Semiempirical Scaling Law for Multiple Vacancy Production in Fast-Heavy-Ion Collisions

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According to the geometrical model [1], the probability of removing an electron from the state with wave function \( \psi_{nlm} \) and quantum numbers \( n, l, \) and \( m \), is given by the expression:

\[
p_{nlm} = \int \int \int |\psi_{nlm}(r)|^2 \eta(b) \, d\rho \, dz \, d\phi,
\]

(1)

where \( r \) is the position vector, \( \eta \) is the so-called geometrical efficiency function of the electronic impact parameter, \( b \), and \( d\rho \, dz \, d\phi \) is the differential volume in cylindrical coordinates. This probability was shown to be a function of the universal variable, \( x \), defined with the equation:

\[
x = \frac{Z_1}{v_1} \frac{V}{\sqrt{G(V)}} \frac{\sqrt{G(V)}}{n}.
\]

(2)

Here, \( V = v_1/v_2 \) is the ratio between the projectile velocity and the bound electron velocity, \( G(V) \) is the Binary Encounter Approximation (BEA) universal ionization function, and \( n \) is the principal quantum number of the electron to be removed. The function \( G(V) \) is a numerical function that reaches a maximum when the velocity matching criteria \( (V=1) \) has been satisfied. While a number of analytical approximations to \( G(V) \) have been proposed, the one used here is given by equations (6a-6c) of ref. [2].

The geometrical model results from Equation (1) have been compared with the experimentally determined average number of M-shell vacancies, \( <n_M> \). The experimental results were extracted from target L x-ray spectra excited by heavy ion collisions and are described in more detail elsewhere [3]. The agreement between \( <n_M> \) determined from the measured L x-ray spectra and the results of the geometrical model was found to be rather poor. However, it was observed that the measured data points, when plotted as a function of \( x \), appear to fall on well defined curves, thus lending credence to the universality of \( x \). Two cases were distinguished, as shown in Figure 1. One case corresponds to single L-shell ionization (solid points), and the other to double L-shell ionization of the target atoms (open points). The curves shown in Figure 1 represent fits to the experimental data obtained using the fitting function:

\[
<n_M> = \frac{a}{1 + \left( \frac{b}{x} \right)^c}.
\]

(3)

Figure 1. Average number of target atom M-shell vacancies deduced from the L x-ray spectra as a function of the universal variable (see text for details). The solid points are for single L-shell ionization, the open points are for double L-shell ionization, and the curves are the functions fitted to the data points.
For single L-shell ionization $a=10.43$, $b=0.3623$, and $c=2.144$ while for double L-shell ionization $a=12.29$, $b=0.2271$, and $c=2.205$. These curves were used to predict the average number of L-shell vacancies, $<n_L>$, in the previously reported K-shell ionizing collisions [4]. The comparison with the measured data shown in Figure 2 is in this case unbiased, since none of these data were used in the evaluation of the fitting parameters. The solid points in Figure 2 are the values of $<n_L>$ extracted from the x-ray data and the open points are calculations of $<n_L>$ taking into account the possible rearrangement processes that would change $<n_L>$ prior to x-ray emission [5]. The similarities in both shape and value of the experimental results for $<n_L>$ from Cu K x-ray spectra and the semiempirical predictions is very encouraging and provides motivation for further tests.

References


Figure 2. Average number of L-shell vacancies produced in a Cu target by 10 MeV/u projectiles as a function of projectile atomic number. The solid points represent the measured values determined from the satellite intensities, whereas, the open circles have been corrected for pre-emission electron rearrangement [5] and so represent target atoms at the time just after the collision. The curve represents semi-empirical predictions based on the geometrical model.