

*Cyclotron Institute Texas A&M University*  
*Nuclear Data Evaluation Center*

**Data-Based Research Project:**  
**Could revisiting the Principles of**  
**a Level Scheme bring new Insight**  
**into High-Spin Physics?**

*N. Nica*

# ***Content***

***I. Experimental Evidence***

***II. Level Scheme Re-Concept***

***III. Insight into High-Spin Physics***

# *I. Experimental Evidence*

# Case study: $^{171}\text{Yb}$ nucleus high spin rotational bands

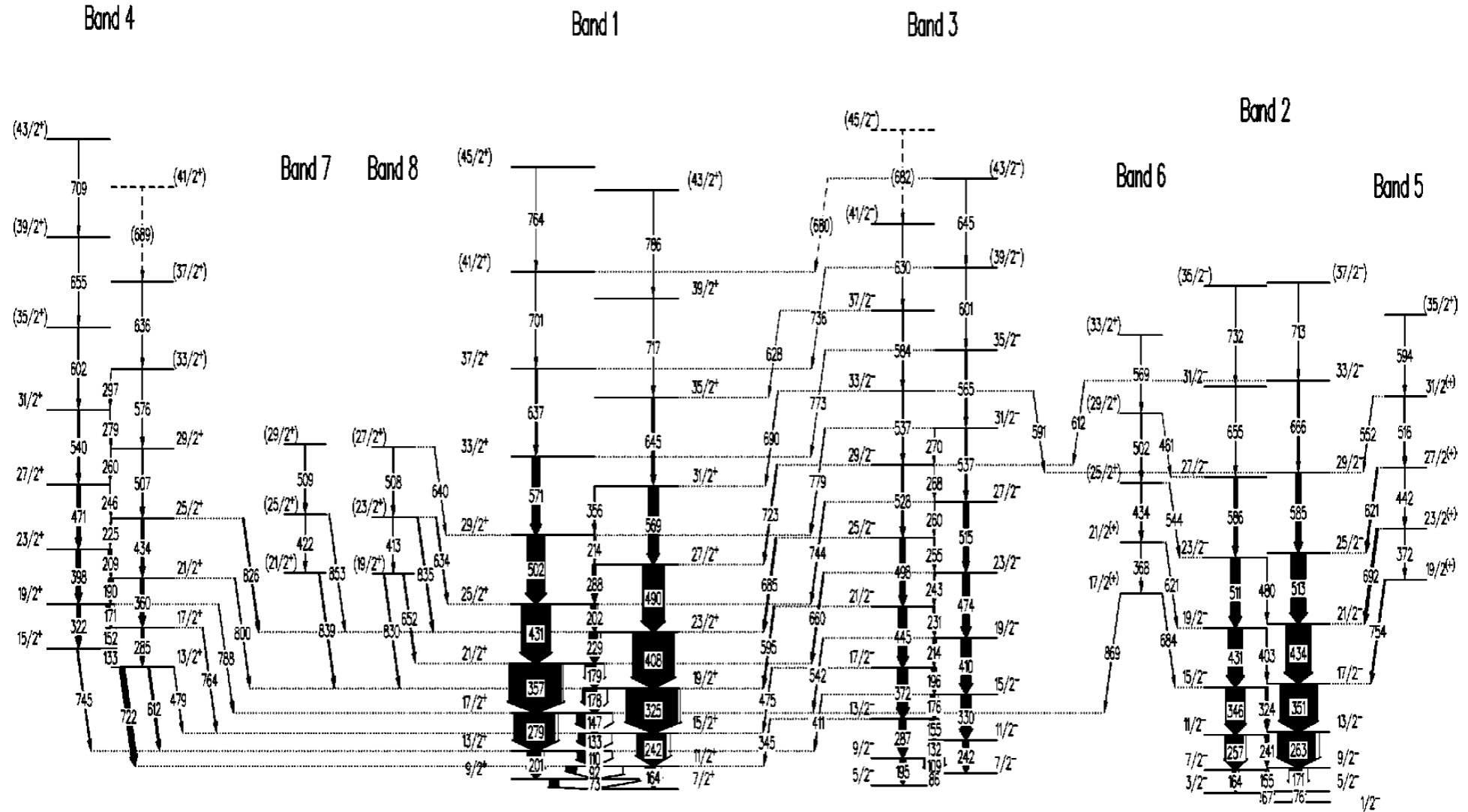


FIG. 5. Level scheme for  $^{171}\text{Yb}$ .

*How the bands can be described?  $(\Delta E_\gamma^x, \Delta E_\gamma^y)$  Differential Coincidence Matrix*

*Bohr-Mottelson Collective Rotor*

$$E(I) = \frac{\hbar^2}{2\mathfrak{I}} I(I+1), \quad c = \frac{\hbar^2}{2\mathfrak{I}}$$

$$E_\gamma = E(I) - E(I-2) = \frac{\hbar^2}{2\mathfrak{I}} (4I-2) = 2c(2I-1)$$

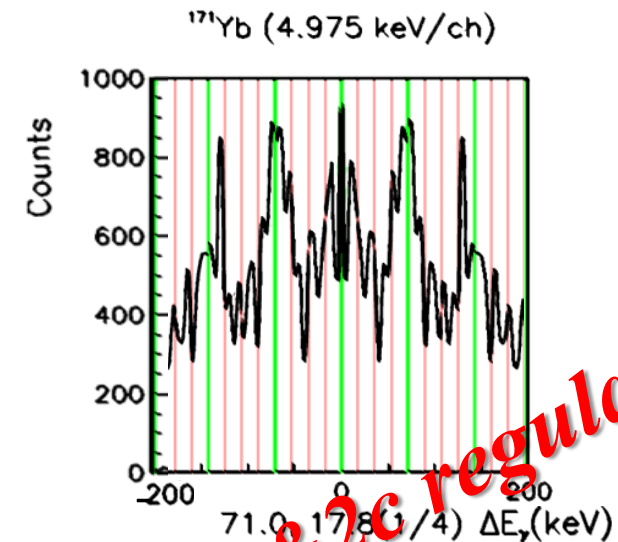
$$\Delta E_\gamma = E_\gamma(I) - E_\gamma(I-2) = 8 \frac{\hbar^2}{2\mathfrak{I}} = 8c$$

*New Parametrization (average behavior)*

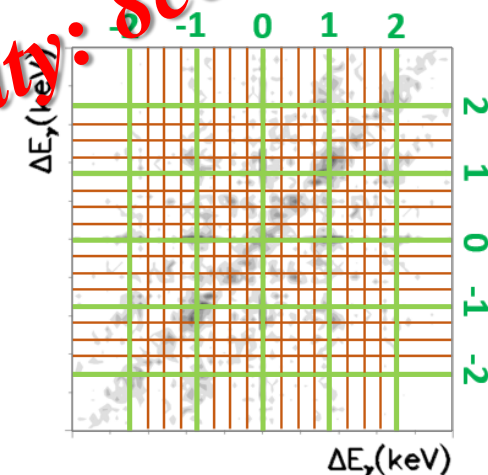
$$E_\gamma = 2c(2I + k - 1), \quad k \text{ integer}$$

- $2c$  Moment of Inertia, Real
- $(2I+k-1)$  Angular Momentum, Integer

*Bitmap Distribution*



*Repeatability:  $8c$  &  $2c$  regular grid!*



# How the bands can be described?

## Bohr-Mottelson Collective Rotor

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<sup>171</sup>Yb Rotational Bands

