The Influence of The Optical Potential Parameters on the Isoscalar Giant Dipole Resonance

B. Dobrescu, S. Shlomo, A. I. Sanzhur, B. Karki and A. Moalem[a]
[a]*Department of Physics, Ben - Gurion University, Beer - Sheva, Israel*

The main experimental tool for studying isoscalar giant resonances is inelastic ∀-particle scattering, as this type of reaction is selective as to exciting isoscalar modes by either eliminating or greatly reducing the interference of other excitations. The angular distributions of inelastic scattered ∀-particles at small angles are characteristic for some of the multipolar modes in the experimentally measured angular distributions and thus can be easily identified in the experimental analysis.

The Distorted Wave Born Approximation (DWBA) has been extensively used in experimental studies in order to give a theoretical description of low-energy scattering reactions and, hence, analyze measured cross sections. In this work the folding model (FM) approach is considered for the evaluation of optical potentials from the Hartree-Fock (HF) ground state density and a density dependent nucleon-∀ interaction potential ($V_{\forall n}$). The parameters of $V_{\forall n}$ are determined by fitting experimentally measured angular distributions for the case of elastic scattering. Angular distributions of inelastically scattered ∀-particles for ISGDR excitation of the target nucleus are obtained using FM-DWBA and both microscopic (based on self-consistent Skyrme-HF Random Phase Approximation (RPA) approach) and collective model (hydrodynamical) transition densities.

The sensitivity on the choice of optical model parameters for the predicted cross sections of the isoscalar giant dipole resonance (ISGDR) and the shifts of centroid energies, in both microscopic HF-RPA and collective model calculations, is investigated. We will consider fits to both the full range and parts (forward angles) of the experimental data set for elastic scattering, this being a procedure which, in general, leads to different values for the parameters entering the nucleon-∀ interaction potential. Numerical minimization in a multidimensional space, as a part of the fitting process, is a delicate task to perform, since one very often encounters a number of local minima. The choice of a local minimum instead of the global one in the fitting operation, producing erroneous values for the parameters of $V_{\forall n}$, will also be considered in the analysis.

Within the folding model, the optical potential $U(r)$ is given by:

$$U(r) = \int d\mathbf{r}' V_{\alpha n}(|\mathbf{r}-\mathbf{r}'|, \rho_0(r'))\rho_0(r') \qquad (1)$$

where $V_{\alpha n}(|\mathbf{r}-\mathbf{r}'|), \rho_0(\mathbf{r}'))$ is the nucleon-∀ interaction, which is generally density dependent. In this work, both real and imaginary parts of the nucleon-∀ interaction are chosen to be of Gaussian form with density dependence:

$$V_{\alpha n}(|\mathbf{r}-\mathbf{r}'|, \rho_0(r')) = -V(1+\beta_V \rho_0^{2/3}(r'))e^{-\frac{|\mathbf{r}-\mathbf{r}'|^2}{\alpha_V}}$$

$$-iW(1+\beta_W \rho_0^{2/3}(r'))e^{-\frac{|\mathbf{r}-\mathbf{r}'|}{\alpha_W}} \qquad (2)$$

The ground state particle density of the target nucleus, $\Delta_0(r')$, is obtained by performing self-consistent Skyrme HF calculations. We use SL1 parameterization of the Skyrme effective interaction which is associated with a value of 230 MeV for the nuclear matter incompressibility coefficient. The parameters V, $\exists_V$, $\forall_V$ and W, $\exists_W$, $\forall_W$ in Eq. (2) are determined by a fit of the elastic scattering data with the code PTOLEMY.

For a state with the multipolarity L and excitation energy E, the radial form of the transition potential is given by:

$$\delta U(r,E) = \int d\mathbf{r}'[V_{\alpha n}(|\mathbf{r}-\mathbf{r}'|, \rho_0(r')) +$$

$$\rho_0(r')\frac{\partial V_{\alpha n}(|\mathbf{r}-\mathbf{r}'|, \rho_0(r'))}{\partial \rho_0(r')}]\delta\rho_L(\mathbf{r}', E) \quad (3)$$

where *Δ_L($\mathbf{r'}$, E) is the transition density for the considered excited state.

Within the microscopic approach, both the ground state density and the transition density which enters Eqs. (1) and (3) are obtained from self-consistent HF-RPA calculations. In the macroscopic approach, the transition densities are assumed to have energy-independent radial shapes and are obtained from the HF ground state density using the collective model.

