Memory Effects on Descent From Nuclear Fission Barrier

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The goal of the present work is to study the influence of the memory effects on the descent of the nucleus from the fission barrier to the scission point. Starting from the kinetic equation and taking the collision integral in the ϑ-approximation, we have derived the non-Markovian transport equations for the nuclear shape parameters $\{q_i\}$ in the following form

$$\sum_{j=1}^{N}\left[B_{ij}\ddot{q}_j+\sum_{k=1}^{N}\frac{\partial B_{ij}}{\partial q_k}\dot{q}_j\dot{q}_k\right.$$

$$\left.+\int_{t_0}^{t}dt'\exp\left(\frac{t'-t}{\tau}\right)\kappa_{ij}(t,t')\dot{q}_j(t')\right]=-\frac{\partial E_{\text{pot}}}{\partial q_i},\quad (1)$$

where ϑ is the relaxation time and $E_{\text{pot}} \equiv E_{\text{pot}}(\{q_i\})$ is the adiabatic deformation energy. The inertia tensor $B_{ij} \equiv B_{ij}(\{q_i\})$ and the memory kernel $6_{i,j}(t,t')$ have been evaluated assuming an irrotational and incompressible nuclear Fermi liquid. We point out that the memory integral in Eq. (1) is caused by the Fermi surface distortion (FSD) in momentum space.

To apply Eq. (1) to nuclear fission we assumed that the axially symmetric shape of the nucleus is defined by rotation of the profile function $\Delta=Y(z,\{q_i\})$ around the z-axis in the cylindrical coordinates Δ, z, ν [1]. The profile function $Y(z,\{q_i\})$ was taken in the form of the Lorentz parametrization [2],

$$Y^2(z)=(z^2-\zeta_0^2)(z^2+\zeta_2^2)/Q,$$

$$Q=-[\zeta_0^3(\frac{1}{5}\zeta_0^2+\zeta_2^2)]/R_0^3. \qquad (2)$$

To evaluate both the transport tensors $B_{ij}(\{q_i\})$ and $\kappa_{i,j}(t,t')$ we have solved the Neumann problem for the potential $N \equiv N(\mathbf{r},\{q_i\})$ of the velocity field of the incompressible and irrotational nuclear liquid. The boundary condition on the nuclear surface S was taken in the form $(\mathbf{n}\Lambda N)_S = u_S$, where $\mathbf{n}$ is the unit vector which is normal to the nuclear surface S and u_S is the velocity of the nuclear surface, $u_S=\sum_{i=1}^{N}\left[(\partial Y/\partial q_i)/\sqrt{1+(\partial Y/\partial z)^2}\right]\dot{q}_i$. The adiabatic deformation energy E_{pot} in Eq. (1) was taken from Ref. [2] and the scission line was derived from the condition of the instability of the nuclear shape with respect to the variations of the neck radius as $\partial^2E_{\text{pot}}/\partial\rho_{\text{neck}}^2=0$, where $\rho_{\text{neck}}=\zeta_2/\sqrt{\zeta_0(\zeta_0^2/5+\zeta_2^2)}$ is the neck radius.

We have performed numerical calculations for symmetric fission of the nucleus ^{236}U. The equations of motion (1) were solved with the initial condition corresponding to the saddle point deformation and the initial kinetic energy $E_{\text{kin},0}$ = 1 MeV (initial neck velocity $\dot{\zeta}_2 = 0$). To solve the Neumann problem for the velocity field potential $N = N(\mathbf{r},\{q_i\})$ we have used the method based on the theory of the potential, see Ref. [1]. In Fig. 1 we show the dependence of the fission trajectory, i.e., the dependence of the neck parameter $._2$ on the elongation $._0$, for the fissioning nucleus ^{236}U for two different values of the relaxation time ϑ. We point out that the memory effect is absent at $\vartheta = 0$. The scission line (dot- dashed line in Fig. 1) was obtained from the instability condition as mentioned above. We defined the scission point as the intersection point of the fission trajectory with the scission line. As can be seen from Fig. 1 the memory effect hinders slightly the neck formation and leads to a more elongated scission configuration. To illustrate the memory effect on the observable values we have evaluated the translation kinetic energy of the fission fragments at infinity, E_{kin}, and the prescission Coulomb interaction energy, E_{Coul}. The value of E_{kin} is the sum of the Coulomb interaction energy at scission point, E_{Coul} and the prescission kinetic energy $E_{kin,ps}$. After scission the fission fragments were described in terms of two equal mass spheroids and we assumed that the distance between the centers of mass of two spheroids is equal to the distance between the two halves of the fissioning nucleus at the scission point (two-

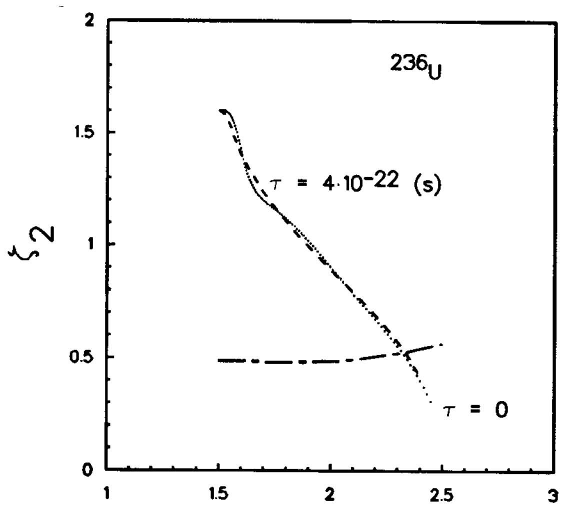

Figure 1: Trajectories of descent from the saddle point of the nucleus ^{236}U in the .0, .2 plane. Dashed line represents the result of the calculation in presence of the memory effects. We have used the relaxation time ϑ = 4 θ 10^{-22}s and the initial kinetic energy E_{kin} = 1 MeV. Dotted line is for the case of Markovian (no memory) motion at ϑ = 0. Dot-dashed line is the scission line.

spheroid parametrization, see Ref. [3]).