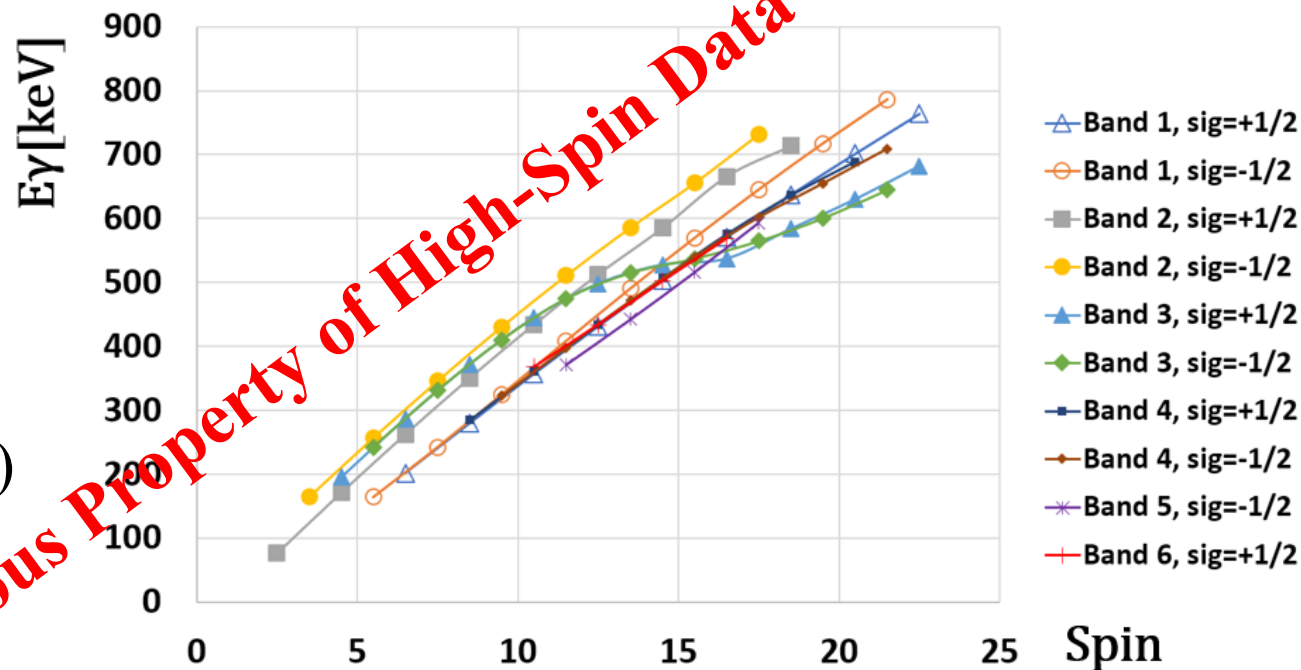


Fig. ( $E_\gamma$ 's versus spins )

-Quasi-linear beam almost parallel and equidistant

-Average behavior:  $2c(2I+k-1)$ ,  $k$  integer

-Determine from fit over all bands'  $\gamma$  rays:

$$2c, k\text{'s, from } \sum (E_\gamma(I)/2c - (2I+k-1))^2 = \min$$

- Same slope ( $2c$ ) for all  $k$ -bands  $\Rightarrow$  same  $\mathcal{J}_{eff}^{(2)} = \frac{\hbar^2}{2c}$

# Case study: $^{171}\text{Yb}$ nucleus high spin rotational bands

$^{171}\text{Yb}$  Generalized Ideal Rotational Bands

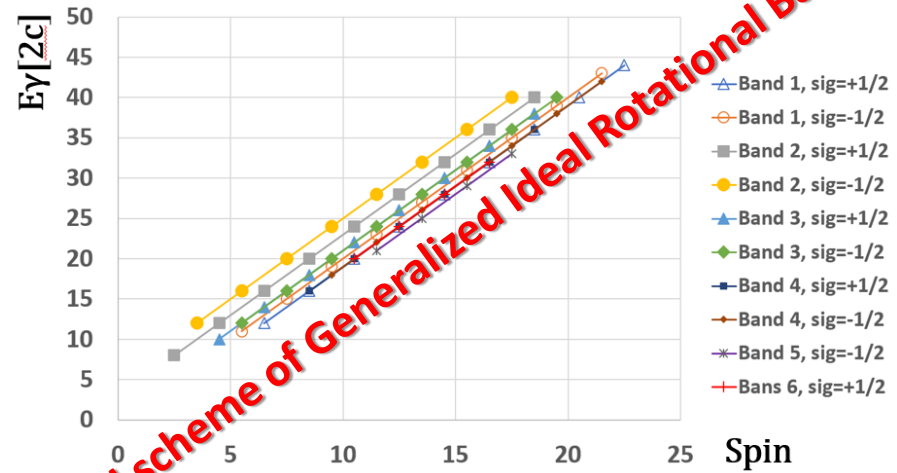


Fig. (Generalized Ideal Rotation Bands)

$^{171}\text{Yb}$  Rotational Bands

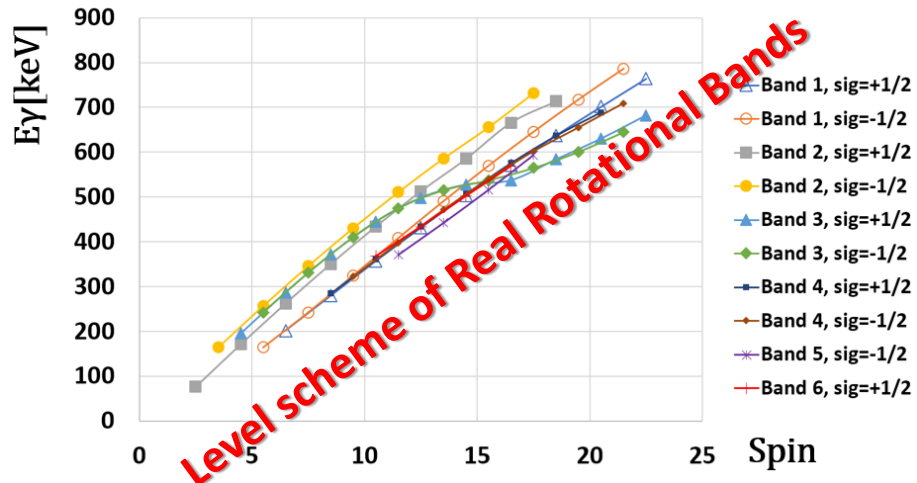


Fig. ( $E_\gamma$ 's versus spins )

$^{171}\text{Yb}$  band fits using  $E_\gamma = 2c(2I+k-1)$  parametrization,  $\Sigma(E_\gamma(I)/2c - (2I+k-1))^2 = \min$

