Semiempirical Scaling Law for Multiple Vacancy Production in Fast-Heavy-Ion Collisions

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According to the geometrical model [1], the probability of removing an electron from the state with wave function ψ_{nlm} and quantum numbers n, l, and m, is given by the expression:

$$p_{nlm} = \iiint |\psi_{nlm}(\mathbf{r})|^2 \, \eta(b) \rho \, d\rho \, dz \, d\phi, \tag{1}$$

where **r** is the position vector, η is the so-called geometrical efficiency function of the electronic impact parameter, b, and $\rho \, d\rho \, dz \, d\phi$ is the differential volume in cylindrical coordinates. This probability was shown to be a function of the universal variable, x, defined with the equation:

$$x = \frac{Z_1}{v_1} V \frac{\sqrt{G(V)}}{n}. \tag{2}$$

Here, $V = v_1/v_2$ is the ratio between the projectile velocity and the bound electron velocity, G(V) is the Binary Encounter Approximation (BEA) universal ionization function, and n is the principal quantum number of the electron to be removed. The function G(V) is a numerical function that reaches a maximum when the velocity matching criteria (V=1) has been satisfied. While a number of analytical approximations to G(V) have been proposed, the one used here is given by equations (6a-6c) of ref. [2].

The geometrical model results from Equation (1) have been compared with the experimentally determined average number of M-shell vacancies, $\langle n_M \rangle$. The experimental results were extracted from target L x-ray spectra excited by heavy ion collisions and are described in more detail elsewhere [3]. The agreement between $\langle n_M \rangle$ determined from the measured L x-ray spectra and the results of the geometrical model was found to be rather poor. However, it was observed that the measured data points, when plotted as a function of x, appear to fall on well defined curves, thus lending credence to the universality of x. Two cases were distinguished, as shown in Figure 1. One case corresponds to singe L-shell ionization (solid points), and the other to double L-shell ionization of the target atoms (open points). The curves shown in Figure 1 represent fits to the experimental data obtained using the fitting function:

$$\langle n_M \rangle = \frac{a}{1 + \left(\frac{b}{x} \right)^c}. \tag{3}$$

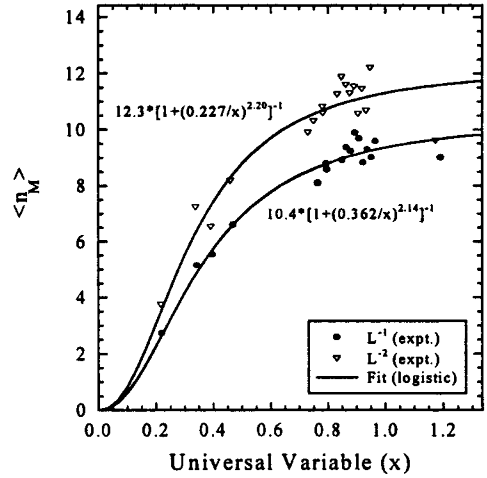

Figure 1. Average number of target atom M-shell vacancies deduced from the L x-ray spectra as a function of the universal variable (see text for details). The solid points are for single L-shell ionization, the open points are for double L-shell ionization, and the curves are the functions fitted to the data points.

For single L-shell ionization a=10.43, b=0.3623, and c=2.144 while for double L-shell ionization a=12.29, b=0.2271, and c=2.205. These curves were used to predict the average number of L-shell vacancies, $<n_L>$, in the previously reported K-shell ionizing collisions [4]. The comparison with the measured data shown in Figure 2 is in this case unbiased, since none of these data were used in the evaluation of the fitting parameters. The solid points in Figure 2 are the values of $<n_L>$ extracted from the x-ray data and the open points are calculations of $<n_L>$ taking into account the possible rearrangement processes that would change $<n_L>$ prior to x-ray emission [5]. The similarities in both shape and value of the experimental results for $<n_L>$ from Cu K x-ray spectra and the semiempirical predictions is very encouraging and provides motivation for further tests.

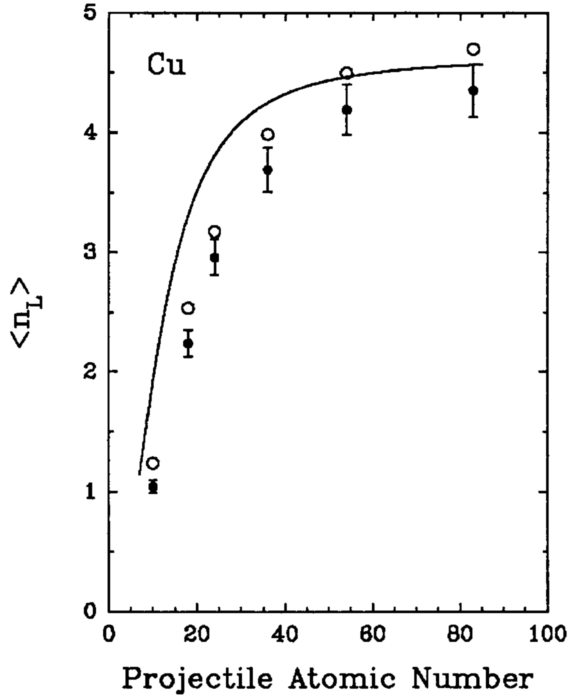

Figure 2. Average number of L-shell vacancies produced in a Cu target by 10 MeV/u projectiles as a function of projectile atomic number. The solid points represent the measured values determined from the satellite intensities, whereas, the open circles have been corrected for pre-emission electron rearrangement [5] and so represent target atoms at the time just after the collision. The curve represents semi-empirical predictions based on the geometrical model.

References

[1] B. Sulik, I. Kádár, S. Ricz, D. Varga, J. Végh, G. Hock, and D. Berényi, Nucl. Instrum. Methods Phys. Res. B **28**, 509 (1987).

[2] James H. McGuire and Patrick Richard, Phys. Rev. A **8**, 1374 (1973).

[3] J.M. Blackadar, V. Horvat, and R.L. Watson, Progress in Research, 1998-1999, Cyclotron Institute, TAMU, p. IV-14.

[4] J.M. Blackadar, V. Horvat, and R.L. Watson, Progress in Research, 1997-1998, Cyclotron Institute, TAMU, p. IV-4.

[5] V. Horvat, Progress in Research, 1998-1999, Cyclotron Institute, TAMU, p. IV-11.