Connection Between Asymptotic Normalization Coefficients, Subthreshold Bound States, and Resonances

A.M. Mukhamedzhanov and R.E. Tribble

Nuclear excited states below the particle emission threshold typically undergo γ-decay to lower lying states. These decays result in the initial states having their own natural width. In the case when γ-emission is the only open decay channel, the natural width, Γ_γ, is typically $\sim eV$. If a particle bound excited state lies very close to the particle threshold, the natural width can result in the tail of the wave function extending above the particle threshold. Due to this tail, the subthreshold bound state can behave like a resonance state in a capture reaction. Such states are often referred to as subthreshold resonance states [1] and they can play an important role in determining reaction rates of interest in nuclear astrophysics. It is shown that the ANC of a subthreshold bound state defines the normalization of both direct radiative capture leading to this state and resonance capture in which the state behaves like a subthreshold resonance. A determination of the appropriate ANC(s) thus offers an alternative method for finding the strength of these types of capture reactions, both of which are important in nuclear astrophysics. We present here useful relations showing the connection between the asymptotic normalization coefficient (ANC) and the fitting parameters in $K-$ and $R-$matrix theory methods which are often used when analyzing low energy experimental data.

Consider the capture of particle b by particle a at very low relative kinetic energy E and assume that there is a subthreshold bound state $c1$ in the system $c = (ab)$. There are three possible mechanisms by which the capture can occur:

(i) direct radiative capture to the ground state c;

(ii) radiative capture to the ground state through the subthreshold resonance;

(iii) direct radiative capture into the subthreshold bound state with γ-emission.

Process (ii) corresponds to non radiative capture of particle b into the subthreshold resonance $c1$. The excited state then undergoes γ-decay to the ground state c. The energy of the emitted photon is

$$E_\gamma = E + \varepsilon_c, \qquad (1)$$

where ε_c is the binding energy of the ground state $c = (ab)$. Note only one gamma is emitted in the process and it occurs after capture into the $c1$ state. Process (iii) results initially in a photon with energy

$$E_\gamma = E + \varepsilon_{c1}. \qquad (2)$$

The subthreshold bound state $c1$ is then deexcited to the ground state c by emitting a photon with energy $\varepsilon_c - \varepsilon_{c1}$. Note that in mechanisms (ii) and (iii) the capture occurs into the same state, but in (ii) this state reveals itself as a resonance, while in (iii) it acts as a real bound state. All three of these capture processes occur in nature and are important in determining reaction rates for nuclear astrophysics.

In previous papers (see [2] and references therein) we have pointed out that the overall normalization of the cross section for a direct radiative capture reaction at low binding energy is entirely defined by the asymptotic normalization coefficient of the final bound state wave function into the two-body channel corresponding to the colliding particles. In this work we show how to extend this to capture into subthreshold resonance states. In what follows we use the system of units in which $\hbar = c = 1$.

Among the important relationships found in our work we would like to present here the astrophysical factor at $E \to 0$ for the capture to the subthreshold resonance:

$$S(E) \overset{E\to 0}{\approx} (2l+1)\frac{\pi^2\kappa_{c1}}{\mu_{ab}^2}\frac{(\eta_{\kappa_{c1}})^{2l+1}}{\Gamma^2(l+1+\eta_{\kappa_{c1}})} \times\frac{\Gamma_\gamma \mid C\mid^2}{(E+\varepsilon_{c1})^2}, \qquad (3)$$

where Γ_γ is the γ width of the subthreshold resonance, η_{c1} is the Coulomb parameter of the bound state $c1$ and κ_{c1} is the wave number of the bound state $c1$. Thus we have shown that the ANC of the subthreshold bound state defines the overall normalization of the the astrophysical factor for the capture into the subthreshold resonance at $E \to 0$.

We derived also the connection between the scattering length and the ANC:

$$\alpha = \frac{1}{\kappa_{c1}^2}\frac{1}{\Gamma^2(1+\eta_{\kappa_{c1}})}\mid C\mid^2 . \qquad (4)$$

Finally we present the relationship between the partial width of the subthreshold resonance $c1$ at $E > 0$ in the R-matrix method and the ANC:

$$\Gamma_{c1} = 2\mathcal{V}_l(E)\tilde{\gamma}_{c1}^2 = \frac{1}{\mu_{ab}}\mathcal{V}_l(E)\frac{W^2_{-\eta_{\kappa_{c1}},l+1/2}(2\kappa_{c1}r_0)}{r_0} \mid C\mid^2, \qquad (5)$$

where $\mathcal{V}_l(E) = 2kr_0P_l(E)$, $P_l(E)$ is the penetration factor, $W_{-\eta_{\kappa_{c1}},l+1/2}$ is the Whittaker function and r_0 is the channel radius in the R-matrix method. When deriving Eq. (5) we used the connection between the ANC and the dimensionless effective reduced width amplitude $\tilde{\theta}_{c1}$ of the subthreshold bound state $c1$:

$$\tilde{\theta}_{c1}^2 = \mu_{ab}r_0^2\tilde{\gamma}_{c1}^2 = \frac{r_0}{2}W^2_{-\eta_{\kappa_{c1}},l+1/2}(2\kappa_{c1}r_0)\mid C\mid^2, \qquad (6)$$

where $\tilde{\gamma}_{c1}$ is the effective reduced width of $c1$. It follows from derived equations that by independently measuring the ANC for the subthreshold bound state, one can calculate the astrophysical factors for direct capture into the subthreshold bound state and subthreshold resonance.

References

[1] C.E. Rolfs and W.S. Rodney, Cauldrons in the Cosmos, Chicago and London: The University of Chicago Press, 1988.

[2] C.A. Gagliardi et al., *Phys. Rev. C59*, 1149 (1999).