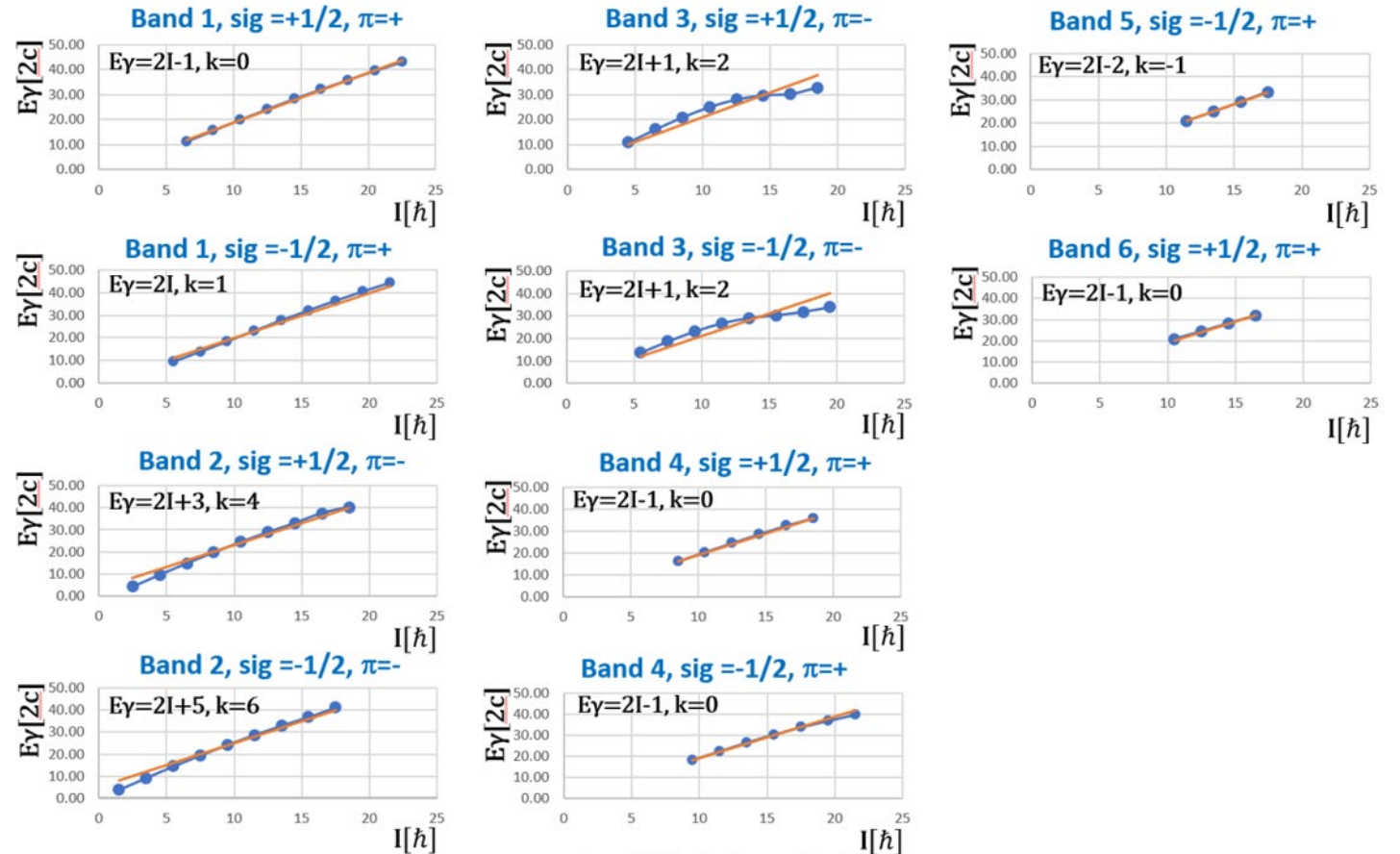


Fig. ( $^{171}\text{Yb}$  bands fit)

$$2c = 17.75 \text{ keV}$$

$$J_{eff}^{(2)} = 56.34 \text{ h}^2/\text{MeV}$$

## ***II. Level Scheme Re-Concept***



# What we got for the average description of $^{171}\text{Yb}$ bands?

## $k$ -Generalized Ideal Rotor bands:

For  $k=0$ , Bohr-Mottelson Ideal Rotor bands: described by the  $2c(I+1)$  rule for even and odd spins

For  $k \neq 0$ ,  $k$ -Generalized Ideal Rotor bands: have the same  $J_{\text{eff}}^{(2)}$  (same  $2c$ ) but are no longer described by the  $2c(I+1)$  rule.

**Q: How to place the  $k$ -generalized ideal rotor bands in the level scheme?**

**A: By adding  $2I+1$  “stairs” of  $2c$  levels to the  $k=0$  band!**

One gets a “parabolic 2D building” on which:

- $k=0$  bands are vertical paths
- $k \neq 0$  bands are tilted paths
- In general, the energy levels can be indexed by three integer numbers,  $(I, m, n)$ , where:
  - $I$  is the nuclear spin,
  - $m$  is the position of the “stair” level relative to the spin “floor”,
  - $n$  the energy of the level, which is a natural number in units of  $2c$ .
- Triple coordinates suggest 3D level scheme!

$^{171}\text{Yb}$  Generalized Ideal Rotational Bands

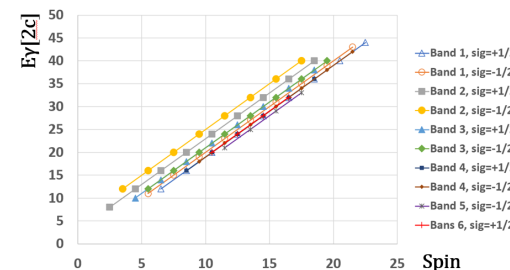


Fig. (Generalized Ideal Rotation Bands)

