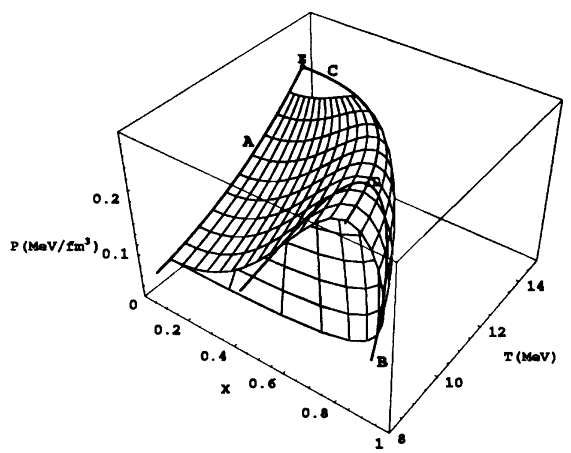


**Figure 1**: Surface of equilibrium in *P,T,X*-space. The upper sheet is the surface of boiling and the lower one is the suface of condensation. The line of the ciritcal pressure is marked by the letter C. The drip line $:_n$=0 is marked by the letter B. The point D notes the maximum possible asymmetry for the bound liquid phase at $:_n$<0 and the point E is the critical point for the symmetric system $X$=0.

because of the increasing of the symmetry energy with temperature.

We have studied the isobaric ($T$, $X$) phase diagrams obtained as a cut of the equilibrium surface of Fig. 1 by the plane $P = P_{ext}$ = const. These diagrams are very useful for obtaining physical insight into the liquid-vapor phase transition in hot asymmetric nuclear matter. The shape of ($T$,$X$)-diagram depends on the value of the pressure $P_{ext}$. The ($T$,$X$)-diagram contains the critical point $T_c$ if the value of pressure $P_{ext}$ exceeds the critical (maximal allowed) pressure $P_c^{(sym)}$ on the aseotropic line in Fig. 1. The corresponding ($T$,$X$)-diagram is shown in Fig. 2. The presence of the critical point on the ($T$,$X$) phase diagrams in Fig. 2 leads to a very specific effect of the retrograde condensation, see also [1]. If one goes along straight line AB in Fig. 2 (at closed volume), the liquid starts to boil at temperature $T_A$. An increase of the temperature leads to an increase of the evaporation. However the evaporation

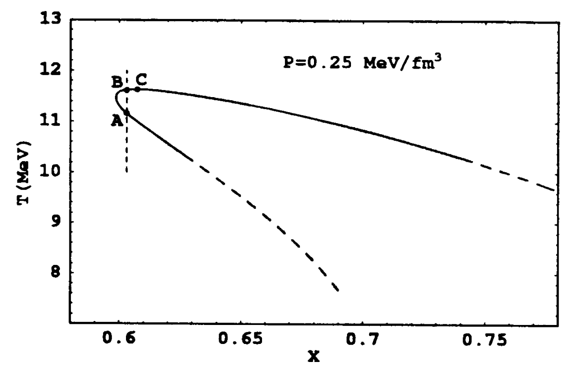


**Figure 2**: The cut of the (P,T,X)-surface of equilibrium by plane $P=P_{ext}> P_c^{(sym)}$ .

begins to decrease at certain temperature $T<T_B$ and the vapor disappears at temperature $T_B<T_C$.