The influence of the memory effects on the fission-fragment kinetic energy, E_{kin}, and the prescission Coulomb interaction energy, E_{Coul}, is shown in Fig. 2. As seen from Fig. 2 the memory effects are negligible at the short relaxation time regime where the memory integral is transformed into the usual friction force. In the case of the usual Markovian motion with friction (dashed line), the yield of the potential energy,)E_{pot}, at the scission point is transformed into both the prescission kinetic

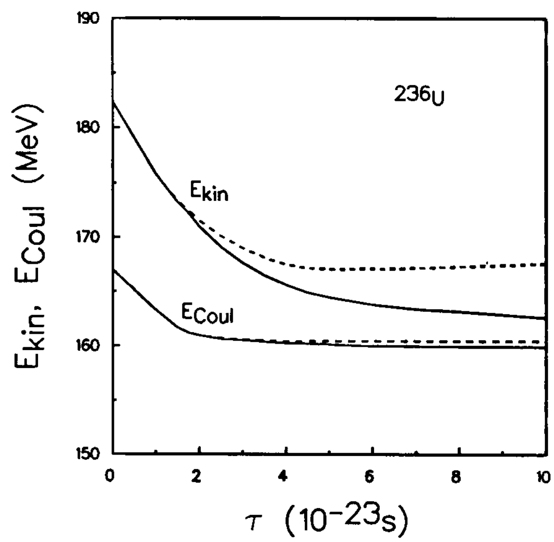

Figure 2: Fission-fragment kinetic energy, E_{kin}, and the Coulomb repulsive energy at the scission point, E_{Coul}, versus the relaxation time ϑ for the nucleus ^{236}U. Solid lines represent the result of the calculation in the presence of the memory effects and dashed lines are for the case of Markovian (no memory) motion with the friction forces. The initial kinetic energy is $E_{kin,0}$ = 1 MeV.

energy, $E_{kin,ps}$, and the time irreversible dissipation energy, E_{dis}, providing)$E_{pot} = E_{kin,ps} + E_{dis}$. In contrast to this case, the non-Markovian motion with the memory effects (solid line) produces an additional time-reversible prescission energy, $E_{F,ps}$ caused by the distortion of the Fermi surface. In this case, the energy balance reads)$E_{pot} = E_{kin,ps} + E_{dis} + E_{F,ps}$. We point out that the two-spheroid parametrization of the fissioning nucleus at the scission point leads to the prescission Coulomb energy E_{Coul} which is about 5 MeV lower (for ^{236}U) than the

Coulomb interaction energy of the scission point shape [4]. Taking into account this fact and using the experimental value of the fission-fragment kinetic energy E_{kin}^{exp} = 168 MeV [4], one can see from Fig. 2 that the Markovian motion with friction (dashed line) leads to the overestimate of the fission-fragment kinetic energy E_{kin}. In the case of the non-Markovian motion with the memory effects (solid line), a good agreement with the experimental data is obtained at the relaxation time of about ϑ= 8θ10^{-23}s.

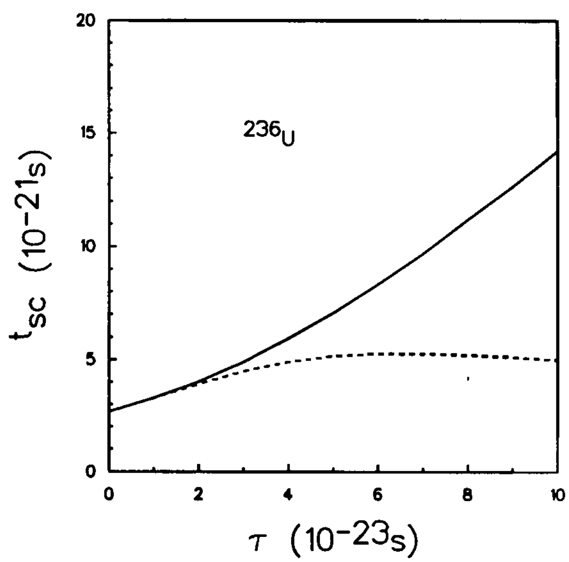

Figure 3: Dependence upon relaxation time ϑ of the saddle-to-fission time, t_{sc}, for the nucleus ^{236}U. Solid line represents the result of the calculation in the presence of the memory effects and dashed line is for the case of Markovian (no memory) motion with the friction forces. The initial kinetic energy is E_{kin} = 1 MeV.

In Fig. 3 we illustrate the memory effect on the saddle-to-scission time t_{sc}. In the case of the non-Markovian motion (solid line), the delay in the descent of the nucleus from the barrier grows with the relaxation time ϑ (at $\vartheta \geq$ 4θ10^{-23}s). This is mainly due to the hindering action of the elastic force caused by the memory integral. The saddle-to-scission time increases by a factor of about 2 due to the memory effect near the "experimental" value of the relaxation time ϑ = 8 θ 10^{-23}s which was derived from the fit of the fission-fragment kinetic energy E_{kin} to the experimental value of E_{kin}^{exp} .

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