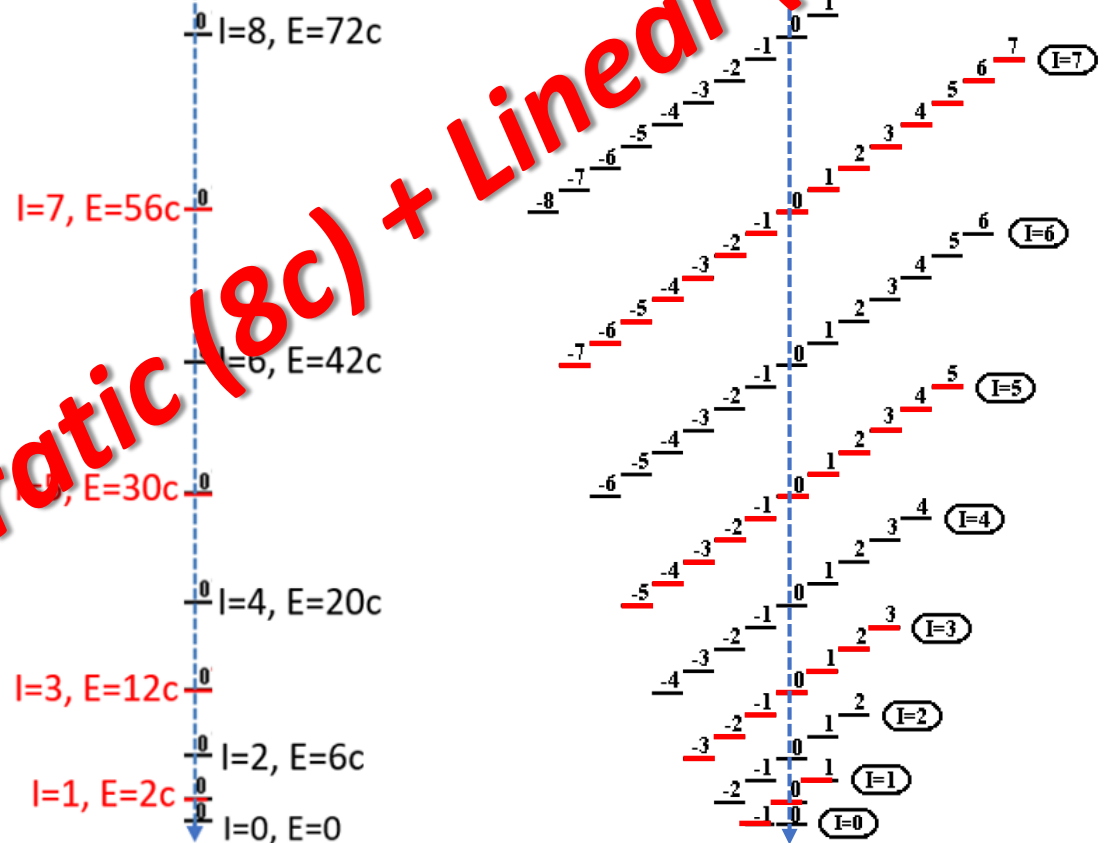


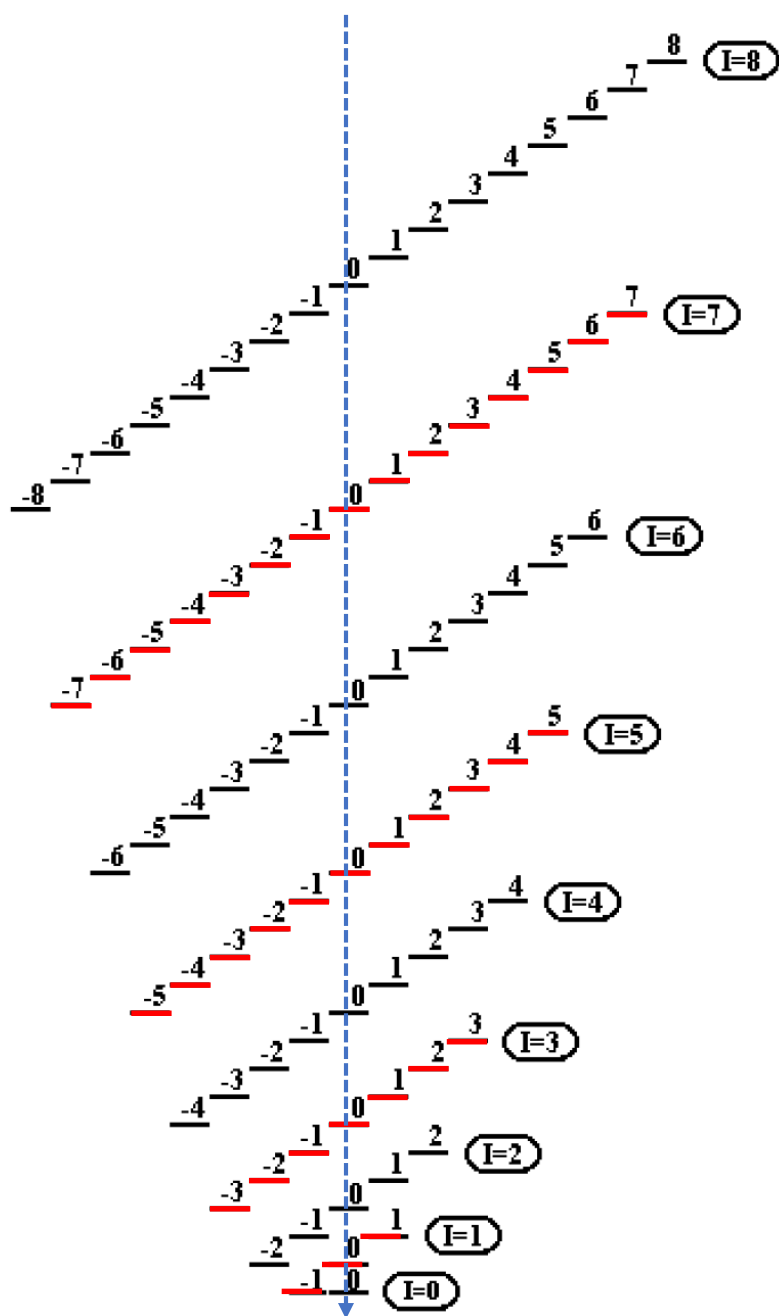
Fig. (Ideal rotor)

Fig. (Opened generalized ideal rotor)

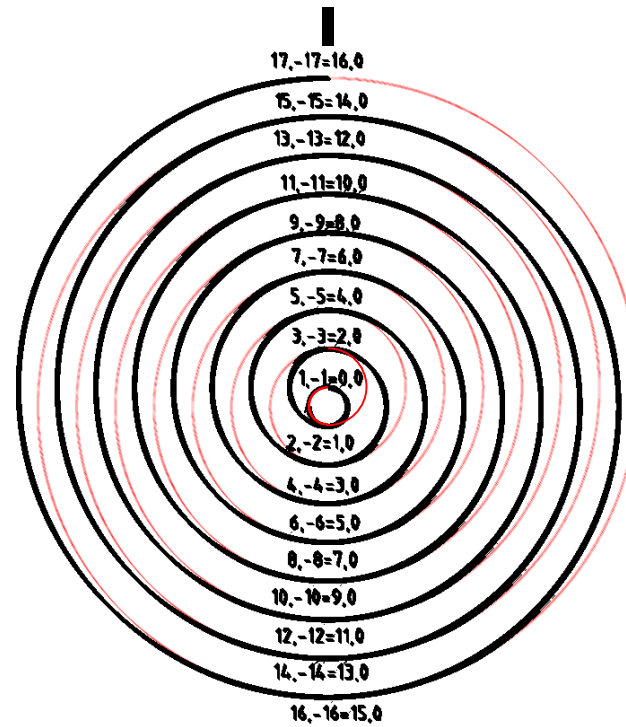
# Parabolic Level Scheme for k bands

$$E_{\gamma}^k(I) = 2c (2I + k - 1), \quad k = \pm 1, \pm 2, \dots, I$$

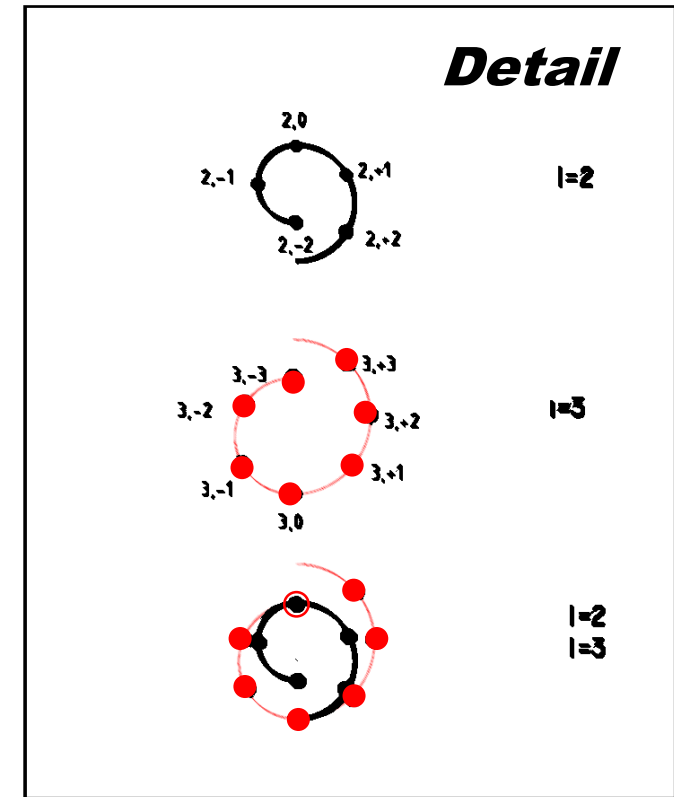
## DOUBLE HELIX



2D view



3D view (from above)



# Double-Helix for even-A and odd-A Nuclei

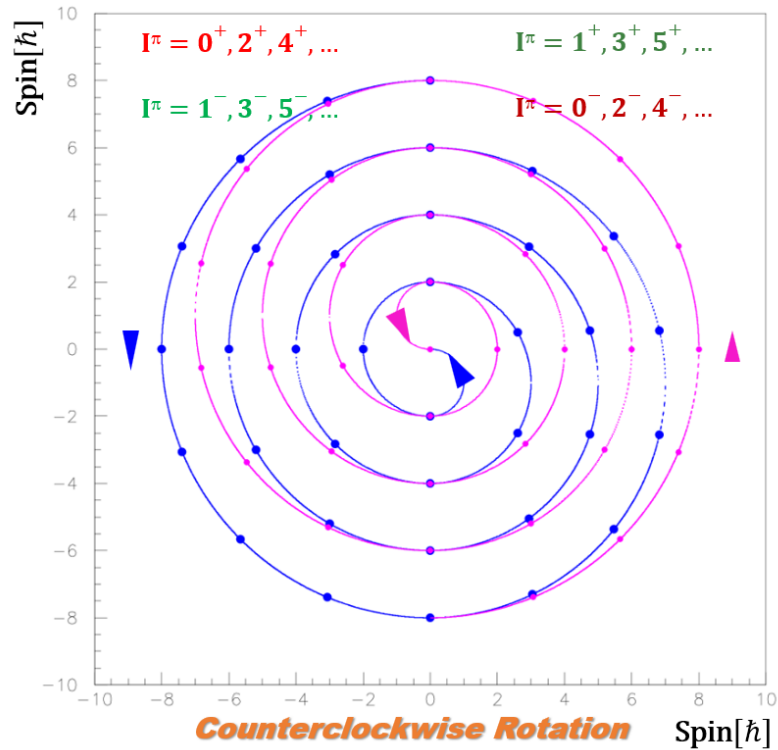


Fig. (Set of 2 helixes for integer spins)

