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Balance function as a unique probe of the quark gluon plasma: experimental overview and outlook

Jinjin(Au-Au) Pan

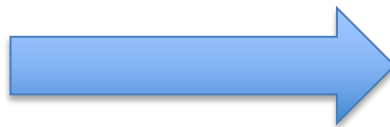
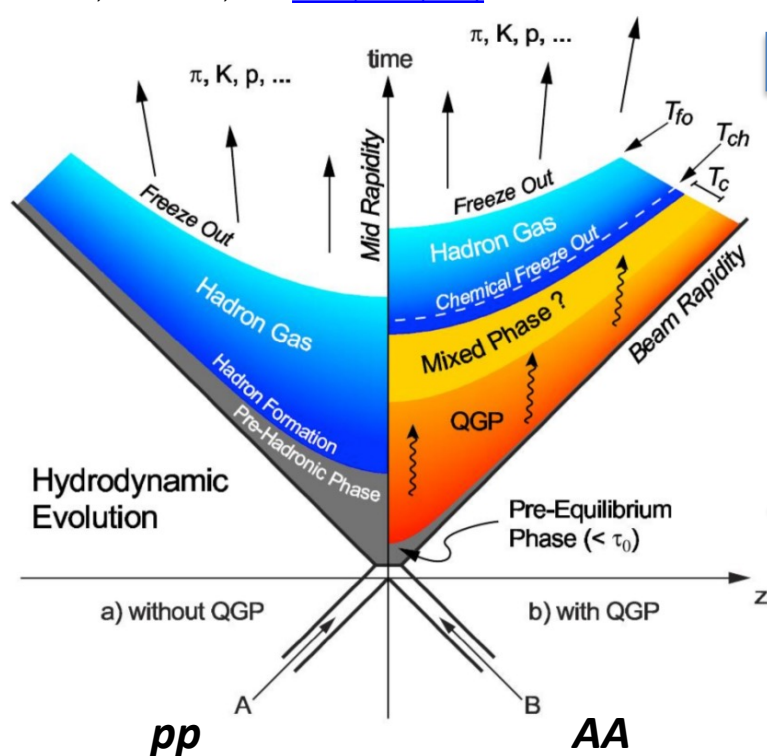
WPCF at FRIB | 07/19/2022

Balance Function – distribution of balancing charges

$$B^{\alpha\beta}(\Delta\vec{p}) = \frac{1}{2} \left[\frac{\rho_2^{\alpha^+\beta^-}(\Delta\vec{p}) - \rho_2^{\alpha^+\beta^+}(\Delta\vec{p})}{\rho_1^{\alpha^+}(\vec{p}^\alpha)} + \frac{\rho_2^{\alpha^-\beta^+}(\Delta\vec{p}) - \rho_2^{\alpha^-\beta^-}(\Delta\vec{p})}{\rho_1^{\alpha^-}(\vec{p}^\alpha)} \right]$$

Bass, Danielewicz, Pratt [PRL 85, 2689 \(2000\)](#)

For each charge +Q, BF identifies the probability where its balancing charge -Q is.



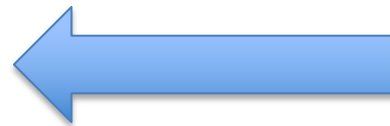
*measure BF of final state
identified hadron pairs*

± — General conserved charges:

- e: electric charge
- S: strangeness
- B: baryon number

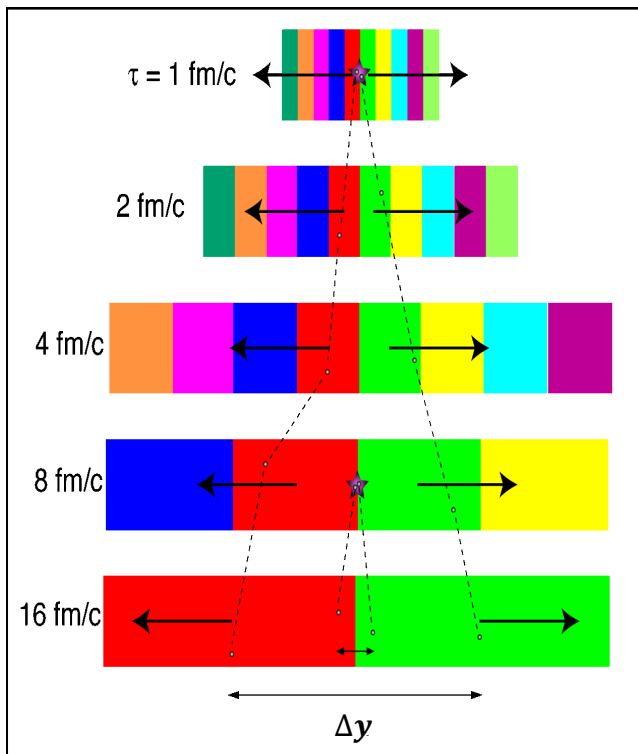


*learn about light and strange
quark production*



Balancing charge separation in Δy

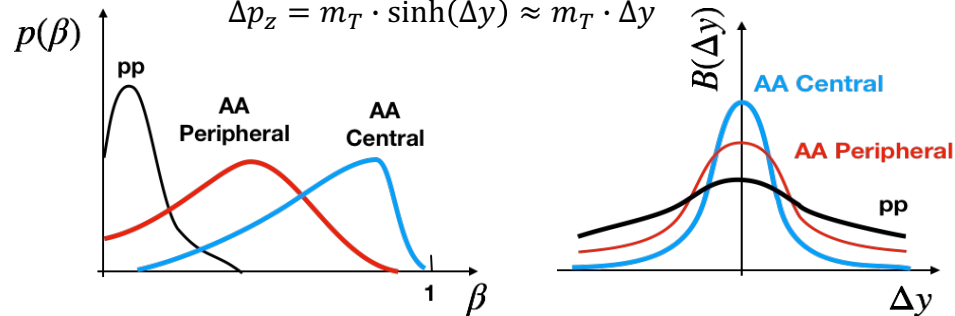
System lifetime



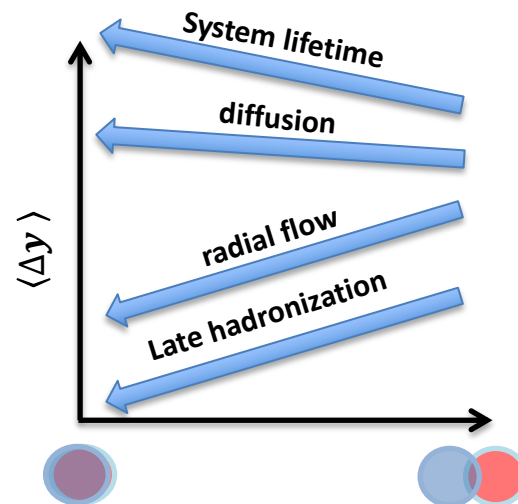
Balancing charge separation

Voloshin [PLB 632 \(2006\) 490-494](#)

$$\Delta p_z = m_T \cdot \sinh(\Delta y) \approx m_T \cdot \Delta y$$



Larger radial flow towards central collisions leads to smaller separation of balancing pairs in Δy



Bass, Danielewicz, Pratt [PRL 85, 2689 \(2000\)](#)

Later hadronization towards central collisions $\rightarrow$ narrower BF of $\pi\pi$, KK , pp

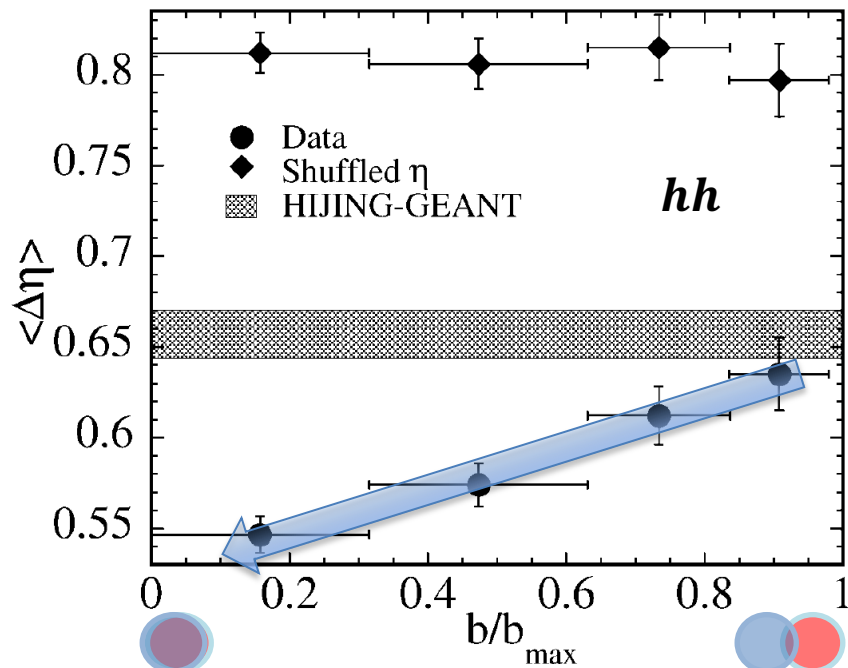


BF of unidentified hadron pairs hh and $\pi\pi$

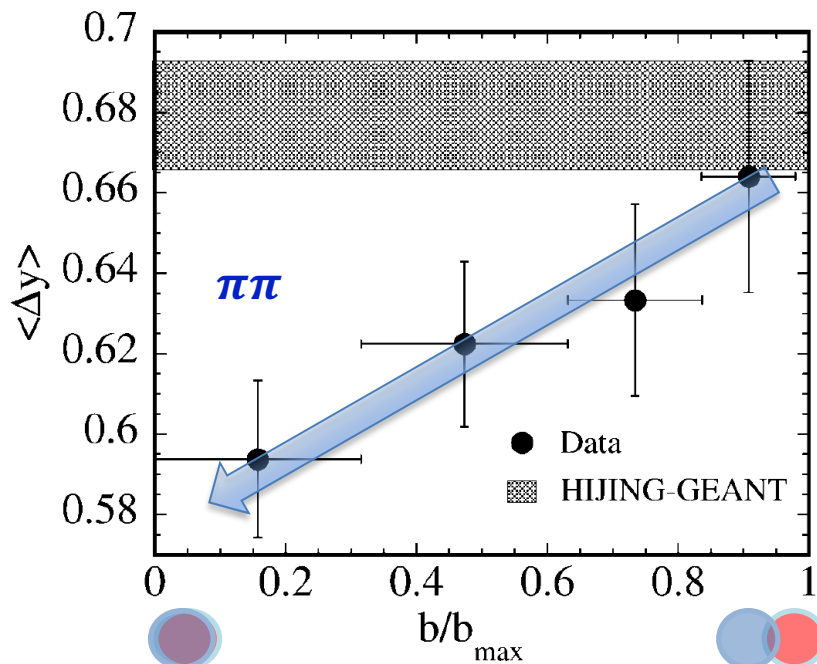
STAR [PRL 90, 172301 \(2003\)](#)