The strength function, $S(E)$, and the microscopic transition density for ISGDR are obtained from studying the response of the nucleus to the excitation operator:

$$\hat{F} = \sum_{i=1}^{A} f(\mathbf{r}_i), \quad (4)$$

$$f(\mathbf{r}) = (r^3 - \eta r)Y_{10}(\hat{\mathbf{r}}) \quad (5)$$

and their expressions, in terms of the RPA Green's function G, are given by:

$$S(E) = \frac{1}{\pi}\mathrm{Im}[Tr(fGf)], \quad (6)$$

$$\delta\rho(\mathbf{r},E) \propto \frac{1}{\sqrt{S(E)}}\int f(\mathbf{r}')\left[\frac{1}{\pi}\mathrm{Im}\,G(\mathbf{r}',\mathbf{r},E)\right]d\mathbf{r}' \quad (7)$$

If one takes the classical view of a collective (hydrodynamical) model, the macroscopic transition density can be expressed by:

$$\delta\rho(\mathbf{r}) \propto \left(10r + 3r^2\frac{\partial}{\partial r} - \eta\frac{\partial}{\partial r}\right)\rho_0(r)Y_{10}(\hat{\mathbf{r}}) \quad (8)$$

with $\eta = \frac{5}{3}\langle r^2 \rangle$ determined by the condition of translational invariance.

Using Eqs. (1) and (2) and HF ground state density, we construct the optical potential and determine the parameters of the nucleon-∀ interaction of Eq. (2) by fitting experimentally measured elastic scattering angular distributions [1]. The procedure was carried out for ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb using various regions of the available experimental data set [2]. In ref. [1] a similar analysis was considered using macroscopic transition densities only and a Woods-Saxon expression for the imaginary part of the optical potential.

If fits were made using only a part of the elastic data (forward angles), then both the extrapolation of the elastic differential cross section at larger scattering angles and the calculated ISGDR cross section could be seriously affected. As a study case for these effects we will consider the nucleus ^{116}Sn.

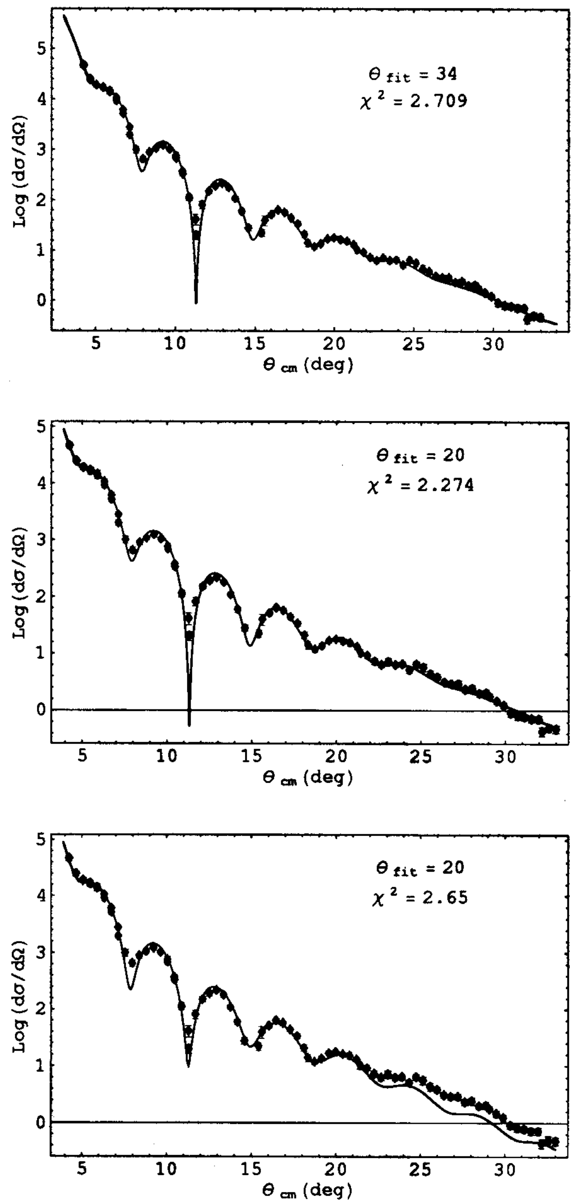

Figure 1: Fits for elastic scattering of 240 MeV ∀-particles, on target ^{116}Sn, within the angular range 0 ÷ 2_{fit} of the available experimental data.

In Fig. (1) are shown fits obtained for two angular ranges, 2_{fit} = 20 and 2_{fit} = 34 (the full data set), respectively. The sets of parameters entering $V_{\forall n}$, presented in Table 1, correspond to different local minima of the quantity Π^2 obtained in the fitting procedure. It can be seen that, even though all the fits are reasonably good for the specific part of the angular interval considered in analysis, an extrapolation of the elastic differential cross section to larger scattering angles can often be misleading. This shortcoming, however, is not so much related to the predictive power of the optical model approach, but rather to the complicated topology of the hypersurface defined by the nucleon-∀ potential energy ($V_{\forall n}$), manifested in a large number of "scattered" local minima.