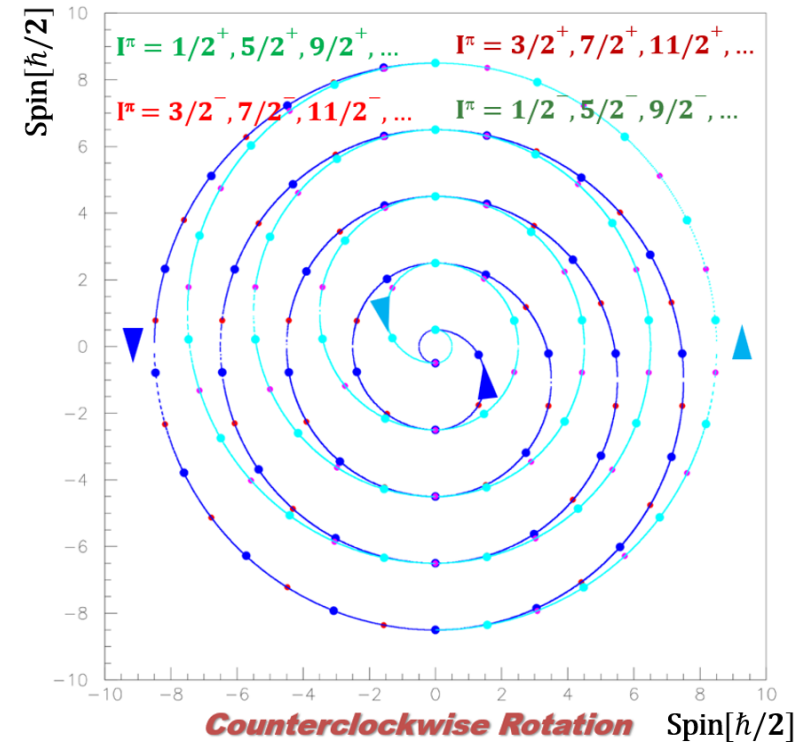


Fig. (Set of 2 helixes for integer half-spins)

# Double-Helix Level Scheme of $^{171}\text{Yb}$ nucleus

## Decomposition of experimental $E_\gamma$ 's of all bands

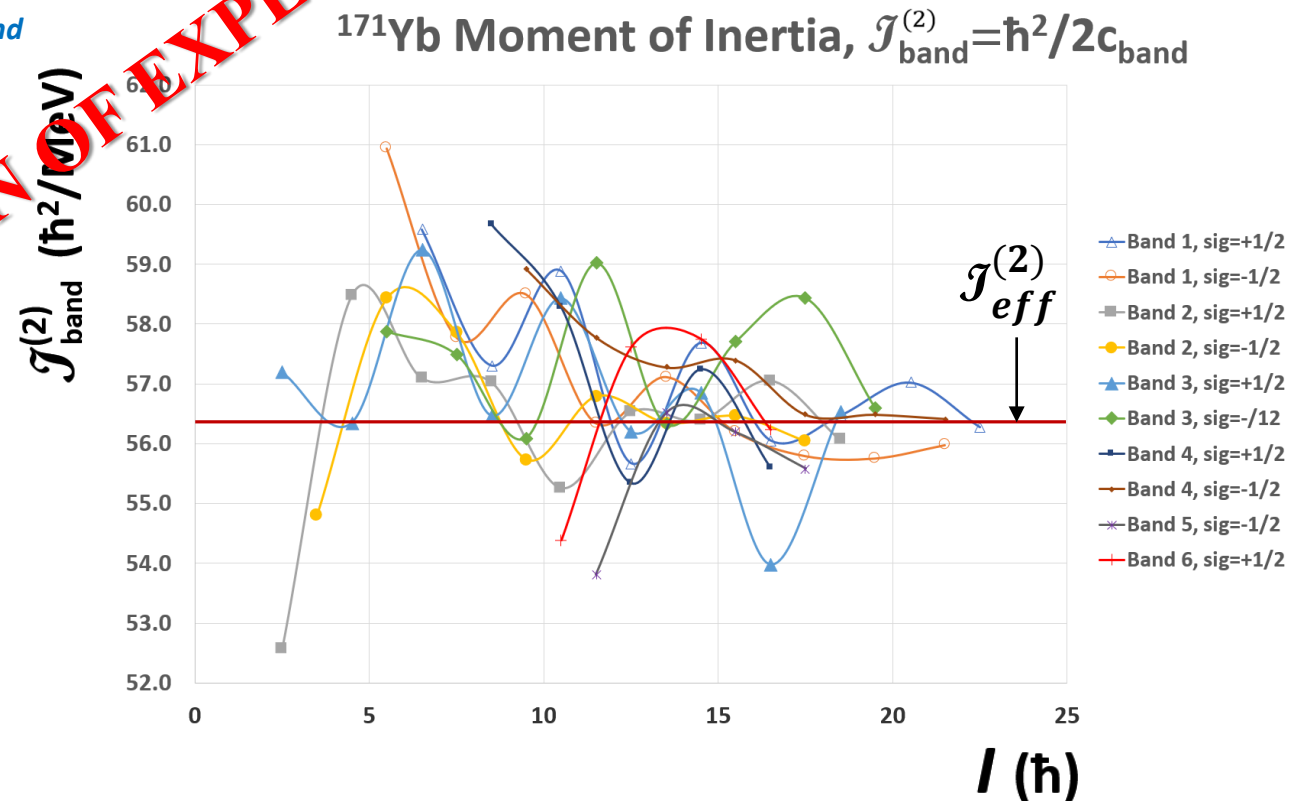
$$E_\gamma = 2c(2I+k-1 + k' + fn)$$

$$E_\gamma = 2c_{\text{band}} \times (2I+k+k'-1), \text{ with } 2c_{\text{band}} = 2c[1+fn/(2I+k+k'-1)],$$

with  $2c_{\text{band}}$  real and  $(2I+k+k'-1)$  integer

- $(2I+k+k'-1)$  generalized angular momentum
- One gets **Bands Moment of Inertia**,  $\mathcal{J}_{\text{band}}^{(2)} = \hbar^2/2c_{\text{band}}$

COMPLETE DESCRIPTION OF EXPERIMENTAL BANDS



# ***<sup>171</sup>Yb nucleus Double Helix Level Scheme***

## **Part I: Level scheme of Generalized Ideal Rotational Bands**

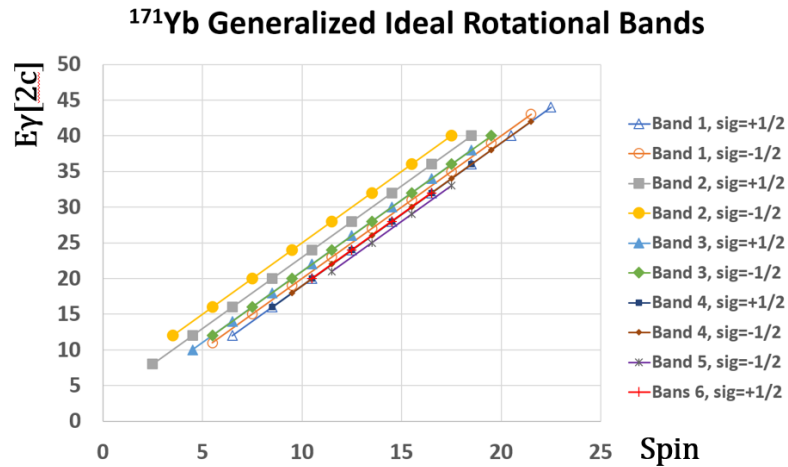


Fig. (Generalized Ideal Rotation Bands)

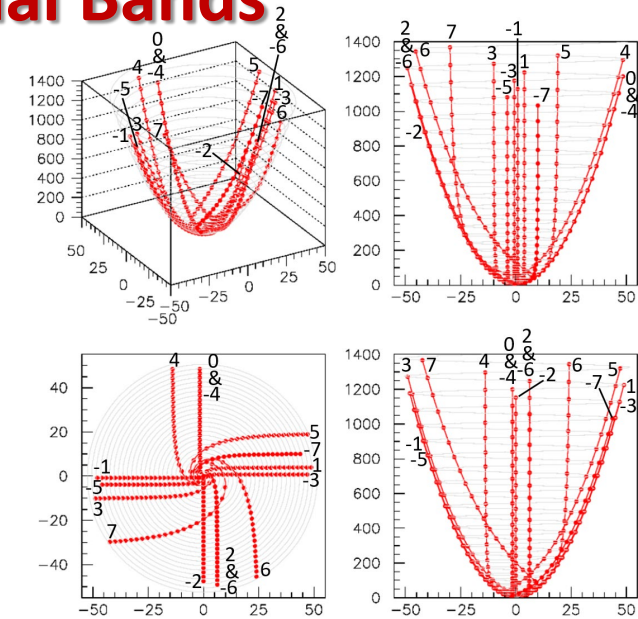
### Average Ideal Rotational Description