Au-Au @ 130 GeV

$0.1 < p < 2.0 \text{ GeV}/c$



$0.1 < p_T, p < 0.7 \text{ GeV}/c$



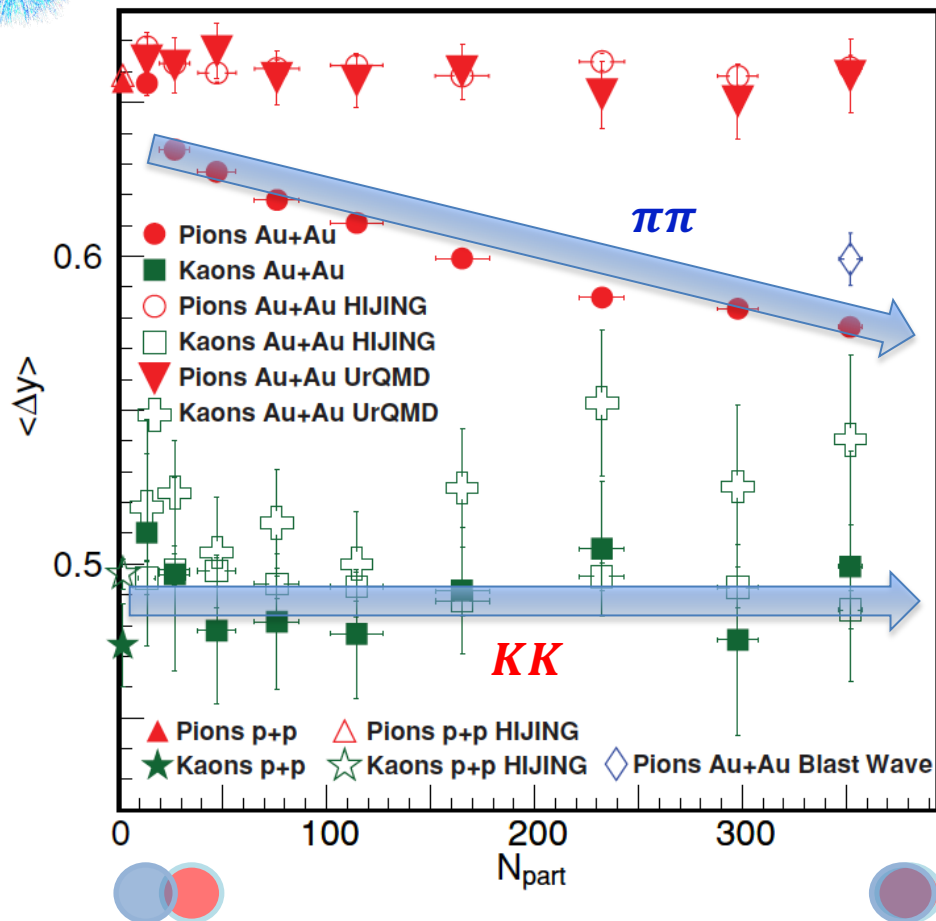
Narrowing of B^{hh} and $B^{\pi\pi}$ towards central Au-Au collisions

-> larger radial flow towards central collisions leads to smaller separation of balancing pairs in $\Delta\eta$ & Δy

-> later hadronization towards central collisions leads to narrower BF



BF of identified hadron pairs $\pi\pi$ and KK



Au-Au @ 200 GeV

STAR [PRC 82, 024905 \(2010\)](#)

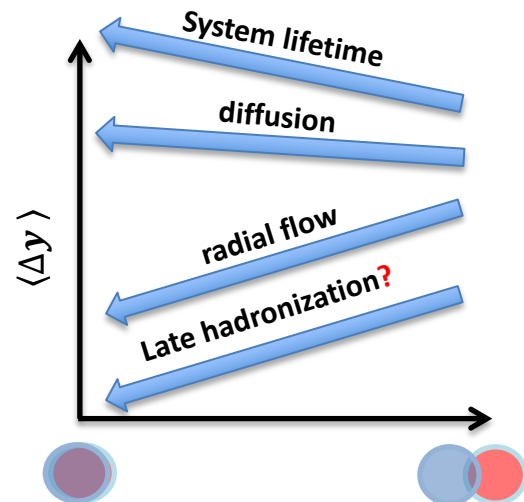
$0.2 < p_T < 0.6 \text{ GeV}/c$

$B^{\pi\pi}(\Delta y)$ narrow towards central collisions, while $B^{KK}(\Delta y)$ no centrality dependence.

-> larger radial flow towards central collisions leads to smaller separation of balancing pairs in Δy for $\pi\pi$ and KK .

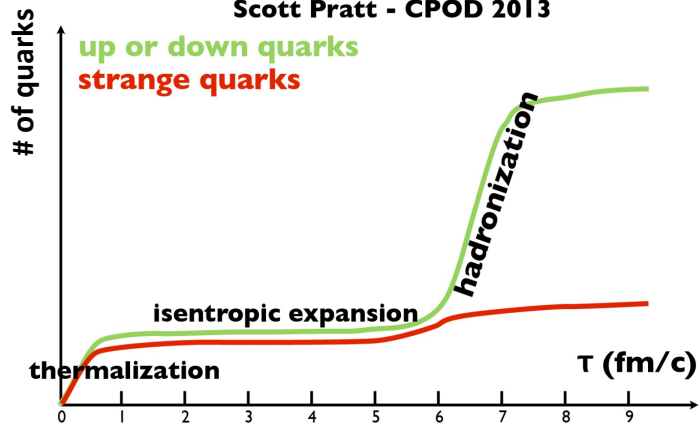
-> $\langle \Delta y \rangle$ for KK is smaller than $\pi\pi$ due to ϕ decay.

-> no late hadronization for KK ?



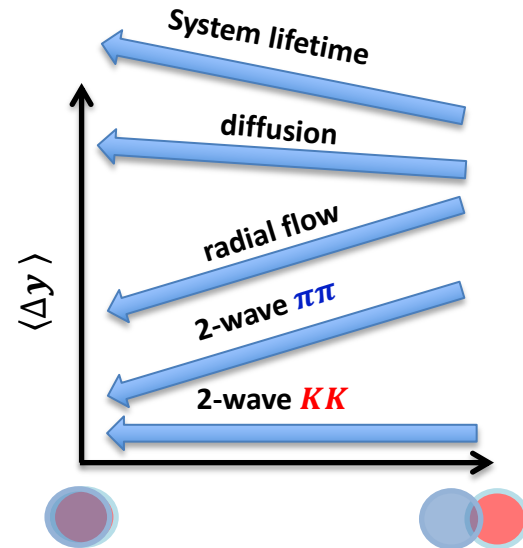
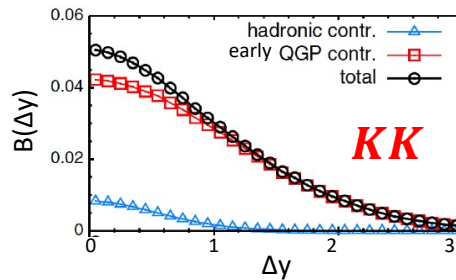
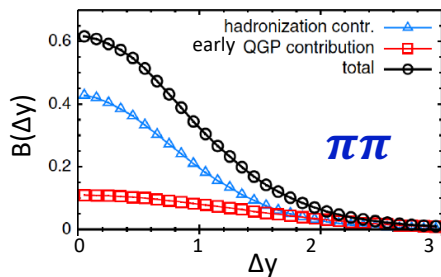
Two-wave Quark Production

Scott Pratt - CPOD 2013



Generalized BF (between different particle species)
-> understand balancing between quark species
-> access to light and strange quark production time

Pratt [PRL. 108, 212301 \(2012\)](#)

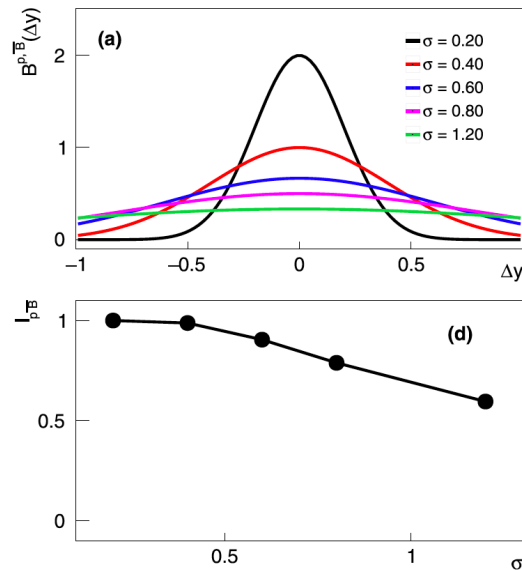
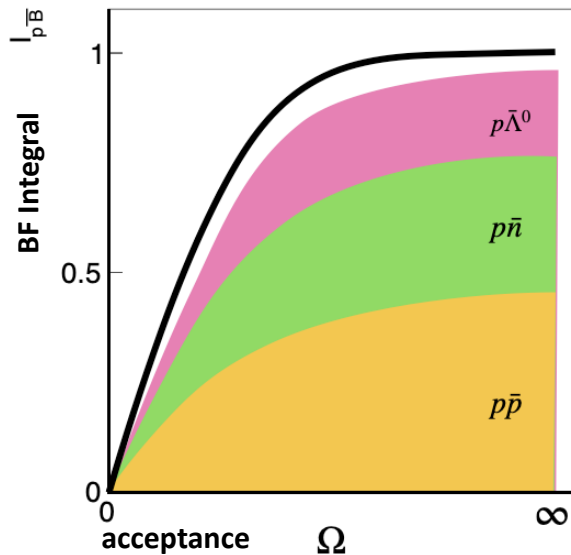


$\pi\pi$: larger 2nd wave up/down quark production -> smaller $\langle \Delta y \rangle$ -> narrow towards central collisions

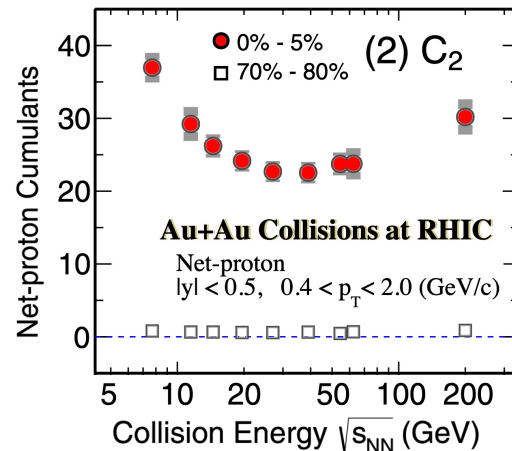
KK : dominant 1st wave strange quark production -> same $\langle \Delta y \rangle$ -> no centrality dependence

Baryon number BF & Net-baryon fluctuation

Pruneau [PRC 100, 034905 \(2019\)](#)



STAR [PRL 126, 092301 \(2021\)](#)



BF Integral

- hadron species pairing probability (never measured before)
- interplay of pair production process, acceptance, and BF width

$$1 - \frac{C_2(\Delta N_p)}{C_2^{Skellam}(\Delta N_p)} = I_{BF}(\Omega)$$

high energy A-A collisions
in the limit $\langle N_p \rangle = \langle N_{\bar{p}} \rangle$

Differential baryon number BF in BES -> critical point search

Two-particle Number Correlation Function

2-Particle Cumulant

$$C_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta) = \rho_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta) - \rho_1^\alpha(\vec{p}^\alpha) \cdot \rho_1^\beta(\vec{p}^\beta)$$

$\alpha, \beta - \mathbf{h}, \pi, \mathbf{K}, \mathbf{p}, \Lambda$ and $\Xi \dots$

α – reference particle

β – associate particle

ρ_1, ρ_2 – single particle & pair number density per event

Measurement:

M – Measured

$$C_{2,M}^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta) \approx \boxed{\varepsilon_1^\alpha(\vec{p}^\alpha) \cdot \varepsilon_1^\beta(\vec{p}^\beta)} \cdot C_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta)$$

Require separate efficiency estimation for single particles

assuming efficiency factorizes

$$\varepsilon_2^\alpha(\vec{p}^\alpha, \vec{p}^\beta) \approx \varepsilon_1^\alpha(\vec{p}^\alpha) \cdot \varepsilon_1^\beta(\vec{p}^\beta)$$

Normalized 2-Particle Cumulant

$$R_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta) = \frac{C_{2,M}^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^\alpha(\vec{p}^\alpha) \cdot \rho_1^\beta(\vec{p}^\beta)} = \frac{\rho_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^\alpha(\vec{p}^\alpha) \cdot \rho_1^\beta(\vec{p}^\beta)} - 1$$

Efficiencies cancel in the ratio -> robust observable

Balance Function (BF) Definition

Associated particle distribution
(per trigger yield)

$$A^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta) = \frac{\rho_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^\alpha(\vec{p}^\alpha)} - \rho_1^\beta(\vec{p}^\beta) = \rho_1^\beta(\vec{p}^\beta) \cdot R_2^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta)$$

BF of positive reference particle α^+

$$B^{\alpha^+\beta}(\vec{p}^\alpha, \vec{p}^\beta) = A^{\alpha^+\beta^-} - A^{\alpha^+\beta^+} = \frac{\rho_2^{\alpha^+\beta^-}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^{\alpha^+}(\vec{p}^\alpha)} - \rho_1^{\beta^-}(\vec{p}^\beta) - \frac{\rho_2^{\alpha^+\beta^+}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^{\alpha^+}(\vec{p}^\alpha)} + \rho_1^{\beta^+}(\vec{p}^\beta)$$

