Relativistic Approaches to Structure Functions of Nuclei

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The major tool to compute nuclear structure functions, as measured in inclusive electron scattering on nuclei is a perturbation series in the interaction between the struck nucleon and the remaining spectator nucleus. The first term of this series, the so-called Impulse Approximation, is widely used in an analysis of data.

In the non-relativistic regime there exists an alternative expansion of the structure function, written in commutators of the residual interaction, and originally proposed by Gersch, Rodriguez and Smith (GRS) [1]. The nuclear structure function is then given by a series in inverse powers of the 3-momentum transfer $|\boldsymbol{q}|$ as

$$F(y_W, |\boldsymbol{q}|) = \sum_{j=0}^{\infty} \left(\frac{M}{|\boldsymbol{q}|} \right)^{j+1} F_j(y_W), \qquad (1)$$

where

$$y_W = -\frac{|\boldsymbol{q}|}{2} + \frac{Mv}{|\boldsymbol{q}|}, \qquad (2)$$

is the West scaling variable [2], v is the energy transfer and M is the nucleon mass. The leading, zero order, term is

$$F_0(y_W) = \frac{1}{4\pi^2} \int_{|y_W|}^{\infty} n(p) p dp, \qquad (3)$$

where $n(p)$ is the nucleon momentum distribution.

Although the GRS expansion shows a better convergence than the standard expansion in the powers of interaction [3], it has been only rarely used in nuclear physics due to the absence of a proper relativistic extension. The latter was found in Ref. [4]. It corresponded to a replacement of the struck nucleon Feynmann propagator

$$G_N = \frac{1}{(p+q)^2 - M^2 + i\eta} \qquad (4)$$

by the modified propagator

$$\widetilde{G}_N = \frac{1}{(p+q)^2 - p^2 + i\eta}, \qquad (5)$$

where $q = (\nu, \boldsymbol{q})$ is the 4-momentum transferred to the struck nucleon and p is the 4-momentum of the struck-nucleon. Yet, the final series involved 4-dimensional integrals, as in any relativistic perturbative expansion. The 3-dimensional reduction was therefore necessary for any practical use of this series. The latter constitutes a considerable problem in the standard perturbation series due to negative energy poles in nucleon propagators.

It appears, however, that our relativistic extension of the GRS series has a significant advantage over standard relativistic perturbative expansions. Indeed, it follows from a comparison between Eqs. (4) and (5) that the modified struck nucleon propagator has only one pole in the complex p_0-plane. This allows us to perform p_0-integration (3-dimensional reduction) in all terms of the relativistic GRS series. As a result we find a new expansion for the relativistic structure function [5]

$$F(y_G,|\boldsymbol{q}|)=\sum_{j=0}^{\infty}\left(\frac{M}{|\boldsymbol{q}|}\right)^{j+1}F_j(y_G,|\boldsymbol{q}|). \qquad (6)$$

This expansion is very similar to the non-relativistic one (1), where only the non-relativistic scaling variable y_W is replaced by a relativistic one [3,4]

$$y_G=-\frac{Q^2}{2|\boldsymbol{q}|}+\frac{(M-\Delta)\nu}{|\boldsymbol{q}|}, \qquad (7)$$

where $Q^2 = \boldsymbol{q}^2 - \nu^2$, and) = E_{A-1}-E_A is the nucleon separation energy.

The leading, zero order, term of this series is

$$F_0^{rel}(y_G,\boldsymbol{q})=\frac{1}{4\pi^2}\int_{|y_G|}^{\infty}pdp\int_0^{\overline{E}_{\max}}\mathrm{P}(p,E)dE$$

$$+\theta(y_G)\int_0^{y_G}pdp\int_{\overline{E}_{\min}}^{\overline{E}_{\max}}\mathrm{P}(p,E)dE, \qquad (8)$$

where $P(p,E)$ is the nuclear spectral function $\int_0^{\infty}\mathrm{P}(p,E)dE=n(p)$, and

$$\overline{E}_{\substack{\max\\ \min}}(q,y_G,p)=\frac{(y_G\pm p)|\boldsymbol{q}|}{\nu}. \qquad (9)$$

An essential distinction with the non-relativistic case, Eq. (3), is that the upper limit of integration, E_{max} in Eq. (8) is always finite, even in the limit $\boldsymbol{q}\to\infty$. Therefore the relativistic structure function is never given by the momentum distribution, but only by the spectral function.

We also demonstrated that all higher order terms of the series can be summed into a closed expression, containing only the struck-nucleon spectator scattering amplitude, determined by the 3-dimensional Lippmann-Schwinger equation [5]. Thus the relativistic nuclear structure function can be essentially mapped on the non-relativistic one with an appropriate resealing of the scaling variable. This essentially simplifies a treatment of relativistic structure functions and opens a new way for an account of the final state interaction.

References

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