#### Parametrization:

$$E_{\gamma} = 2c(2I + k - 1)$$

$2c$  *Real*,  $(2I + k - 1)$  *Integer*

$2c, k$  from least-squares fit



## **Part II: Level scheme of Real Rotational Bands**

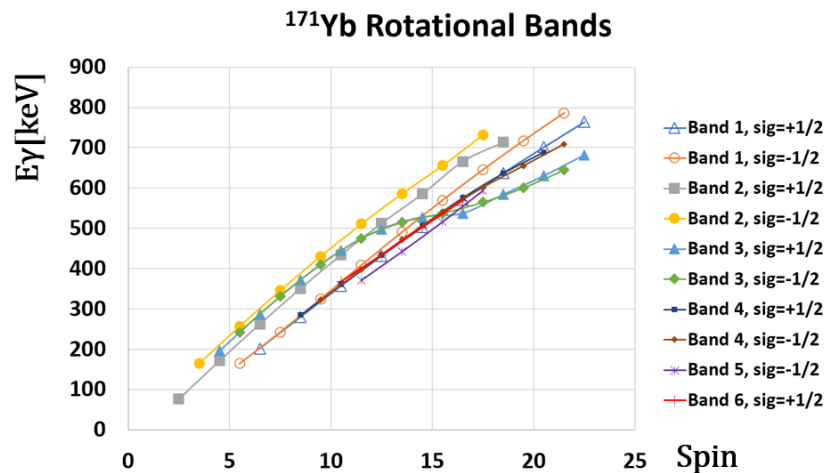


Fig. (Eγ's versus spins )

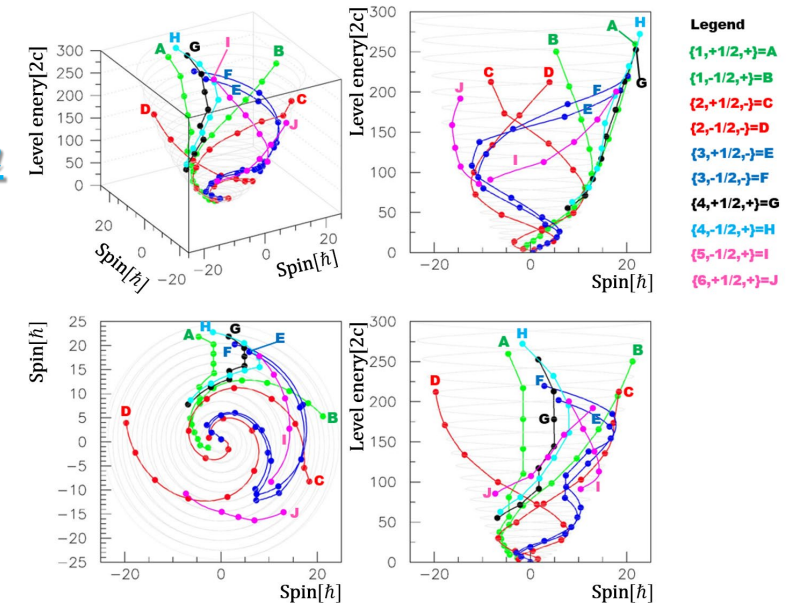
### Real Bands Rotational Description

#### Decomposition:

$$E_{\gamma} = 2c_{band}(2I + k + k' - 1)$$

$2c_{band}$  *Real*,  $(2I + k + k' - 1)$  *Integer*

$$2c_{band} = 2c[1 + fn/(2I + k + k' - 1)]$$



### *III. Insight into High-Spin Physics*



# Elementary Helix Loop with $\gamma$ transition

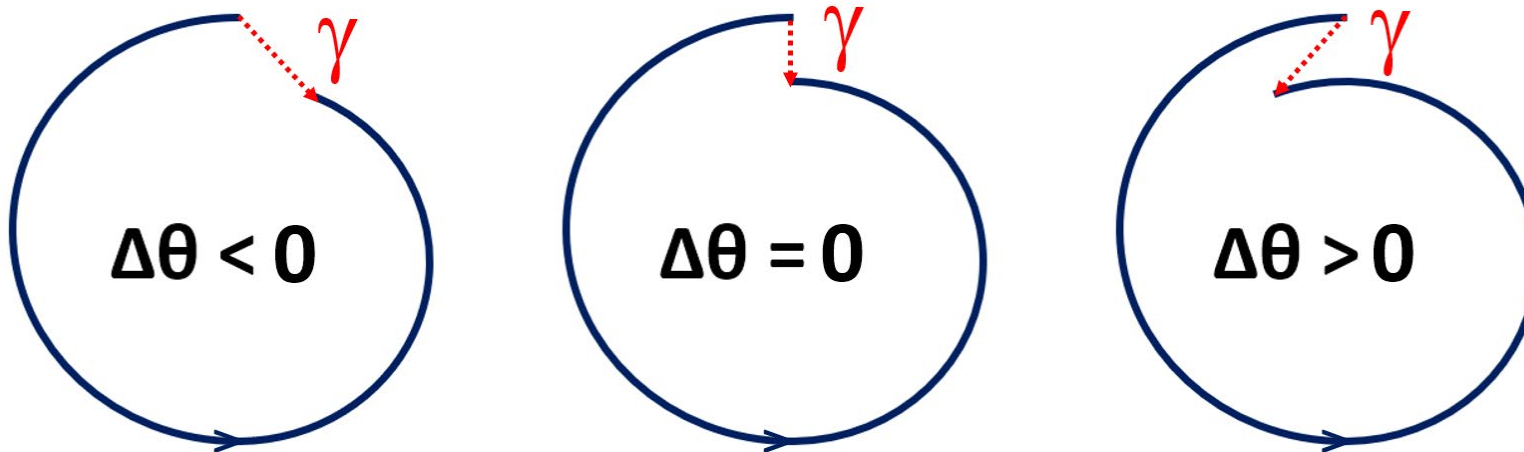
$$E\gamma = 2c_{band}(2I + k + k' - 1):$$

$\Delta\theta = 0, 2\pi$  Elementary Helix Loop Rotation *due to  $2I$  Macroscopic Collective Motion*

- $\gamma$ -decay paths: *along vertical diameter*

$\Delta\theta \neq 0$  Band's Apparent Rotation on the Helix *due to  $k + k'$  Microscopic S.P. Motion*

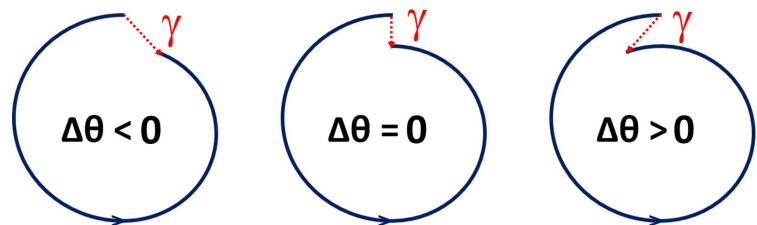
- $\gamma$ -decay paths:  $\Delta\theta < 0$  clockwise precession
- $\gamma$ -decay paths:  $\Delta\theta > 0$  counterclockwise precession



*Phase angle of levels on helicoid:*  $\theta(I, m) = \sum_{I, m} (I + \frac{m}{I})\pi$

*Phase shift between two consecutive band levels:*  $\Delta\theta(I) = \theta(I) - \theta(I-2) - 2\pi$

Fig. ( $\Delta\theta$  apparent band rotation on the helicoid)