BF of negative reference particle α^-

$$B^{\alpha^-\beta}(\vec{p}^\alpha, \vec{p}^\beta) = A^{\alpha^-\beta^+} - A^{\alpha^-\beta^-} = \frac{\rho_2^{\alpha^-\beta^+}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^{\alpha^-}(\vec{p}^\alpha)} - \rho_1^{\beta^+}(\vec{p}^\beta) - \frac{\rho_2^{\alpha^-\beta^-}(\vec{p}^\alpha, \vec{p}^\beta)}{\rho_1^{\alpha^-}(\vec{p}^\alpha)} + \rho_1^{\beta^-}(\vec{p}^\beta)$$

Balance function

$$\begin{aligned} B^{\alpha\beta}(\vec{p}^\alpha, \vec{p}^\beta) &= \frac{1}{2} [B^{\alpha^+\beta}(\vec{p}^\alpha, \vec{p}^\beta) + B^{\alpha^-\beta}(\vec{p}^\alpha, \vec{p}^\beta)] \\ &= \frac{1}{2} [\rho_1^{\beta^-}(\vec{p}^\beta) \cdot R_2^{\alpha^+\beta^-}(\vec{p}^\alpha, \vec{p}^\beta) - \rho_1^{\beta^+}(\vec{p}^\beta) \cdot R_2^{\alpha^+\beta^+}(\vec{p}^\alpha, \vec{p}^\beta) + \rho_1^{\beta^-}(\vec{p}^\beta) \cdot R_2^{\alpha^-\beta^+}(\vec{p}^\alpha, \vec{p}^\beta) - \rho_1^{\beta^+}(\vec{p}^\beta) \cdot R_2^{\alpha^-\beta^-}(\vec{p}^\alpha, \vec{p}^\beta)] \end{aligned}$$

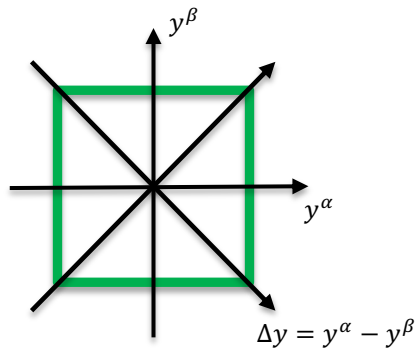
Balance Function Measurement

$$\begin{aligned}
 & B^{\alpha\beta}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta) \\
 &= \frac{1}{2} [\rho_1^{\beta-}(y^\beta, \varphi^\beta) \cdot R_2^{\alpha+\beta-}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta) - \rho_1^{\beta+}(y^\beta, \varphi^\beta) \cdot R_2^{\alpha+\beta+}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta) \\
 &+ \rho_1^{\beta+}(y^\beta, \varphi^\beta) \cdot R_2^{\alpha-\beta+}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta) - \rho_1^{\beta-}(y^\beta, \varphi^\beta) \cdot R_2^{\alpha-\beta-}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta)]
 \end{aligned}$$

- measure $R_2^{\alpha\beta}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta)$ for interested p_T range
- $\rho_1^{\beta}(y^\beta, \varphi^\beta)$ (assuming constant for mid-rapidity) calculated from previous p_T spectra measurements

$$0.2 \leq p_T^{\pi, K} \leq 2.0 \text{ GeV}/c$$

$$0.5 \leq p_T^p \leq 2.5 \text{ GeV}/c$$

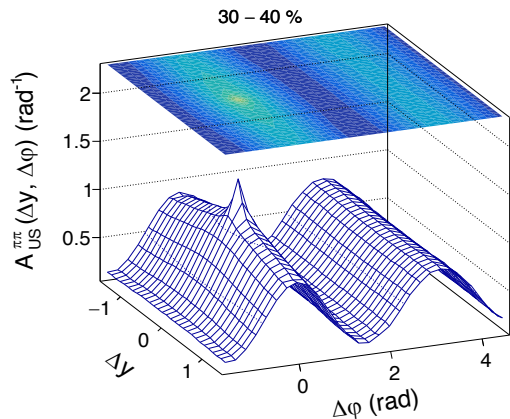


$$B^{\alpha\beta}(\Delta y, \Delta \varphi) = \int B^{\alpha\beta}(y^\alpha, \varphi^\alpha, y^\beta, \varphi^\beta) dy^\beta d\varphi^\beta$$

average over y^β

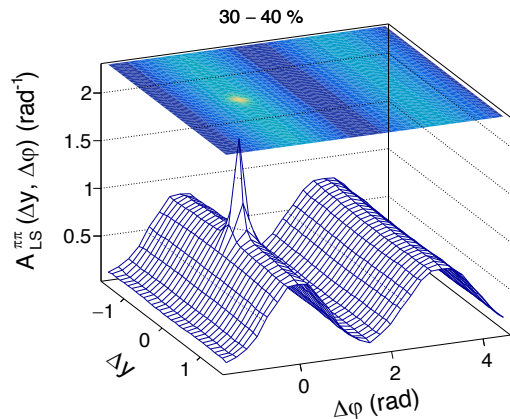
BF measures charge-dependent (CD) correlations

$$A_{US}^{\alpha\beta} = \frac{1}{2}(A^{\alpha^+\beta^-} + A^{\alpha^-\beta^+})$$



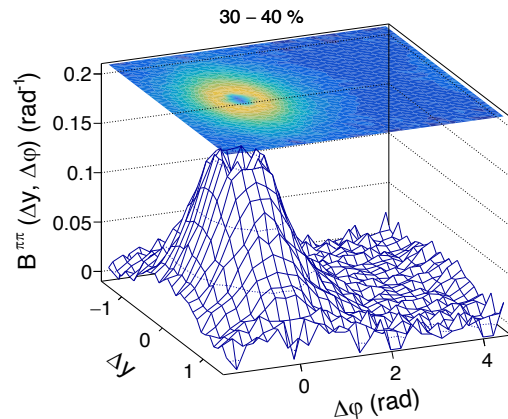
Remove charge independent effects

$$A_{LS}^{\alpha\beta} = \frac{1}{2}(A^{\alpha^+\beta^+} + A^{\alpha^-\beta^-})$$

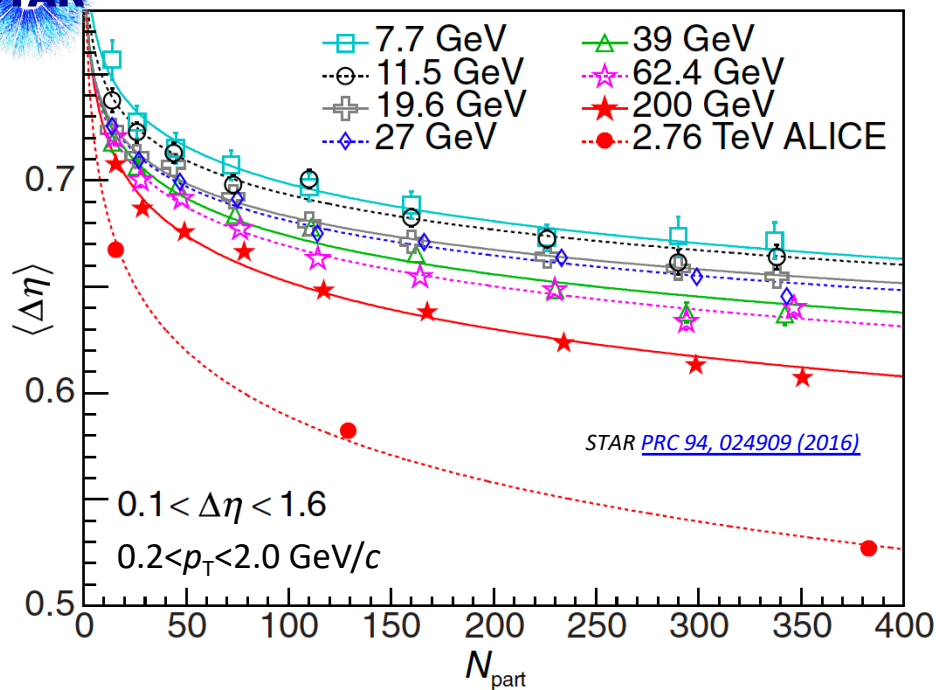


Keep effects related to balancing pairs

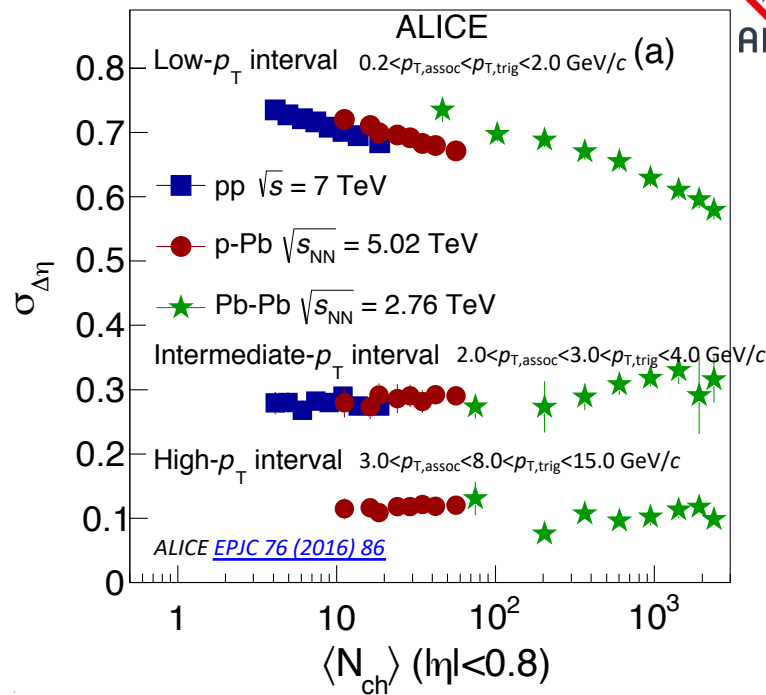
$$B^{\alpha\beta}(\Delta y, \Delta\phi) = A_{CD}^{\alpha\beta} = A_{US}^{\alpha\beta} - A_{LS}^{\alpha\beta}$$



BF of unidentified hadrons: STAR BES I & ALICE



- $B^{hh}(\Delta\eta)$ narrow towards central Au-Au and Pb-Pb collisions
-> larger radial flow in central leads to smaller $\langle \Delta\eta \rangle$ separation
-> larger 2nd wave up/down quark production in central -> smaller $\langle \Delta\eta \rangle$
- lower energy (7.7 GeV): narrow towards central collisions -> QGP



- **Low p_T :**
 - pp, p-Pb: similar widths at overlapping multiplicities -> similar origin in BF
 - p-Pb and Pb-Pb: different at overlapping multiplicities -> different origin
- **Intermediate & high p_T :**
 - narrower & no multiplicity dependence -> initial hard parton scattering & subsequent fragmentation
 - similar values for all multiplicities over all three systems -> similar dynamics

BF of full species matrix $(\pi^\pm, K^\pm, p/\bar{p}) \times (\pi^\pm, K^\pm, p/\bar{p})$

$\pm$ — General conserved charges:

- e: electric charge
- S: strangeness
- B: baryon number

✓: previous works

✓: ALICE 2021 [arXiv:2110.06566](https://arxiv.org/abs/2110.06566)

		$B^{\alpha\beta}(\Delta y, \Delta\varphi)$	$h^\pm$	$\pi^\pm$	$K^\pm$	$p/\bar{p}$
e		$h^\pm$	✓			
e		$\pi^\pm$		✓✓	✓	✓
e	S	$K^\pm$		✓	✓✓	✓
e	B	$p/\bar{p}$		✓	✓	✓

