Coulomb Energy Differences in Mirror Nuclei Revisited

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Coulomb displacement energies (CDE) of analogue states (mirror nuclei),)E_C, provide a stringent test for nuclear models. Nolen and Schiffer (NS) pointed out [1] that a calculation of)E_C within the framework of the independent particle model (mean field approach) leads to disagreement with experimental values. Using a Wood-Saxon potential well which reproduced the experimental charge distribution, Nolen and Schiffer found)E_C to be ~ 7% smaller than the experimental values. This discrepancy between the experimental and the theoretical evaluation of)E_C in mirror nuclei, referred to as the NS anomaly (NSA), has been the subject of many investigations in which various correction terms, namely, the exchange term, vacuum polarization, electromagnetic spin-orbit interaction, proton-neutron mass difference, finite size effect of the proton, center of mass motion, Auerbach-Kahana-Weneser effect, polarization of the core by the valence particle and the Thomas-Ehrman effect, were considered [2]. It has been established that the net contribution of all these correction terms is too small to explain the NSA [2]. The contribution of charge symmetry breaking (CSB) interaction was found [2] to be ~ 3% of)E_C, i.e., accounting for only half of the discrepancy between theory and experiment.

It was first pointed out by Shlomo and Love [3] that, contrary to earlier estimates, the effect of long range correlations (LRC) on)E_C is not negligible. Using the particle- vibration model and taking into account multipole excitations up to $L = 5$, significant contributions of ~ 1 - 3% to)E_C were obtained. Therefore, to explain the NSA, it is necessary to go beyond the non-relativistic mean field (NRMF) approximation and include the contributions due to the LRC effects and the CSB interaction. It is important to point out that long range ground state (RPA) correlation effects can account for the discrepancies between Skyrme Hartree-Fock (SHF) predictions and experimental results for the charge root-mean-square (RMS) radii, r_c, associated with anomalous kink in the mass dependence of r_c, (fluctuation in the isotope shift) [4]. Therefore a consistent description of)E_C and the anomalous kinks in r, is achieved by including the LRC contribution to the NRMF results.

In recent years, relativistic mean field (RMF) theory has been extremely successful in describing various facets of nuclear structure properties. In particular, they have proved to be very successful in reproducing the anomalous kink in r_c in various chains of nuclei. The anomalous kink in r_c in Cr isotopes that required invocation of LRC (zero point oscillations) for explanation in the NRMF theory [4] also finds a natural explanation [5] in the RMF approach. Because of these successes of the RMF theory, we revisit the problem of Coulomb energy difference in mirror nuclei and calculate the CDE within the RMF theory using the recent parameter sets that proved successful in reproducing the anomalous kink in r_c.

The Coulomb energy difference in mirror nuclei is given by

$$\Delta E_C = B(N+1, Z) - B(N, Z+1). \tag{1}$$

where $B(N, Z)$ is the binding energy for a nucleus with N neutrons and Z protons. In this work the calculation is based on the self-consistent determination of the energy $E(A)$ of the core nucleus with A nucleons ($N = Z$) and then determining the energy difference

$$\Delta = \mathrm{E}_p - \mathrm{E}_n, \tag{2}$$

where ε_p and ε_n are the single-particle energies of the proton and the neutron that has to be added to or removed from the core. Note that Δ forms the major part of ΔE_C. Due to the self-consistent calculation of $E(A)$, Δ includes the contributions of the AKW and the Thomas-Ehrman effects. A number of small but significant contributions like the exchange term, vacuum polarization, electromagnetic spin-orbit interaction, proton-neutron mass difference, finite size effect of the proton, center of mass motion and polarization of the core by the valence particle are to be added to Δ in order to compare it with ΔE_C. Suzuki et al. [6] included these correction terms to the Skyrme-Hartree-Fock (SHF) evaluation of Δ with the parameter set SGII. With the inclusion of the contributions due to CSB interaction, they found that the theoretical values are smaller than the experimental one by ~ 2%. Marcos et al. [7] carried out RMF calculation of ΔE_C, taking Δ from SHF as a reference model for comparison with predictions from the RMF. Their results (with parameter set labeled R2) underestimate the SHF values considerably, typically by ~ 5%.