## $^{171}\text{Yb}$ Phase Shifts

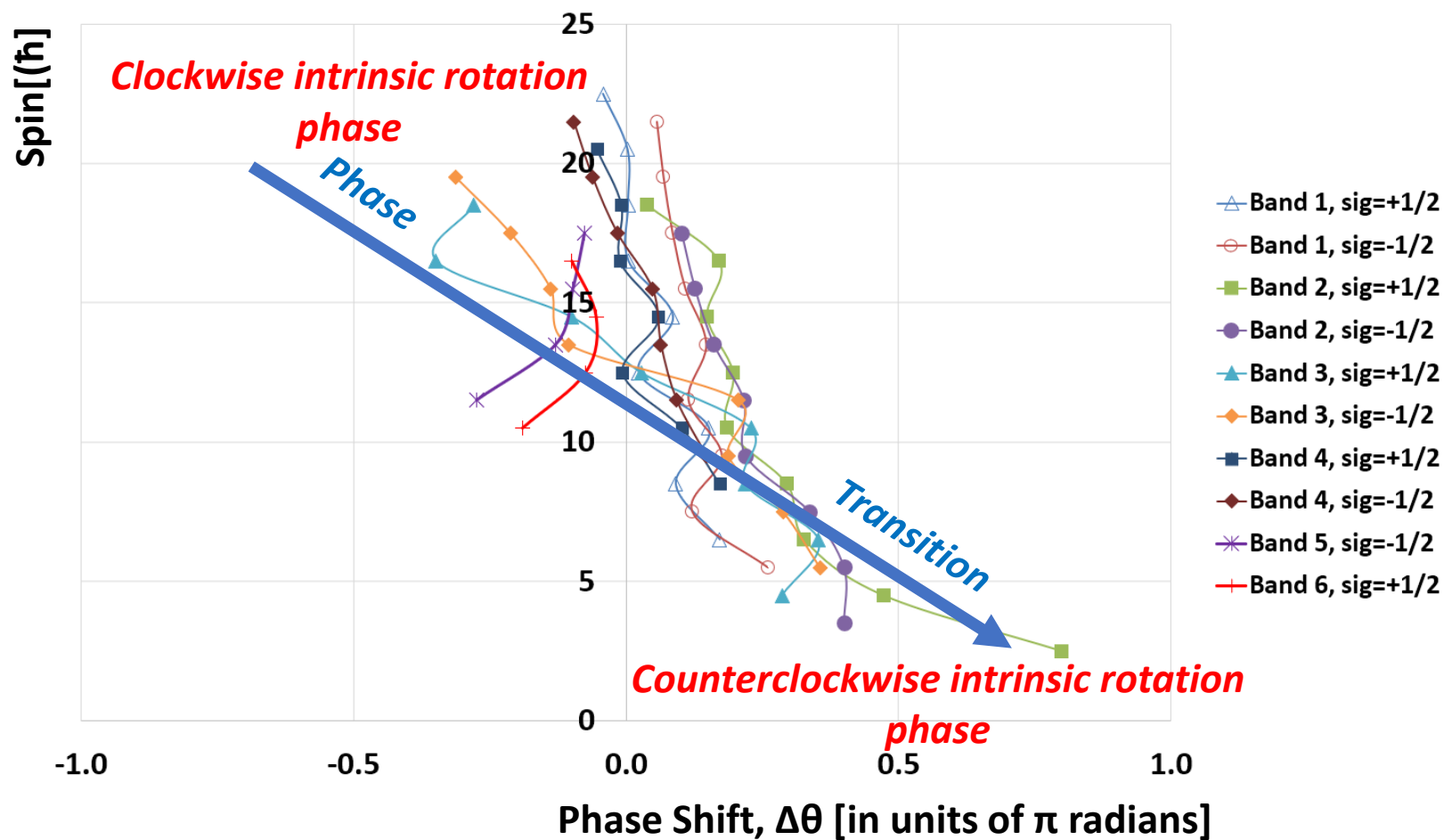


Fig. ( $^{171}\text{Yb}$  Phase Shifts)



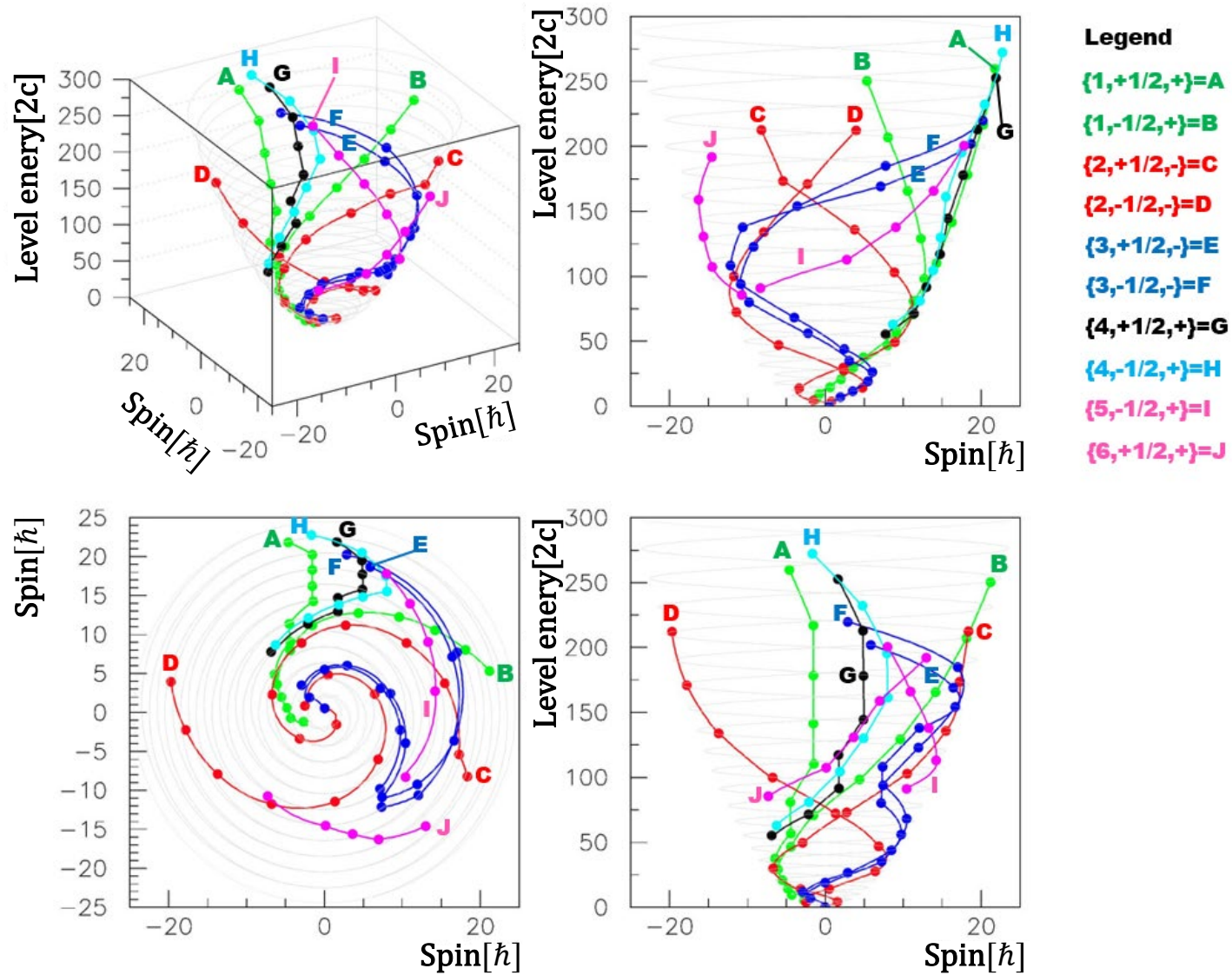
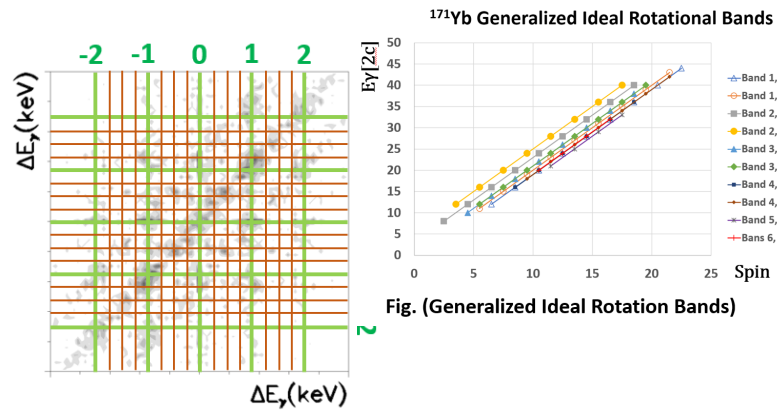
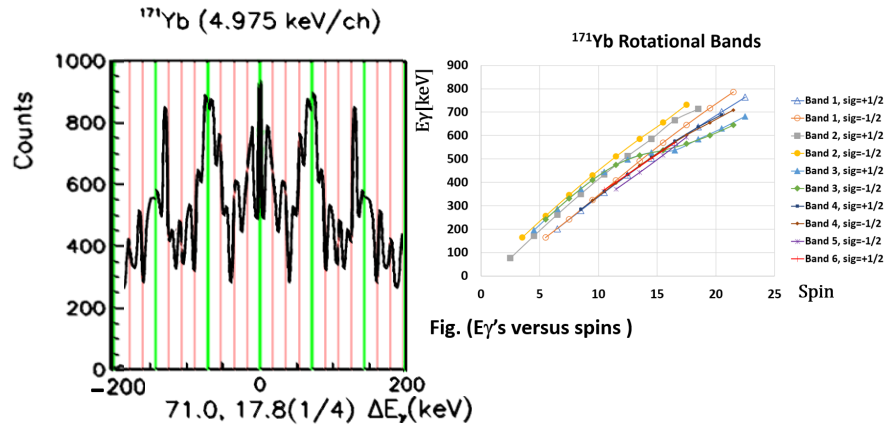