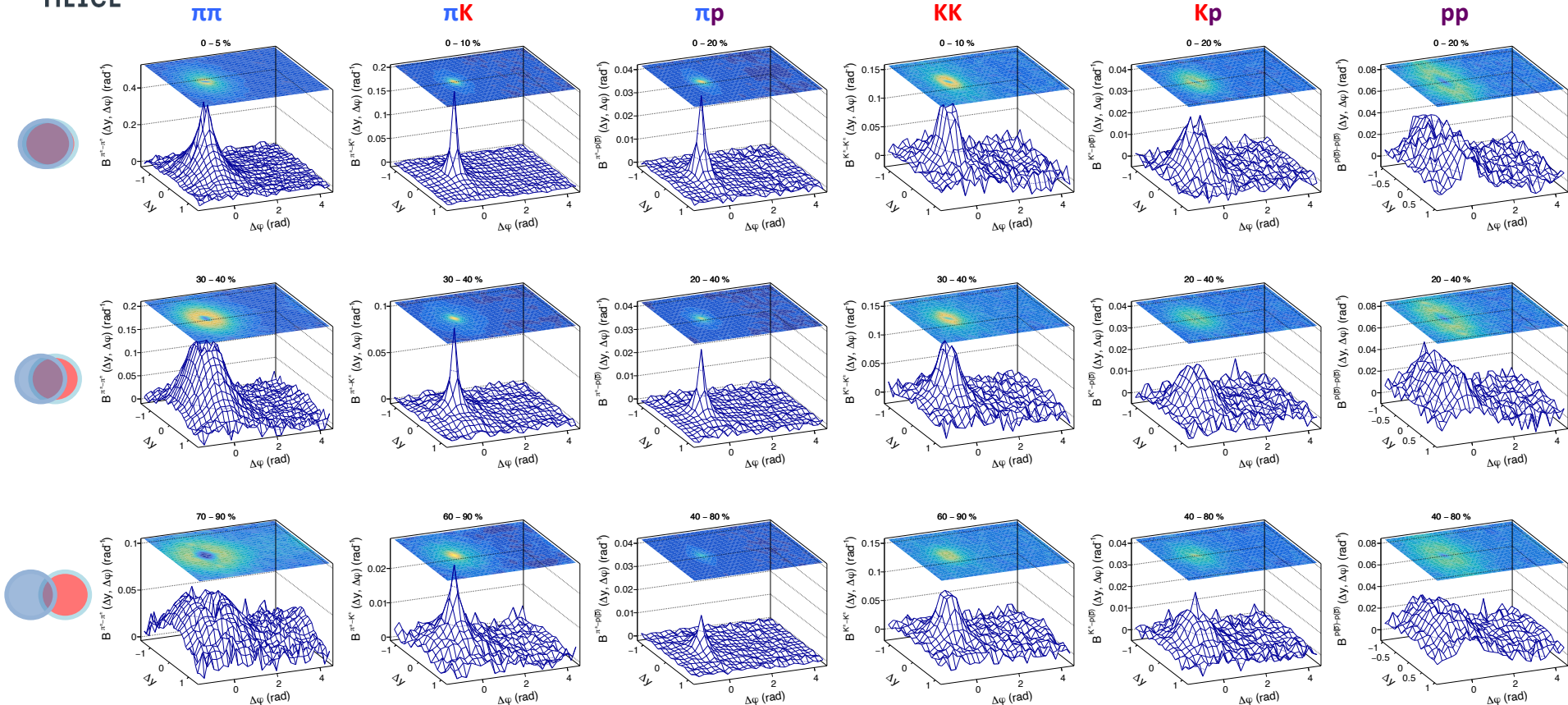
1st BF measurement of full species matrix of $(\pi^\pm, K^\pm, p/\bar{p}) \times (\pi^\pm, K^\pm, p/\bar{p})$.



ALICE

PID BF – Full Species Matrix

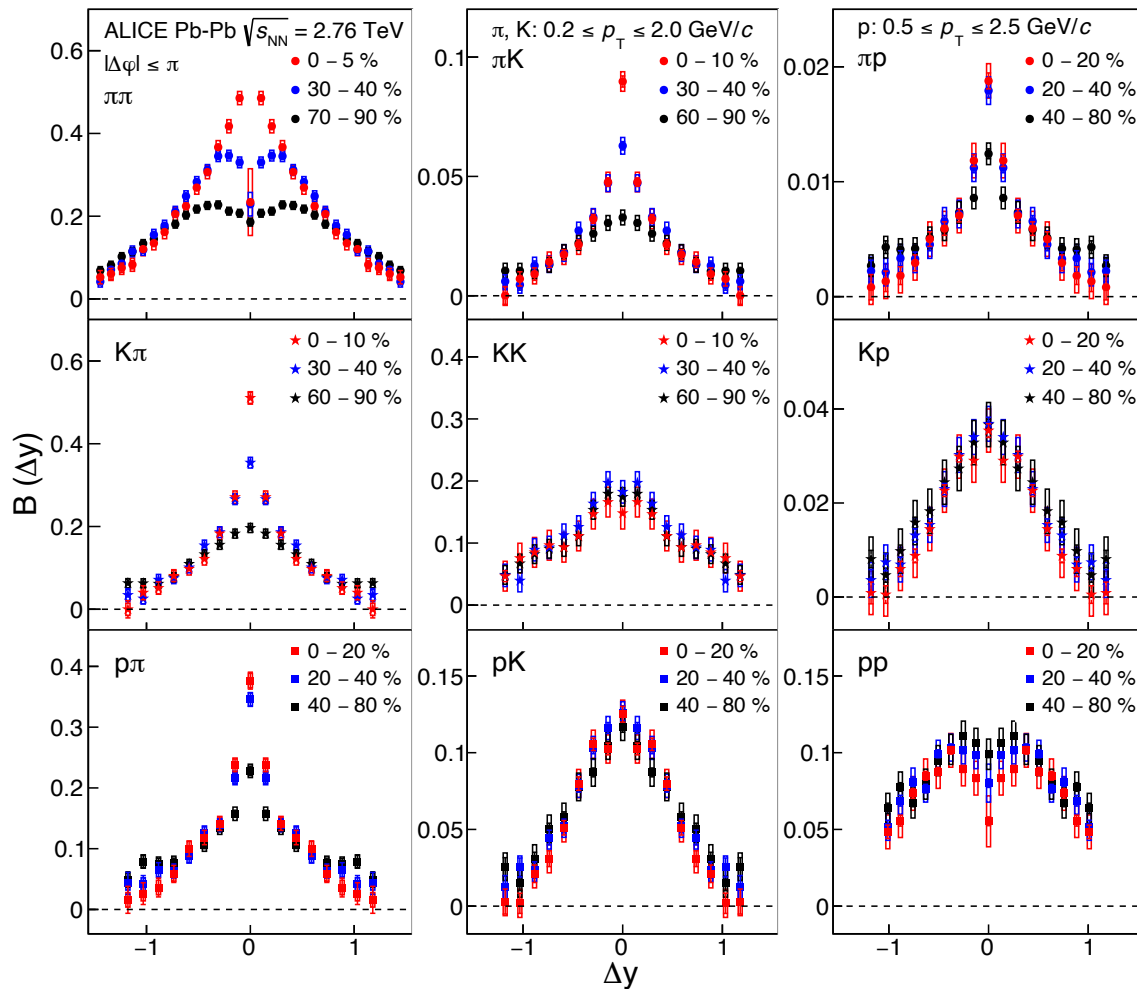
 $0.2 \leq p_T^{\pi,K} \leq 2.0 \text{ GeV}/c$
 $0.5 \leq p_T^p \leq 2.5 \text{ GeV}/c$

 JP PhD dissertation, [arXiv:1911.02234](https://arxiv.org/abs/1911.02234)
 ALICE 2021 [arXiv:2110.06566](https://arxiv.org/abs/2110.06566)




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1D BF Δy Projections

JP PhD dissertation, [arXiv:1911.02234](https://arxiv.org/abs/1911.02234)ALICE 2021 [arXiv:2110.06566](https://arxiv.org/abs/2110.06566)

Note: different scale

$\pi\pi$: clear centrality dependence

KK : no centrality dependence

-> consistent with radial flow and two wave quark production

BF including π :

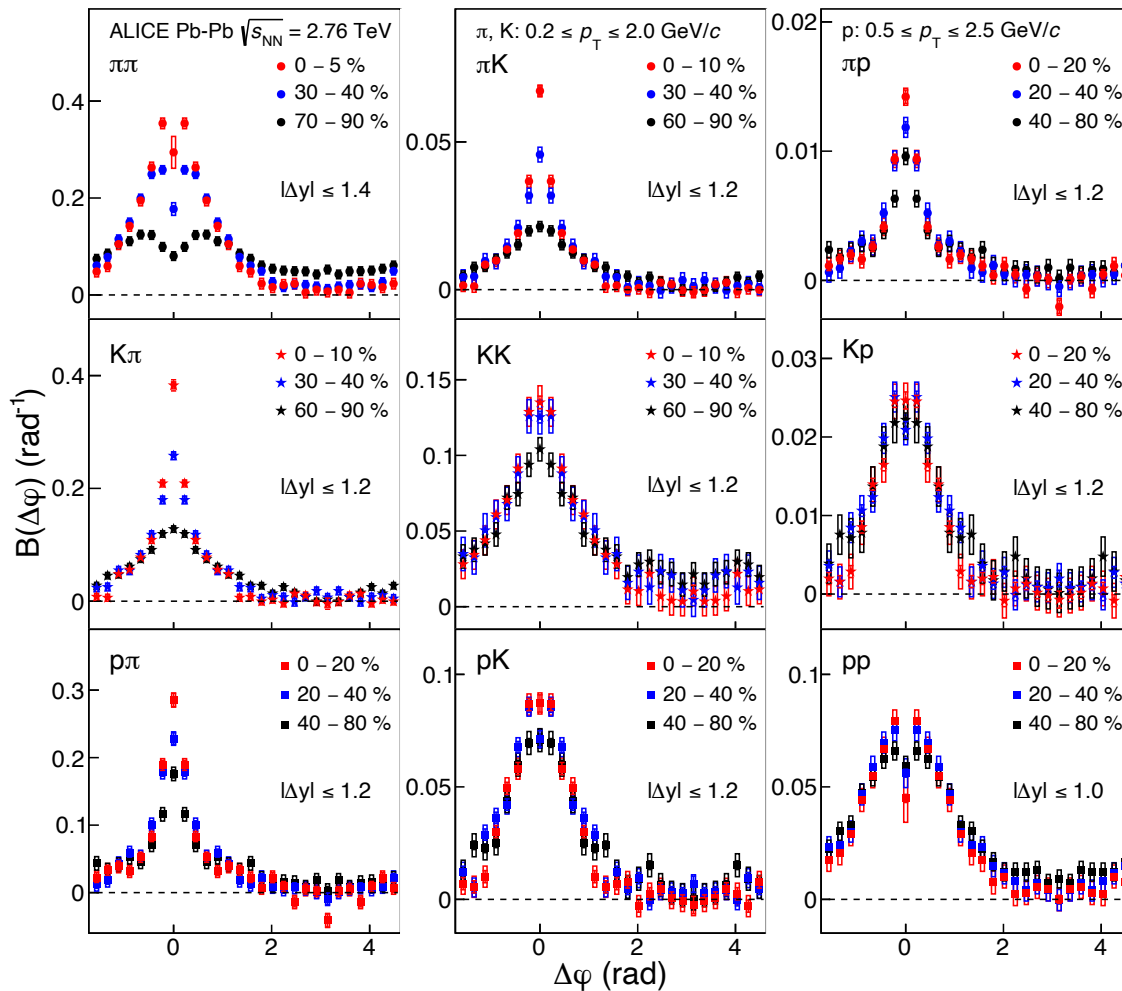
Clear centrality dependence

BF including K, p :

no / little centrality dependence

-> different production mechanisms for π, K, p

1D BF $\Delta\varphi$ Projections



Note: different scale

BF including π :