Having determined the parameters of the nucleon-∀ interaction, we calculate the cross sections of inelastically scattered ∀-particles for the ISGDR (E1T0) excitation of the target nucleus, using the transition potential (3) and both the microscopic HF-RPA transition densities, obtained from Eq. (5), and the collective model transition densities given by Eq. (8).

θ_{fit} (deg)	V (MeV)	W (MeV)	$\beta_V = \beta_W$ (fm^2)	α_V (fm^2)	α_W (fm^2)	χ^2
34	38.47	8.48	-1.9	3.5	5.9	2.709
20	42.67	8.39	-1.9	3.3	6.0	2.274
20	29.05	145.92	-1.9	4.3	1.6	2.650

Table 1: Parameterizations of the nucleon-∀ interaction obtained by fitting different parts of elastic scattering data set for ^{116}Sn, within the angular range 0 ÷ 2_{fit}.

Figure (2) shows the reconstruction of the E1T0 EWSR in ^{116}Sn from the inelastic ∀-particle cross section. The results presented here have been obtained using the $V_{\forall n}$ potentials which correspond to the two local minima for Π^2 in the 2_{fit} = 20 fit. The values of the parameters V, $\exists_V$, $\forall_V$ and W, $\exists_W$, $\forall_W$ are given in the last two rows of Table 1. The middle panel of the figure illustrates double differential E1T0 cross section at the first maximum obtained with RPA transition density, used as our "experimental" data for inelastic scattering.

The solid (dashed) line in the lower panel of the figure represents the first peak of E1T0 angular distribution when microscopic (collective model) transition densities are used,

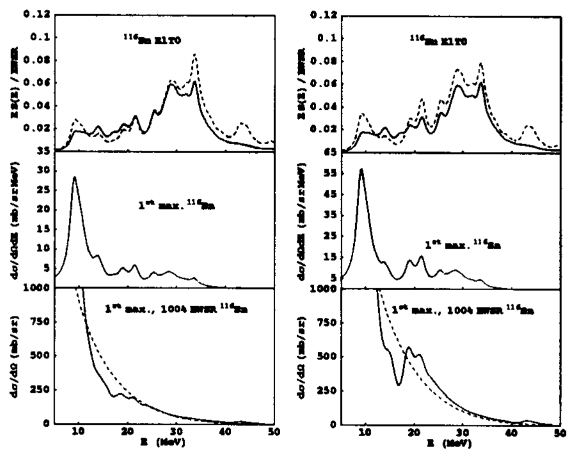

Figure 2: Reconstruction of the ISGDR EWSR from inelastic ∀-particle cross section for ^{116}Sn. For explanation of figure, see text.

normalized to 100% of the E1T0 EWSR. The solid (dashed) line in the upper panel shows the ratio between the curve in the middle panel and the solid (dashed) line in the lower panel. This ratio is the percentage of E1T0 EWSR per unit energy in ^{116}Sn reconstructed from the inelastic cross section analysis using microscopic HF-RPA (solid line) and collective model (dashed line) transition densities, respectively.

It was found that the calculated ISGDR cross-section is quite sensitive to the imaginary part of the interaction potential $V_{\forall n}$. If fits to the elastic data were made at forward angles then, using two sets of parameters for $V_{\forall n}$, which yielded reasonably good fits for the chosen angular interval, the results obtained for the inelastic cross-section could differ by a factor of 2 or more. However, it must be emphasized that even for fits over small angular intervals the results can still be meaningful if the numerical minimization procedure is properly accomplished, in order to get rid of the misleading local minima.

θ_{fit} (deg)	E_c^{coll} (MeV)	E_c^{RPA} (MeV)	χ^2
34	31.60	28.72	2.709
20	31.64	28.72	2.274
20	29.85	28.38	2.650

Table 2: ISGDR centroid energies in ^{116}Sn, calculated for both HF-RPA and collective model transition densities. The parameters of nucleon-∀ interaction, used in numerical calculations, are given in Table 1.

The effect of various parameterizations of the potential $V_{\forall n}$, on the ISGDR centroid energies is summarized in Table 2. It can be seen that the method of extraction of the strength distribution, S(E), from experimental data using collective model transition densities is more sensitive to the parameters entering $V_{\forall n}$ as compared to the method based on HF-RPA transition densities. Studies concerning the influence of optical parameters on other types of giant resonances are currently underway.

References

[1] H. L. Clark, Y. -W. Lui and D. H. Youngblood, Nucl. Phys. **A687**, 80c (2001).

[2] B. Dobrescu *et al.*, to be published.