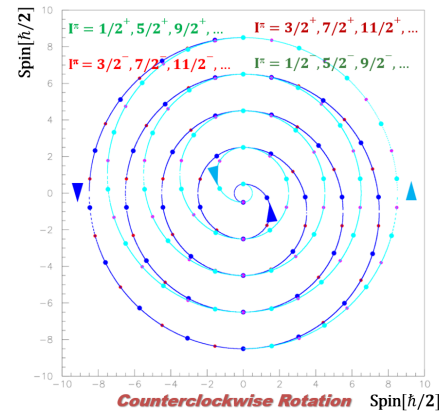
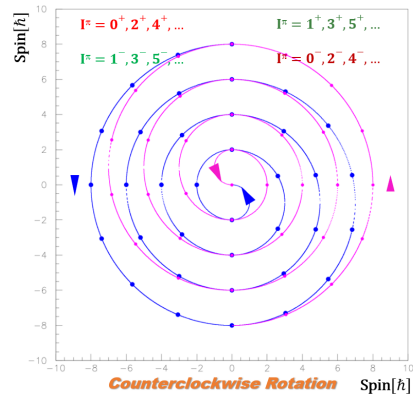
Fig. (Double helix of  $^{171}\text{Yb}$  nucleus)

# Double Helix Level Scheme Summary

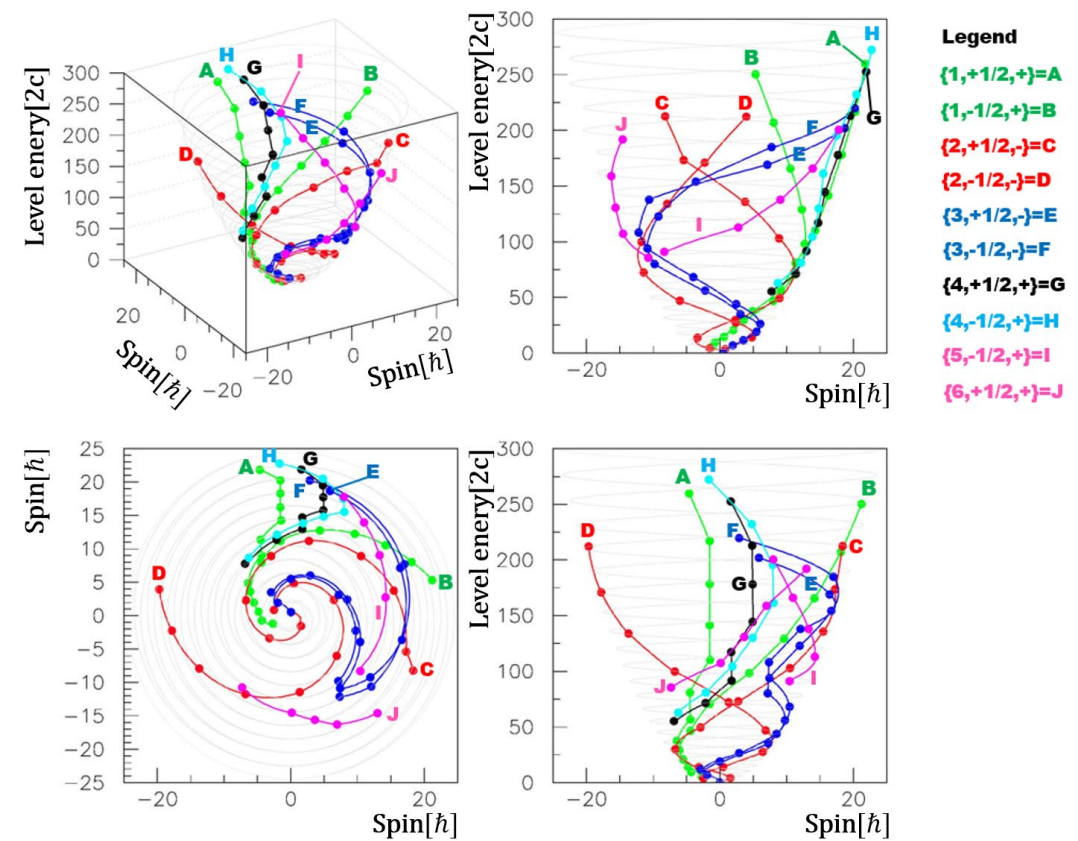
## I Repeatability



## II Double Helix

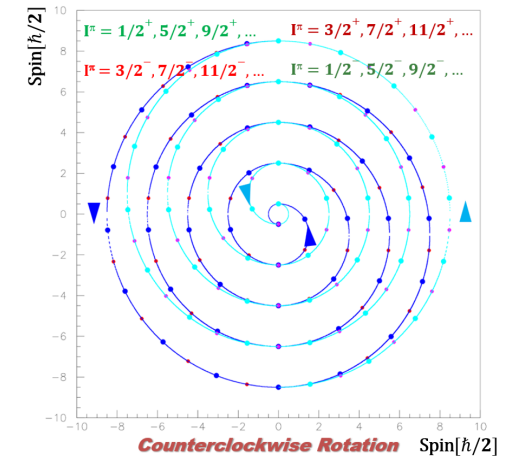
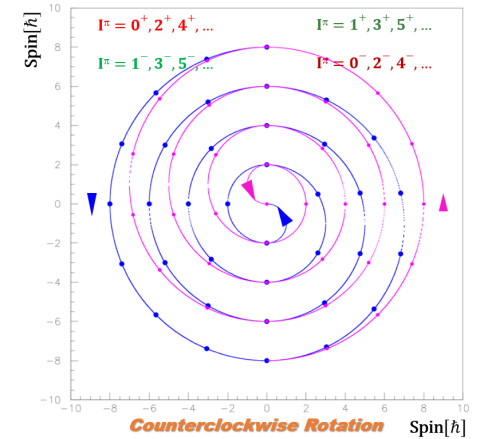


## III Double Helix Level Scheme



# Double Helix Level Scheme Insight: *Investment in Nuclear Structure Building*

- *Double Helix is the geometrical place of the discrete set of spin states available for the rotational motion of the nucleus, which defines a Semiclassical Meta-Trajectory*
- *On average, one can assume that Nuclear Matter itself follows the Semiclassical Meta-Trajectory on Double Helix, with the actual levels selected by the rotational bands' paths*
- *Semi-classically, through Repeatability Nuclear Matter's Double Helix Motion can be seen as a Vortex Motion*
- *This can indicate vorticity in the liquid drop and relax the irrotational flow hypothesis on Bohr-Mottelson model*





# *DOUBLE HELICOID LEVELS SCHEME (DHLS)*