Clear centrality dependence

BF including K , p :

no / little centrality dependence

-> different production mechanisms for π , K , p

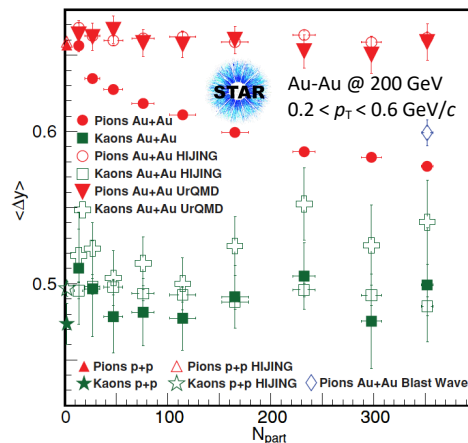
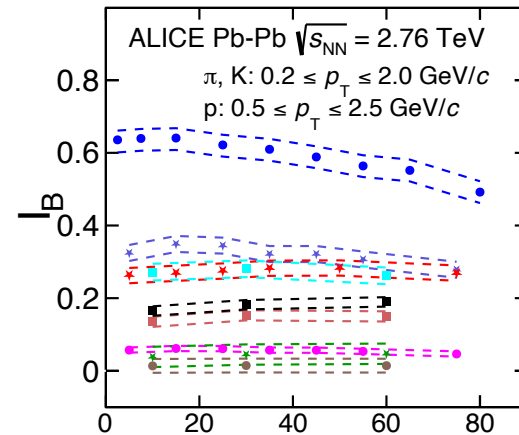
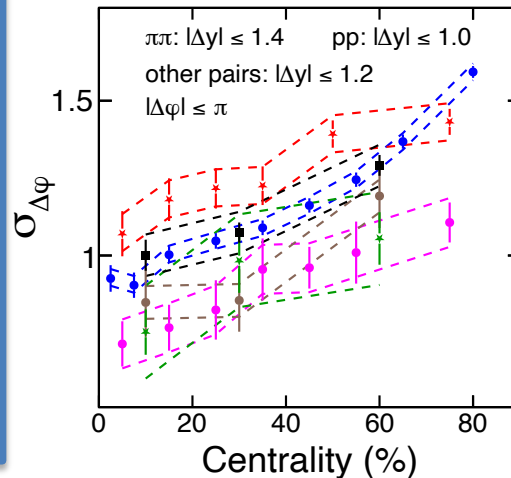
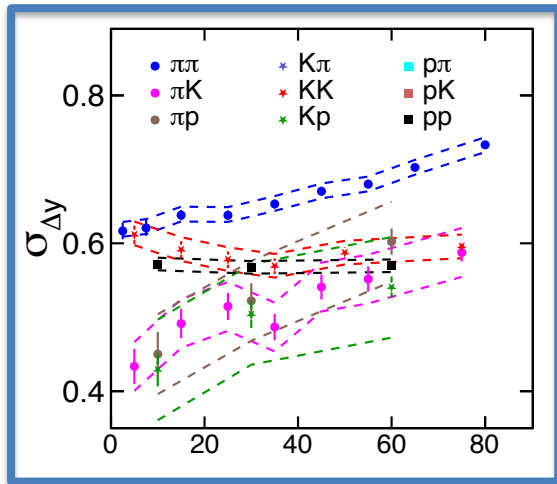


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BF RMS Widths and Integrals

JP PhD dissertation, [arXiv:1911.02234](https://arxiv.org/abs/1911.02234)

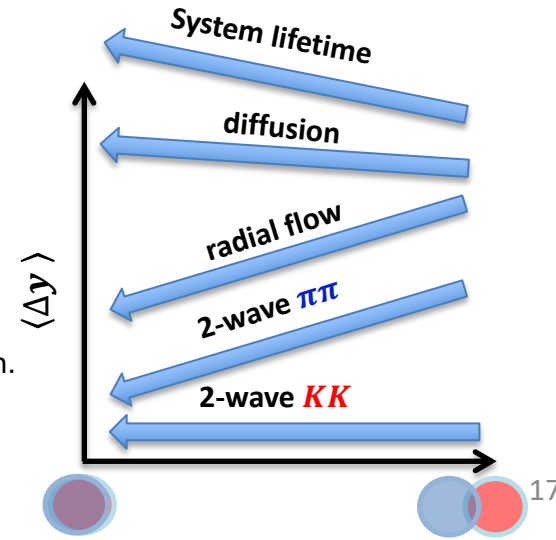
ALICE 2021 [arXiv:2110.06566](https://arxiv.org/abs/2110.06566)



STAR [PRC 82, 024905 \(2010\)](https://arxiv.org/abs/1005.0249)

B(Δy) RMS Widths:

- **KK** & **pp** no centrality dependence; **$\pi\pi$** & cross-species pairs narrow towards central collisions
 - Similar values for all cross-species pairs.
 - Qualitatively consistent with radial flow & two-wave quark production
- > detailed modeling required to distinguish them.



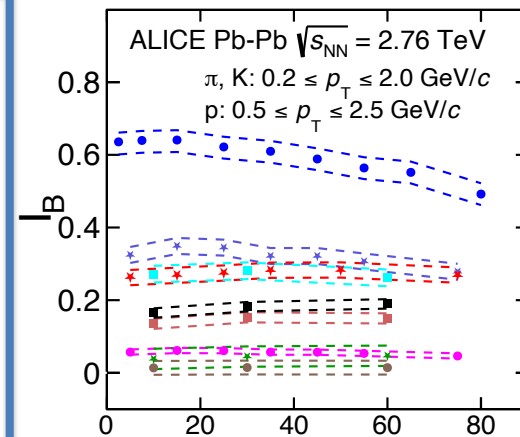
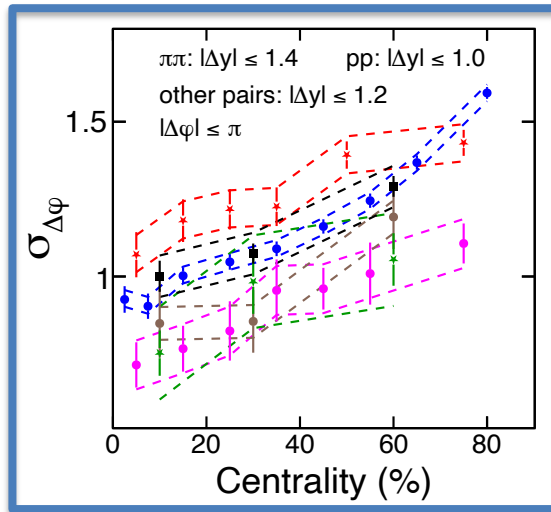
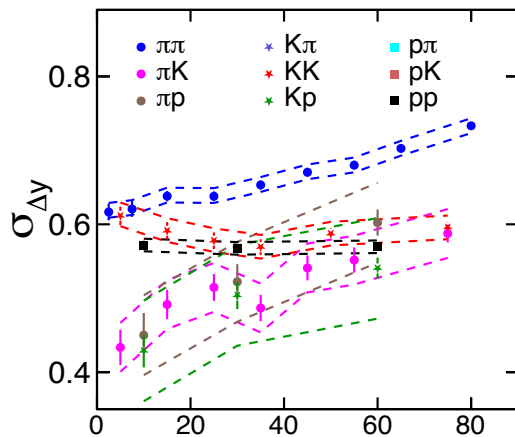


ALICE

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ALICE 2021 [arXiv:2110.06566](https://arxiv.org/abs/2110.06566)



$B(\Delta\phi)$ RMS Widths:

- Different values for different species pairs
-> radial flow affects pairs of different mass differently.
- Widths for pp is same with $\pi\pi$ due to different Δy range. Other effects?
- All species pairs narrow towards central collision -> qualitatively radial flow > diffusion.
- More detailed information on radial flow profile in context of hadron species pairs.

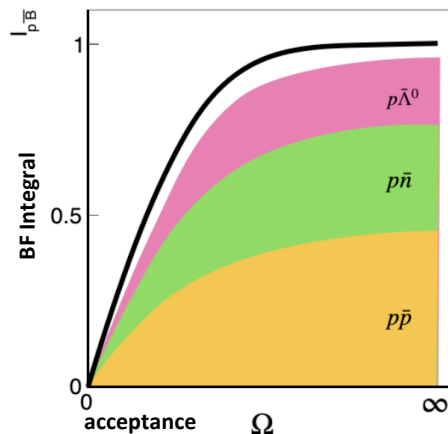
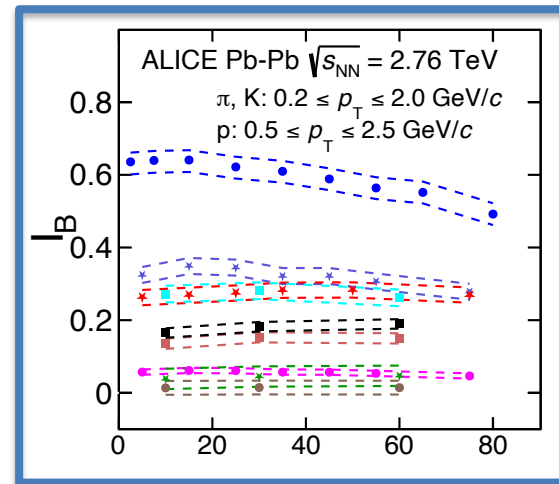
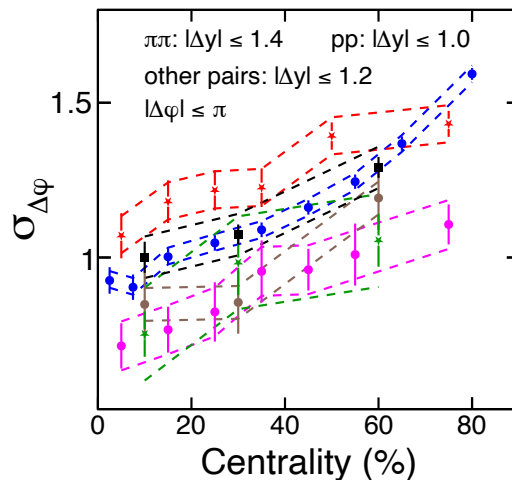
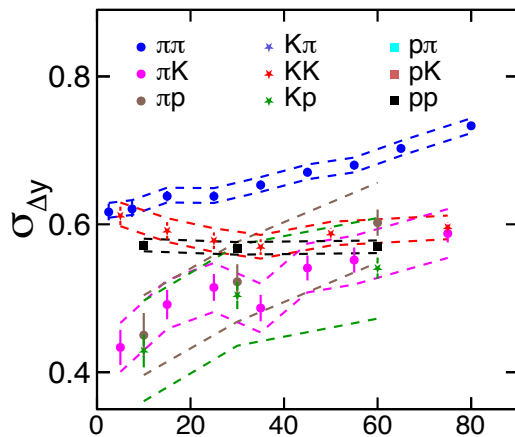


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ALICE 2021 [arXiv:2110.06566](https://arxiv.org/abs/2110.06566)



Balance Function Integrals:

- 1st measurement of hadron species pairing probability.
- Sum of integrals of **pi** trigger, **K** trigger, **p** trigger BFs ~ 0.65 .
- Minimal centrality dependence for most pairs, but **pi pi** increasing towards central collisions
 -> $B(\pi\pi)$ losses beyond acceptance more for peripheral than central collisions.

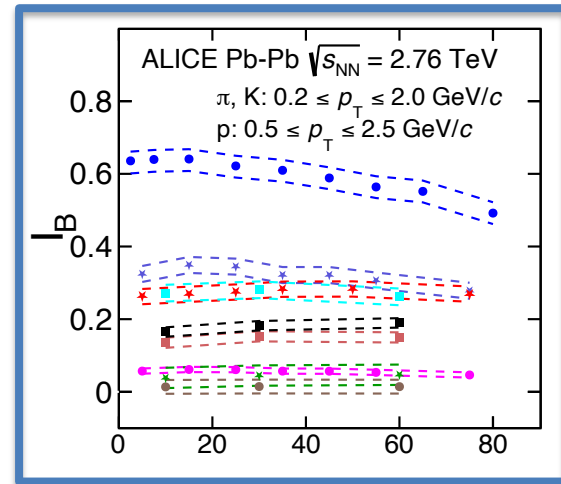
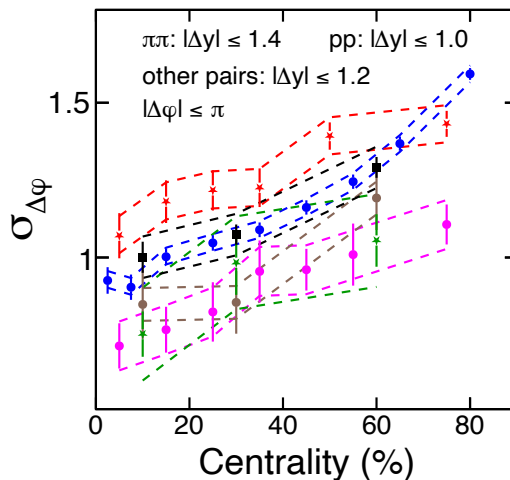
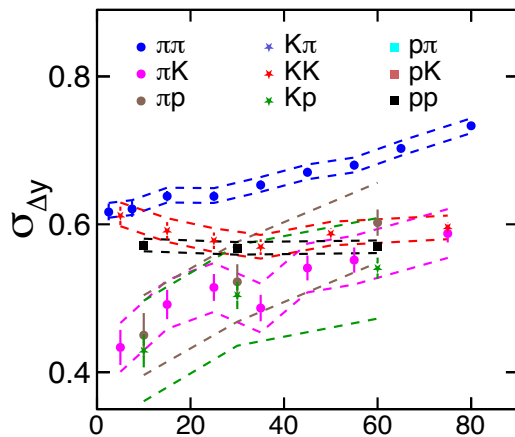