The ($T$, $X$) phase diagrams allow us also to study the shape of the caloric curve for the case of isobaric heating. We have considered the case of the evaporation in a closed (but not fixed) volume at the fixed pressure $P < P_c^{(sym)}$. Introducing the volume fractions, $8^{liq}$ and $8^{vap}$ of the liquid and vapor phases we have derived the excitation energy per particle, $E^*/A$, as

$$\frac{E^*}{A} = \frac{\lambda^{liq}\mathrm{E}^{liq} + \lambda^{vap}\mathrm{E}^{vap}}{\lambda^{liq}\rho^{liq} + \lambda^{vap}\rho^{vap}} - \left(\frac{\mathrm{E}^{liq}}{\rho^{liq}}\right)_{T=0}, \quad (4)$$

where $\mathcal{E}^{liq}$ and $\mathcal{E}^{vap}$ are, respectively, the energy densities of the liquid and vapor phases, taken

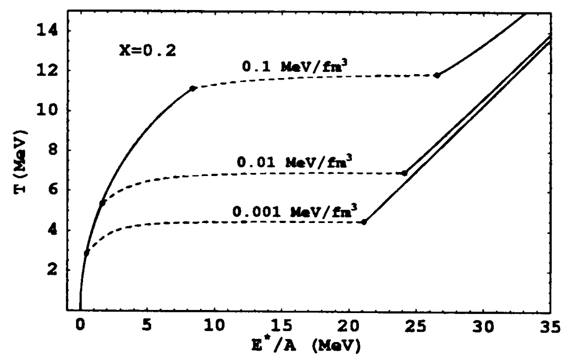


**Figure 3**: Caloric curves for isobaric heating of the asymmetric nuclear matter for different pressure $P$ shown near the curves at fixed asymmetry parameter $X = 0.2$.

for the corresponding values of the particle density $\Delta$ and the asymmetry parameter $X$. Caloric curves determined by Eq. (4) for $X = 0.2$ and pressures $P = 10^{-3}$, $10^{-2}$ and $10^{-1}$ MeV/fm$^3$ are presented in Fig. 3. The solid line at low values of the excitation energy $E^*/A$ corresponds to the heating of the degenerate Fermi liquid with $E^*/A \sim T^2$. The solid line at high excitation energy $E^*/A$ describes the classical Boltzmann's gas with $E^*/A = (3/2)T$. The region of two phase co-existence is displayed by the dashed line. The caloric curve is a continuous function of $E^*/A$ and has a break in its derivative at two points connecting the two-phase region with the corresponding single-phase regions. This indicates a phase transition of the first kind (boiling). As seen from the Fig. 3, the plateau region corresponds to the two-phase region. The value of plateau temperature increases with the increase of pressure. The experimental observation shows a nearly flat caloric curve with a temperature of about 7 MeV [2]. If one could assume the process of isobaric heating for the description of the experimental data, the order of magnitude of the pressure should be $10^{-2}$ MeV/fm$^3$ for this process.

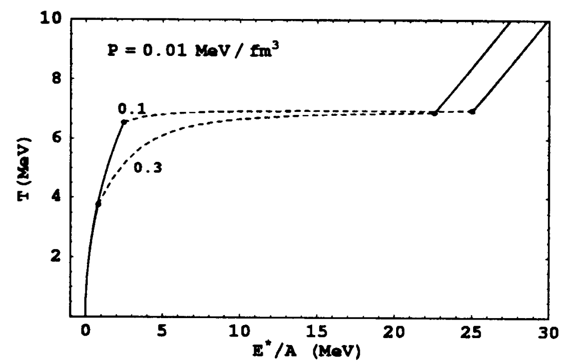


**Figure 4**: The same as in Fig. 3 for the fixed pressure $P$ = 0.01 MeV/fm$^3$ and the different asymmetry parameter $X$ shown near the curves.

The asymmetry dependence of the shape of the caloric curve is displayed in Fig. 4 by plotting two curves at $X$ = 0.1 and 0.3. The figure shows that the plateau temperature is slightly sensitive to the asymmetry parameter. At low asymmetry the two-phase region of the caloric curve is more flat and it is shifted to the lower values of $E^*/A$ as compared to the case of high asymmetry.

**References**

[1] H. Müller and B. Serot, Phys. Rev. C **52**, 2072 (1995).

[2] J. Cibor *et al.*, Phys. Lett **B473**, 29 (2000).