We perform calculations with the NL3 parameter set [5] that includes nonlinear self-coupling of the Φ-meson. We have also repeated the calculations with the NL-SV2 parameter set [8]; here the self-coupling of vector meson is included. It has been shown [8] that vector self-coupling might be important for an accurate description of the nuclear shell effects, particularly, for nuclei near the drip line. We have carried out relativistic Hartree mean field calculations for six pairs of mirror nuclei, with ^{16}O, ^{40}Ca and, ^{56}Ni as the core.

The value of ΔE_C is determined predominantly by the RMS value of charge distribution of the core and that of the valence proton, in particle or hole state. The charge radii of the core nuclei ^{16}O, ^{40}Ca and ^{56}Ni are reproduced to within ~ 0.5% accuracy with the NL3 set and ~ 1% accuracy with the NL-SV2 set. The R2 parameter set also explains the experimental data extremely well. The SHF calculation for the nuclei ^{16}O and ^{40}Ca with SGII parameter set [6] also reproduces their experimental charge radii reasonably.

System	NL3	NL-SV2	R2^a	SGII^a	ΔE_C (EXP^b)
^{15}O - ^{15}N	3.925	3.876	3.667	3.808	3.536
^{17}F - ^{17}O	3.575	3.525	3.290	3.665	3.543
^{39}Ca - ^{39}K	7.352	7.381	7.249	7.450	7.313
^{41}Sc - ^{41}Ca	7.011	7.090	6.868	7.289	7.278
^{55}Ni - ^{55}Co	9.504	9.500	-	—	9422
^{57}Cu - ^{57}Ni	8.895	9.376	-	—	9.499

^{a} Ref. [7], ^{b} Ref. [9]

Table 1: The RMF and Skyrme-HF results for $\Delta = \varepsilon_p - \varepsilon_n$ and the experimental Coulomb energy differences ΔE_C in mirror nuclei (MeV).

In Table 1, the values of the Coulomb energy difference $\Delta = \varepsilon_p - \varepsilon_n$, calculated in our model are compared with those from the R2 set and also from the SGII set. In the earlier RMF calculation with the R2 parameter set, it is seen that the discrepancy with the SGII results is ~5%. With the NL3 parameter set, the said discrepancy is reduced to ~2.5%. Thus one can find that there is an overall improvement by approximately a factor of two with the NL3 parameter set. One also notes that with the inclusion of self-coupling of vector meson (NL-SV2 set), the results for) are further improved, though marginally.

Information on the valence neutron and proton wave functions can be deduced from magnetic form factor obtained in backward electron scattering. Since their extension has a seminal role in the determination of Coulomb energy, their calculations and comparison with the available experimental data may throw more light on the intricate nature of the NSA. With the NL3 parameter set, the rms radii of the valence neutron orbitals in ^{17}O and ^{41}Ca are larger than the experimental values by ~ 4%, leading to a decrease of) by ~ 2%. Therefore, increasing the values of) for the A = 17 and 41 mirror nuclei obtained in the RMF calculation with the NL3 parameter set by ~ 2%, we obtain an agreement within ~ 1% with the results obtained with the SGII interaction.

The net contributions [2,6] due to the exchange term, vacuum polarization, electromagnetic spin-orbit interaction, proton- neutron mass difference, finite size effect of the proton, center of mass motion and polarization of the core by the valence particle, decrease the calculated CDE by about 0.45, 0.20, 0.40 and 0.30 MeV for the A=15, 17, 39 and 41 mirror nuclei, respectively. Thus, the discrepancy between the mean-field (SHF or RMF) results for the CDE and the corresponding experimental values is reduced to about 3-5%. As pointed out earlier, this remaining discrepancy can be accounted for by including the contributions due to CSB interaction and LRC effects [3]. Our investigation also shows that, although the RMF with the NL3 parameter set reproduces the kink in the isotope variation of r_c, the values obtained for) are too small to account for the experimental values of the CDE without the addition of the contribution due to the LRC effects.

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