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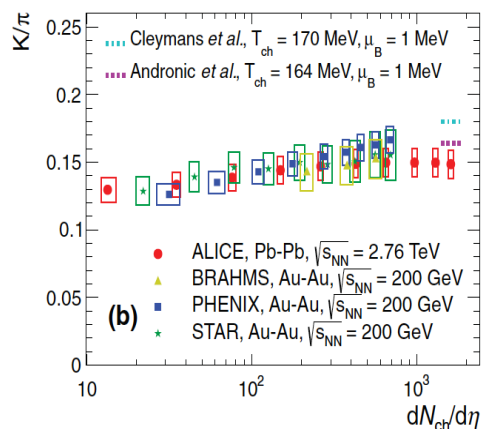
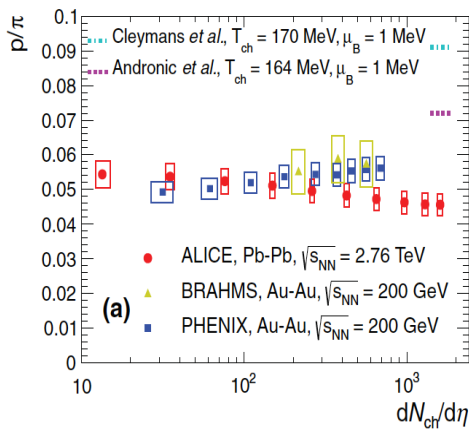
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ALICE 2021 [arXiv:2110.06566](https://arxiv.org/abs/2110.06566)



ALICE [PRC 88, 044910 \(2013\)](https://arxiv.org/abs/1304.0449)

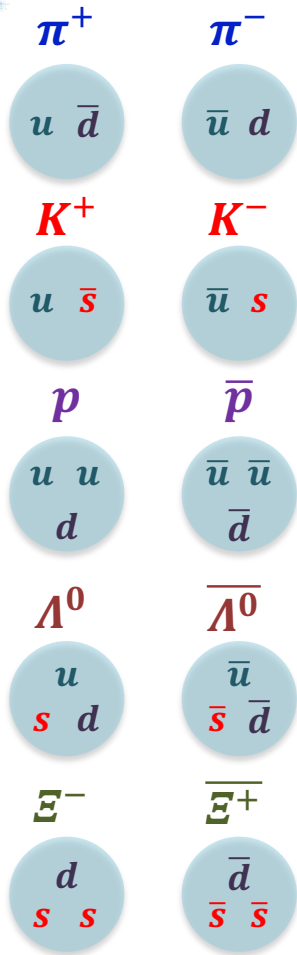


Balance Function Integrals:

- 1st measurement of hadron species pairing probability.
- Sum of integrals of **pi** trigger, **K** trigger, **p** trigger BFs ~ 0.65 .
- Minimal centrality dependence for most pairs, but **pi pi** increasing towards central collisions
 -> $B(\pi\pi)$ losses beyond acceptance more for peripheral than central collisions.
- Hadron species pairing probabilities very different from single hadron ratios.
 e.g. **K pi** not larger than **K K** by **pi/K** ratio; **p p** larger than **p K**.
 -> better constraint for models.



Balance Function experimental outlook



$\pm$ — General conserved charges:

- **e**: electric charge
- **S**: strangeness
- **B**: baryon number

✓: previous works

✓: ALICE 2021 [arXiv:2110.06566](https://arxiv.org/abs/2110.06566)

★: ALICE, STAR work in progress

		$B^{\alpha\beta}(\Delta y, \Delta\phi)$	$h^\pm$	$\pi^\pm$	$K^{\pm 0}$	$p/\bar{p}$	$\Lambda^0/\bar{\Lambda}^0$	$E^-/\bar{E}^+$
e		$h^\pm$	✓					
e		$\pi^\pm$		✓✓	✓	✓		
e	S	$K^{\pm 0}$		✓	✓✓	✓	★	★
e	B	$p/\bar{p}$		✓	✓	✓	★	★
S	B	$\Lambda^0/\bar{\Lambda}^0$			★	★	★	★
e	S	$E^-/\bar{E}^+$			★	★	★	★

uncharted territory

- 3D $B^{\alpha\beta}(\Delta y, \Delta\phi, \Delta p_T)$
- differential $B^{\alpha\beta}(\Delta p_T)$
- $B^{\alpha\beta}$ w.r.t event plane
- $B^{\alpha\beta}$ in jets

Summary & Conclusions

- **Generalized BF (between different particle species) key observable**
 - > understand balancing between quarks
 - > have access to the timing
 - > equivalent to net-baryon C_2 for critical point search
- **GBF (Pb-Pb @ 2.76 TeV) ALICE 2021 [arXiv:2110.06566](https://arxiv.org/abs/2110.06566):**
 - **Three 1st** -> challenge models
 - 1st GBF measurement of full species matrix of $\pi^\pm$, $K^\pm$, $p/\bar{p}$.
 - 1st 2D differential measurement of PID BF.
 - 1st measurement of hadron species pairing probability.
 - **$B(\Delta y)$ Widths:**
 - qualitatively consistent with radial flow and two-wave quark production
 - > a detailed model required to distinguish them.
 - **$B(\Delta\phi)$ Widths:**
 - qualitatively radial flow > diffusion.
 - more info on radial flow profile in context of hadron pairs.
 - **BF Integrals:**
 - minimal centrality dependence.
 - hadron pairing probabilities different from single hadron ratios.
 - > better constraint for models.



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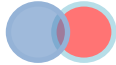
TEXAS A&M
UNIVERSITY

*Thanks to DOE for partially
funding some of the works
shown in this talk.*

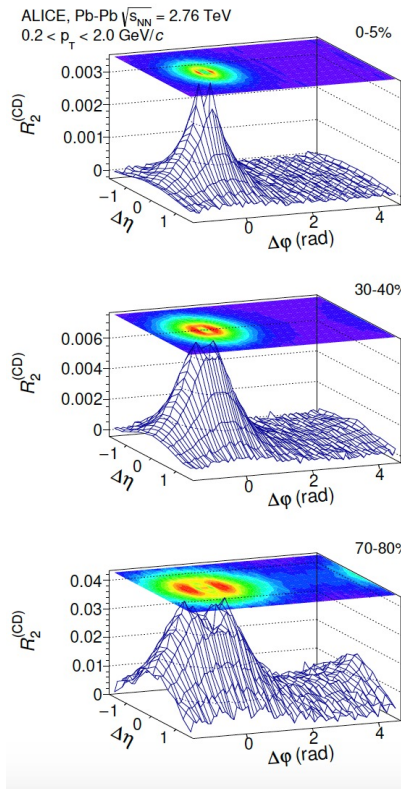
Back-up



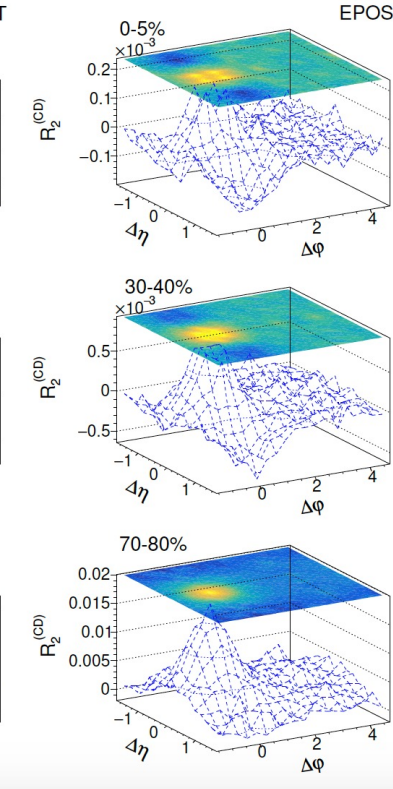
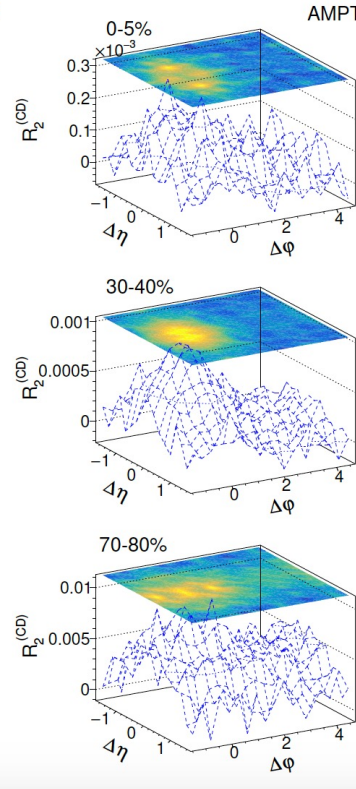
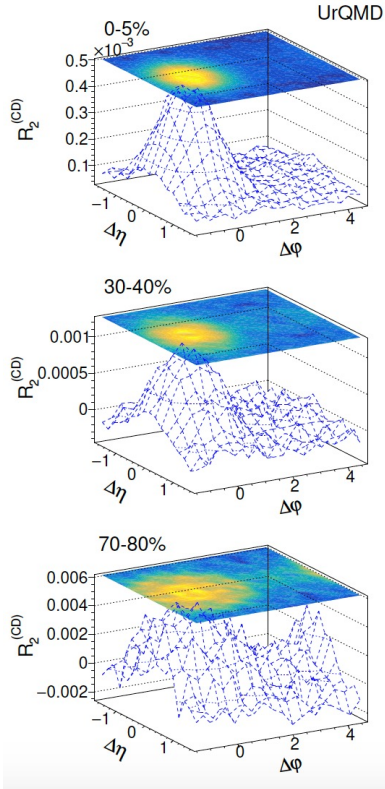
ALICE



$R_2^{(CD)}$ vs. models — unidentified hadrons



PRC 100, 044903 (2019)

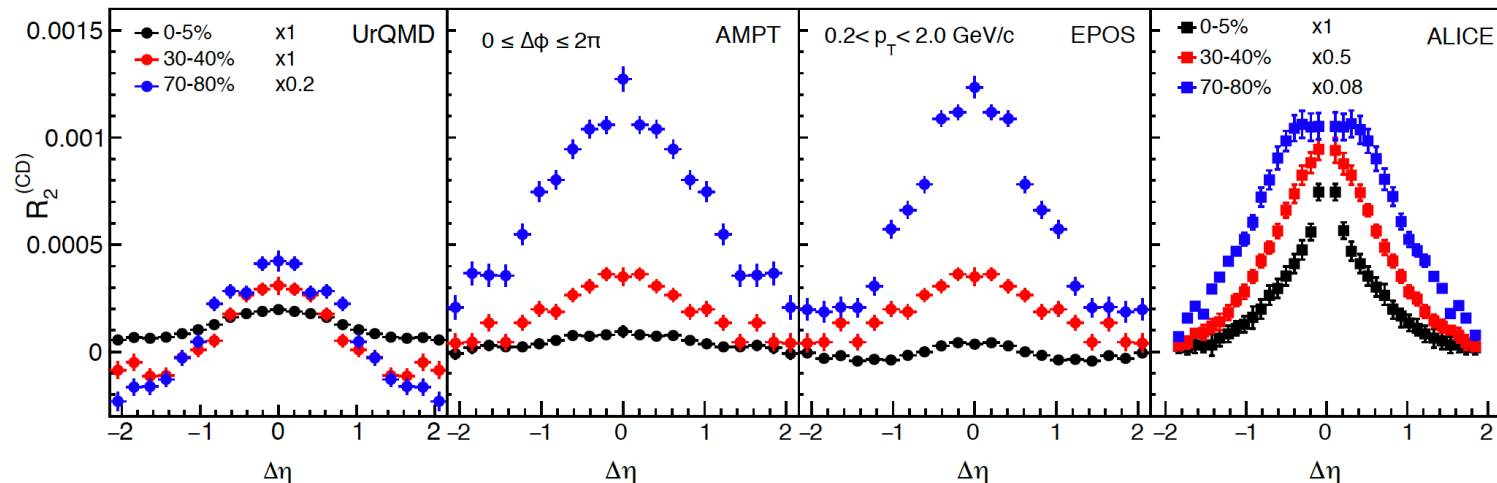


Basu, Gonzalez, JP, et al. [Phys. Rev. C 104, 064902 \(2021\)](#)

