Superallowed $0^+ \rightarrow 0^+$ Beta Decay and V_{ud}: Current Status

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Within the Standard Model, the Cabibbo-Kobayashi-Maskawa (CKM) matrix relates the quark eigenstates of the weak interaction with the quark mass eigenstates. The matrix should be unitary, and experimental tests of that expectation constitute important tests of the Standard Model itself. The leading element of the CKM matrix, V_{ud}, only depends on quarks in the first generation and so it is the element that can be determined most precisely. Currently, the most demanding test of unitarity is the one that incorporates the elements of the matrix's first row: *viz,*

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1.$$

The value of V_{ud} can be derived from experimental results for nuclear superallowed $0^+ \rightarrow 0^+$ beta decays, neutron decay, and pion beta decay. We have surveyed current world data (to August 1998) for all three decay modes and examined the implications of the results [1-4].

The $0^+ \rightarrow 0^+$ nuclear transitions have several advantages over the other decay modes: they are generally strong transitions that depend only on the vector part of the weak interaction; and, in the allowed approximation, the nuclear matrix elements for the transitions are given by the expectation value of the isospin ladder operator, which is independent of any details of nuclear structure, being simply an SU(2) Clebsch-Gordan coefficient. Thus the experimentally determined *ft*-values are expected to be very nearly the same for all $0^+ \rightarrow 0^+$ transitions between states of a particular isospin, regardless of the nuclei involved. Naturally, there are corrections to this simple picture arising from electromagnetic effects, but these corrections are small — of order 1% — and calculable. Thus, if we write δ_R as the nucleus-dependent part of the radiative correction, Δ_R as the nucleus-independent part, and δ_C as the isospin symmetry-breaking correction, then the experimental *ft*-value for transitions between T=1 analog states can be expressed as follows:

$$ft(1+\delta_R)(1-\delta_C) \equiv \mathcal{F}t = K/[2G_V^2(1+\Delta_R)] = \text{constant}$$

where G_V is the weak vector coupling constant and K is a numerical constant. Thus, to extract V_{ud} from experimental data, the procedure is to determine the $\mathcal{F}t$-values for a variety of different nuclei having the same isospin, and then to test if they are self consistent. If they are, their average is used to determine a value for G_V and, from it, V_{ud}.

To date, superallowed $0^+ \rightarrow 0^+$ transitions have been measured to ±0.1% precision or better in the decays of nine nuclei ranging from ^{10}C to ^{54}Co. World data on Q-values, lifetimes and branching ratios — the results of over 100 independent measurements — yield the *ft*-value results given in Table I. After incorporating average values obtained from independent calculations for the correction terms [2], we arrive at the $\mathcal{F}t$-values also shown in the table. In a real sense, both experimentally and theoretically, the $\mathcal{F}t$-values in the table represent the totality of current world knowledge. Evidently, there is no statistically significant evidence of inconsistencies in the data

Table I: Results from world data.

Parent	ft (s)	$\mathcal{F}t$ (s)
^{10}C	3038.7(45)	3072.9(48)
^{14}O	3038.1(18)	3069.7(26)
^{26m}Al	3035.8(17)	3070.0(21)
^{34}Cl	3048.4(19)	3070.1(24)
^{38m}K	3049.5(21)	3071.1(27)
^{42}Sc	3045.1(14)	3077.3(23)
^{46}V	3044.6(18)	3074.4(27)
^{50}Mn	3043.7(16)	3073.8(27)
^{54}Co	3045.8(11)	3072.2(27)
	Average $\mathcal{F}t$	3072.3(9)

(χ^2/ν = 1.1), thus verifying the expectation of CVC at the level of 3×10^{-4}, the fractional uncertainty quoted on the average $\mathcal{F}t$-value.

This average $\mathcal{F}t$-value, when combined with results from muon decay [5], yields the value

$$|V_{ud}| = 0.9740 \pm 0.0005$$

and a unitarity sum of

$$\sum_i V_{ui}^2 = 0.9968 \pm 0.0014 \qquad \text{[nuclear]},$$

which fails to meet unitarity by 2.2 standard deviations. If this result can be made more definitive, it would have profound implications, most likely in requiring an extension to the Standard Model [2].

To determine the robustness of the non-unitarity result, we have examined the reliability of the small calculated corrections [2] that have been applied to the data. To restore unitarity, the calculated radiative corrections (δ_R or Δ_R) for all nine superallowed transitions would all have to be shifted downwards by 0.3%, or the δ_C corrections all shifted upwards by 0.3%, or some combination of the two. Such changes would constitute a substantial fraction of the total values of these small quantities, and are, in our opinion, very unlikely. Nevertheless, further experimental support for the accuracy of the calculations would be very valuable.

On the one hand, free neutron decay has an advantage over nuclear decays since there are no nuclear-structure dependent corrections to be calculated. On the other hand, it has the disadvantage that it is not purely vector-like but has a mix of vector and axial-vector contributions. Thus, in addition to a lifetime measurement, a correlation experiment is also required to separate the vector and axial-vector pieces. Both types of experiment present serious challenges and, as a result, the unitarity test as obtained from current world data,

$$\sum_i V_{ui}^2 = 1.0007 \pm 0.0042 \qquad \text{[all neutron]},$$

is a factor of three less precise than the test from the nuclear decays. This neutron value agrees with unitarity *and* with the nuclear result. It is interesting to note, however, that the most recent experimental result for the beta-asymmetry from neutron decay [6], which is more precise than any of its predecessors, yields, by itself, a unitarity sum,

$$\sum_i V_{ui}^2 = 0.9919 \pm 0.0030 \qquad \text{[select neutron]},$$

which, like the nuclear result, is several standard deviations below unity.

Like neutron decay, pion decay data has an advantage over nuclear decays in that there are no nuclear structure-dependent corrections to be made. It also has the same advantage as the nuclear decays in being a purely vector transition, in its case $0^- \rightarrow 0^-$, so no separation of vector and axial-vector components is required. Its major disadvantage, however, is that pion beta decay, $\pi^+ \rightarrow \pi^0 e^+ \nu_e$, is a very weak branch, of the order of

10^{-8}. As a result, the unitarity test from current world data has a comparatively large uncertainty:

$$\sum_i V_{ui}^2 = 0.9833 \pm 0.0311 \qquad \text{[pion]}.$$

In summary then, the neutron and pion decay results are, as yet, experimentally less precise than the nuclear ones. The nuclear results have much higher experimental precision but are now constrained by theoretical uncertainties. Further improvements in the unitarity test from $0^+ \rightarrow 0^+$ nuclear decays will only come from experiments [2,3] focused on testing and improving the reliability of the theoretical corrections, particularly of δ_C.

References

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