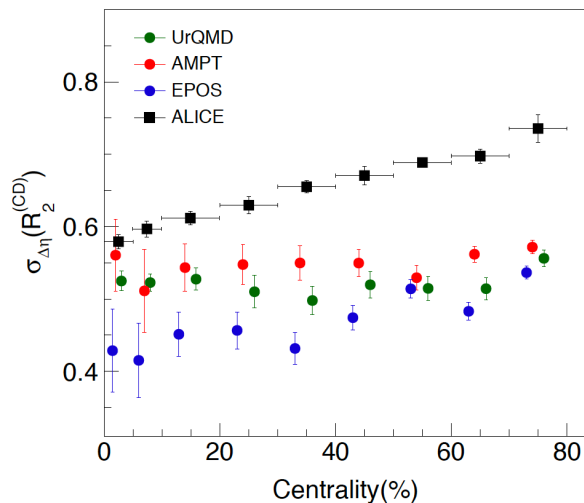
- models qualitatively reproduce near-side peak, but **Not** its amplitude and collision centrality evolution.
- broad dip at $(\Delta y, \Delta \phi) = (0,0)$ in data due to HBT **Not** reproduced by models -> no HBT afterburner
- models qualitatively reproduce away-side tail in peripheral and its suppression in central collisions -> resonance decays, e.g. ρ^0

$R_2^{(CD)}$ vs. models — unidentified hadrons

ALICE, PRC 100, 044903 (2019)



Basu, Gonzalez, JP, et al. [Phys. Rev. C 104, 064902 \(2021\)](#)



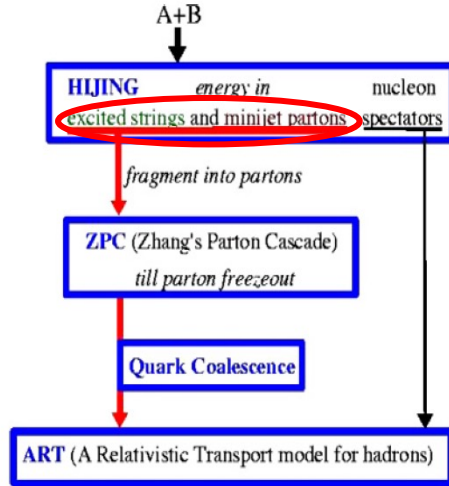
- models **Not** reproduce magnitude and centrality evolution of longitudinal rms
- EPOS: reproduces a narrowing but widths **too narrow** by $\sim 30\%$
 -> corona particle dominance since **No** event-by-event charge conservation in core
 -> average radial flow imparted to corona $\gg$ core
- UrQMD: weak amplitude of near-side peak -> insufficient high-mass resonances
- AMPT: weak amplitude of near-side peak -> incomplete handling of charge conservation



Models Used In This Work

AMPT

Structure of AMPT model with string melting



HIJING

- QCD Lund jet fragmentation
- Hard parton scatterings dominate
- Emphasis on mini-jets in pp, pA & AA

EPOS 3.0

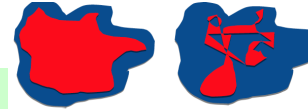
CGC – initial condition

Core – Corona approach

Viscous hydrodynamic expansion

Statistical hadronization (Cooper-Frye)

Final state hadronic cascade (UrQMD model)



Event Generation:

A. Knospe, C. Markert



TEXAS
The University of Texas at Austin

UrQMD

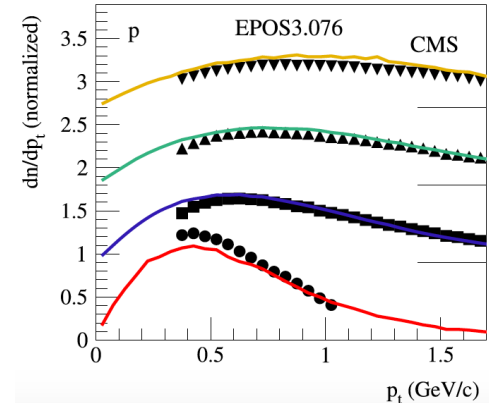
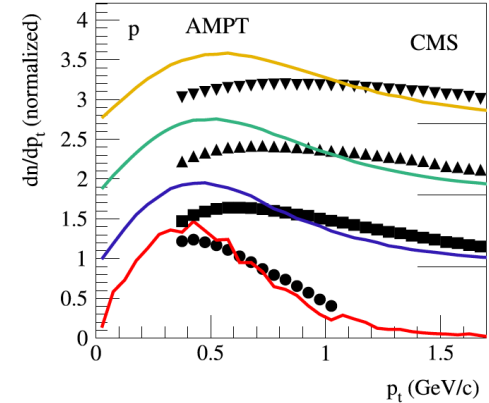
Hadronic relativistic dynamics

Event Generation: W.J. Llope



WAYNE STATE
UNIVERSITY

Werner et al. NPA 931(2014)83–91

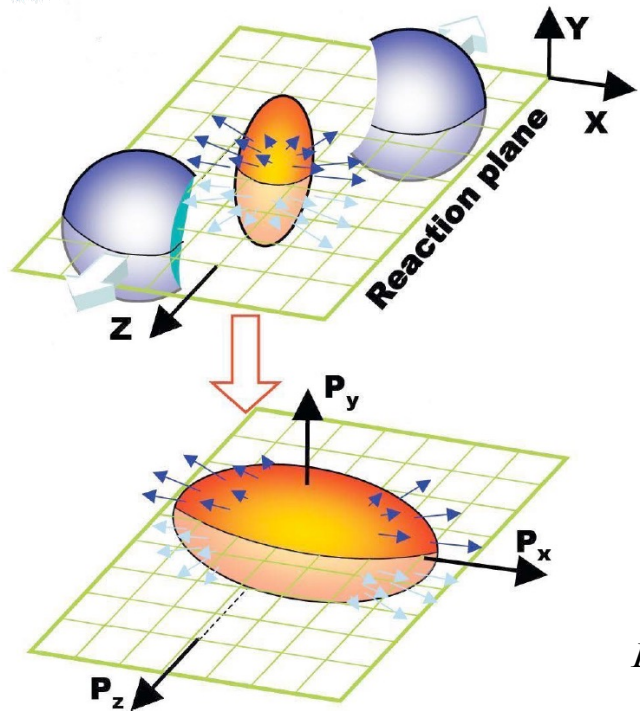




Definition of Anisotropic Flow

- Flow refers to a collective expansion of matter.
- The system follows an anisotropic expansion.
- Anisotropy in the azimuthal particle distribution are studied in terms of the Fourier decomposition.

Voloshin and Zhang. Z.Phys.,C70:665-672,1996.



**Spatial
Anisotropy**

density gradient → pressure
for anisotropic expansion

**Momentum
Anisotropy**

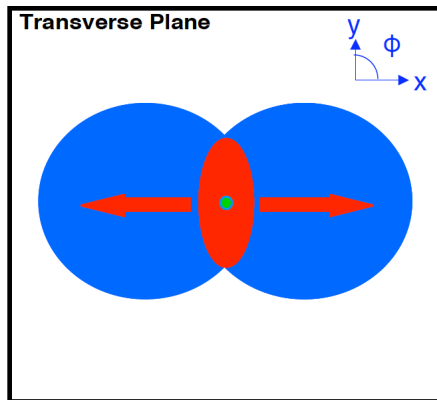
$$E \frac{dN^3}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T dy} (1 + 2v_1 \cos(\phi - \Psi_{RP}) + 2v_2 \cos(2(\phi - \Psi_{RP})) + \dots)$$

$$v_n = \langle \cos[n(\phi - \Psi_{RP})] \rangle$$

directed flow

elliptic flow

reaction plane

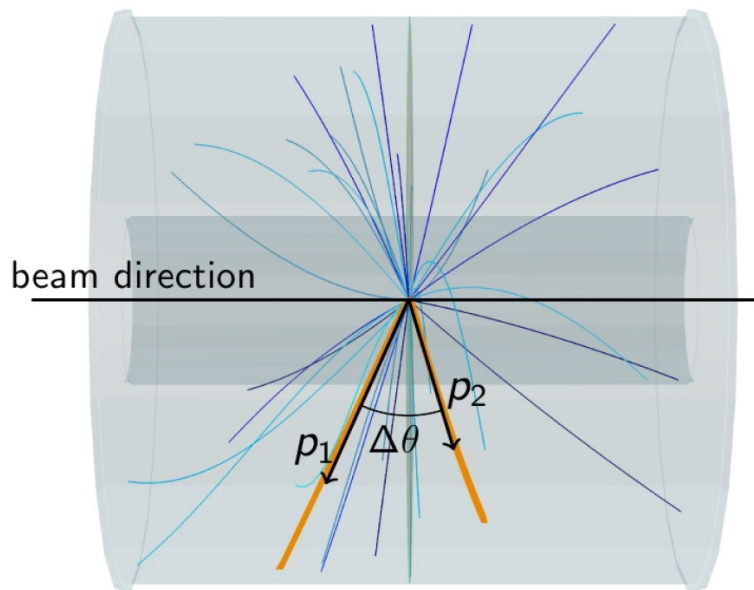


azimuthal angle around the beam axis

Hiroshi Masui (2008)



Momentum Space Variables



Transverse plane

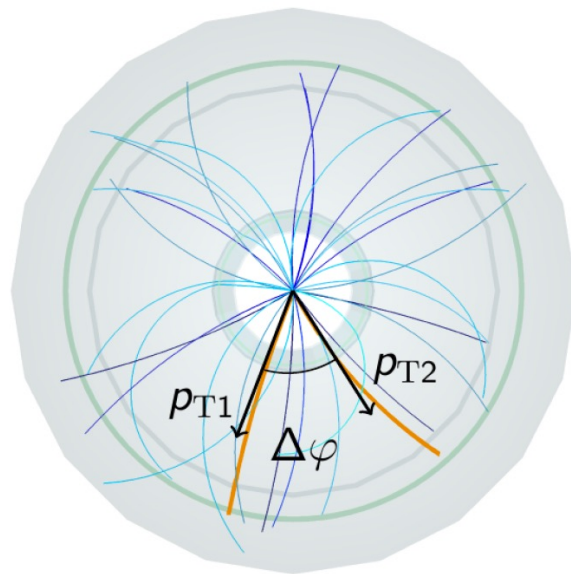


Fig. A. Zaborowska

p — particle momentum

p_T — transverse momentum

φ — azimuthal angle

θ — polar angle

η — pseudorapidity

y — rapidity

$$p_T^2 = p_x^2 + p_y^2$$

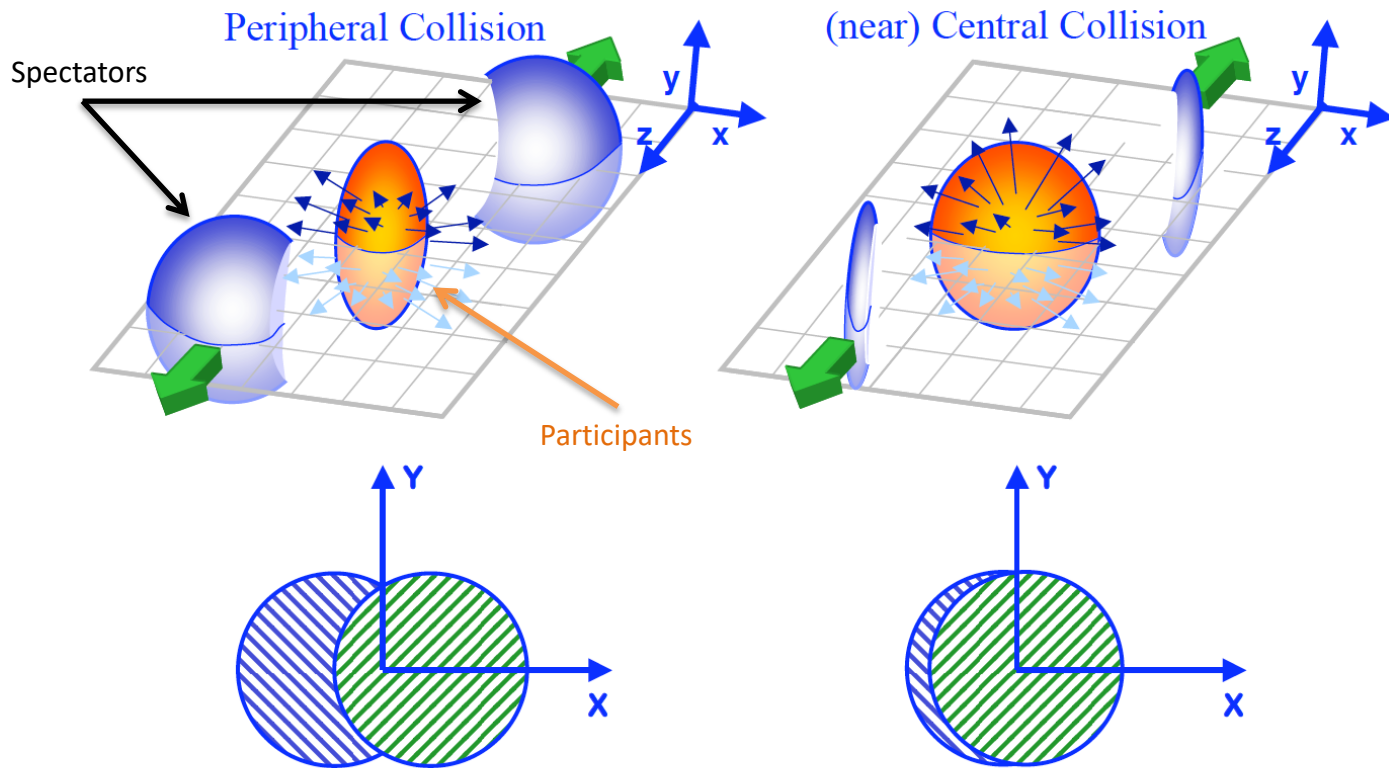
$$\eta = -\ln \left[\tan\left(\frac{\theta}{2}\right) \right] = \frac{1}{2} \ln \frac{|p| + p_z}{|p| - p_z}$$

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$

Lorentz invariant



Centrality of Relativistic Heavy Ion Collisions



Centrality measured by the multiplicity of charged particles

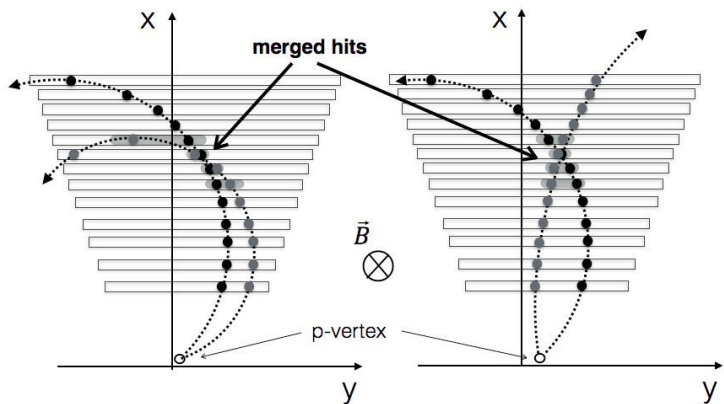


Track Merging Correction

The solution / correction

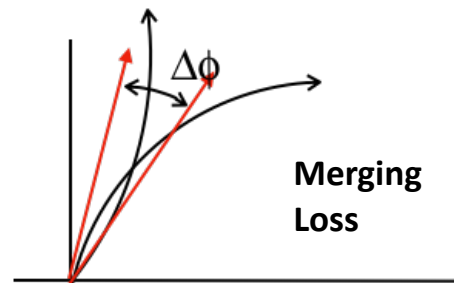
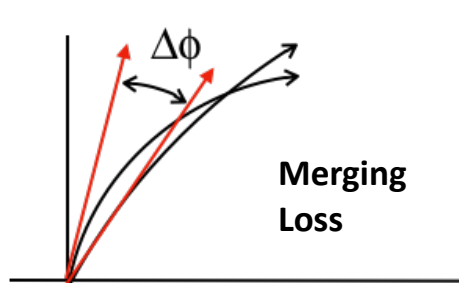
The cause of track merging

tracks with $\Delta y \sim 0$

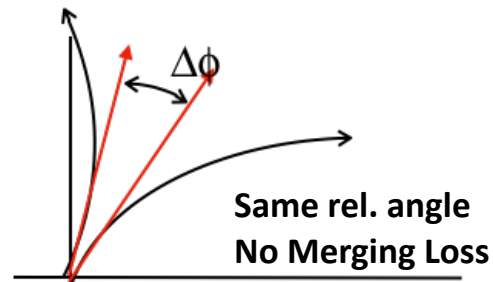
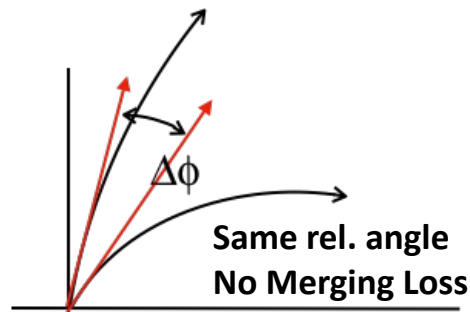


Like-sign: p_T -ordered analysis

Unlike-sign: charge-ordered analysis



At given $\Delta\phi$, count un-merged pairs and use count at $-\Delta\phi$





Correcting The Collision Centrality Bin Width Effects

Gonzalez, Marin, Guevara, JP, Basu, Pruneau, PRC 99, 034907 (2019)

2-Particle Normalized Cumulant (for multiplicity m)

$$R_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m) = \frac{\rho_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m)}{\rho_1(\eta_1, \varphi_1 | m) \rho_1(\eta_2, \varphi_2 | m)} - 1$$

Weighted Average

$$\rho_1^{(Bin,k)}(\eta_1, \varphi_1) = \bar{\rho}_1^{(k)}(\eta_1, \varphi_1) = \frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m) \rho_1(\eta_1, \varphi_1 | m)$$

$$\rho_2^{(Bin,k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \bar{\rho}_2^{(k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m) \rho_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m)$$

Uncorrected

$$R_2^{(Bin,k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \frac{\rho_2^{(Bin,k)}(\eta_1, \varphi_1, \eta_2, \varphi_2)}{\rho_1^{(Bin,k)}(\eta_1, \varphi_1) \rho_1^{(Bin,k)}(\eta_2, \varphi_2)} - 1$$

$$= \frac{\frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m) \rho_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m)}{\left[\frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m) \rho_1(\eta_1, \varphi_1 | m) \right] \left[\frac{1}{Q_k} \sum_{m'=m_{min,k}}^{m_{max,k}} q(m') \rho_1(\eta_2, \varphi_2 | m') \right]} - 1$$

Weighted Average

$$\bar{R}_2^{(k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m) R_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m)$$

$$= \left[\frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m) \frac{\rho_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m)}{\rho_1(\eta_1, \varphi_1 | m) \rho_1(\eta_2, \varphi_2 | m)} \right] - 1$$

$$\rho_1(\eta, \varphi | m) = \langle n \rangle_m P_1(\eta, \varphi | m)$$

$$\rho_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m) = \langle n(n-1) \rangle_m P_2(\eta_1, \varphi_1, \eta_2, \varphi_2 | m)$$

P_1, P_2 — Probability Densities



$$R_2^{(Bin,k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \alpha \frac{\bar{P}_2(\eta_1, \varphi_1, \eta_2, \varphi_2)}{\bar{P}_1(\eta_1, \varphi_1) \bar{P}_1(\eta_2, \varphi_2)} - 1$$

$$\alpha = \frac{\frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m) \langle n(n-1) \rangle_m}{\left(\frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m) \langle n \rangle_m \right)^2}$$



$$\bar{R}_2^{(k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \beta \frac{\bar{P}_2(\eta_1, \varphi_1, \eta_2, \varphi_2)}{\bar{P}_1(\eta_1, \varphi_1) \bar{P}_1(\eta_2, \varphi_2)} - 1$$

$$\beta = \frac{1}{Q_k} \sum_{m=m_{min,k}}^{m_{max,k}} q(m) \frac{\langle n(n-1) \rangle_m}{\langle n \rangle_m^2}$$

Correction



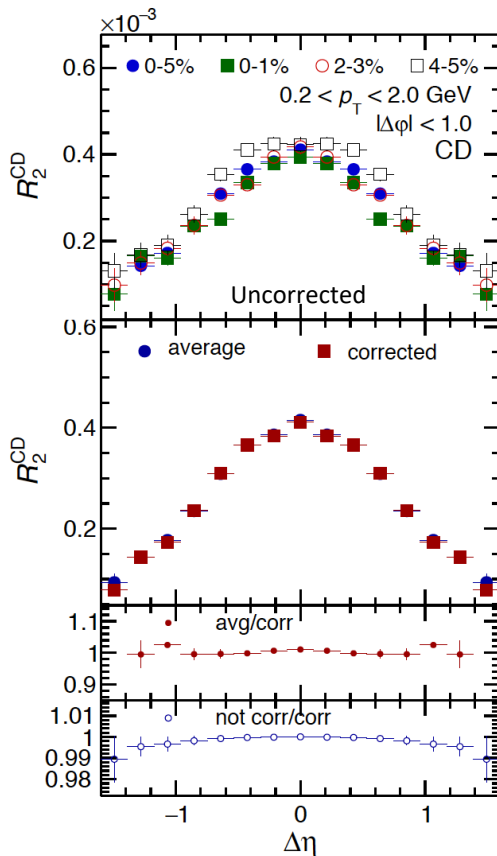
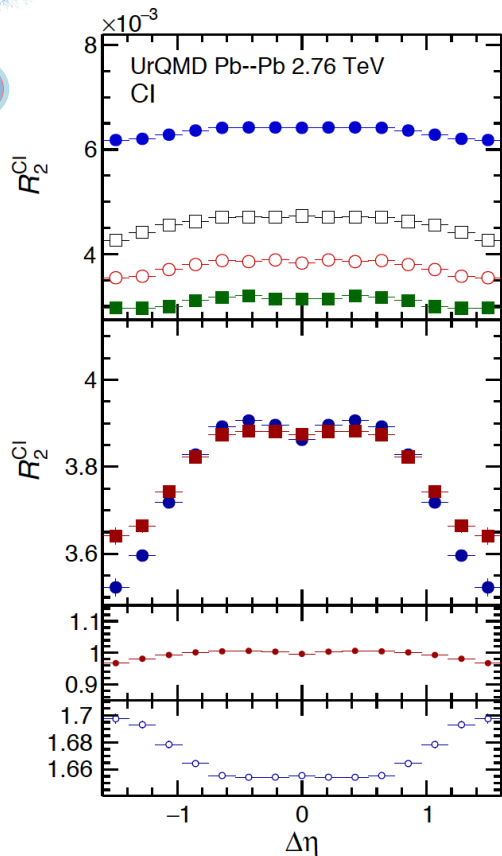
$$\bar{R}_2^{(k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) = \beta \alpha^{-1} (R_2^{(Bin,k)}(\eta_1, \varphi_1, \eta_2, \varphi_2) + 1) - 1$$



Correcting The Collision Centrality Bin Width Effects

UrQMD (100k events) — Unidentified Hadrons

Gonzalez, Marin, Guevara, JP, Basu, Pruneau, PRC 99, 034907 (2019)



$$R_2^{Cl} = \frac{1}{4} [R_2^{+-} + R_2^{++} + R_2^{-+} + R_2^{--}]$$

$$R_2^{CD} = \frac{1}{4} [R_2^{+-} - R_2^{++} + R_2^{-+} - R_2^{--}]$$

- Results corrected agree with those obtained with the weighted mean within 1% for both R_2^{Cl} and R_2^{CD} .
- The correction enables reasonably accurate corrections of the R_2 correlators in the context of HIJING and UrQMD models.
- Given these models provide relatively realistic representations of single and pair particle spectra, the correction method should provide reasonably reliable bin-width corrections of R_2 correlation functions measured at any heavy ion collider.