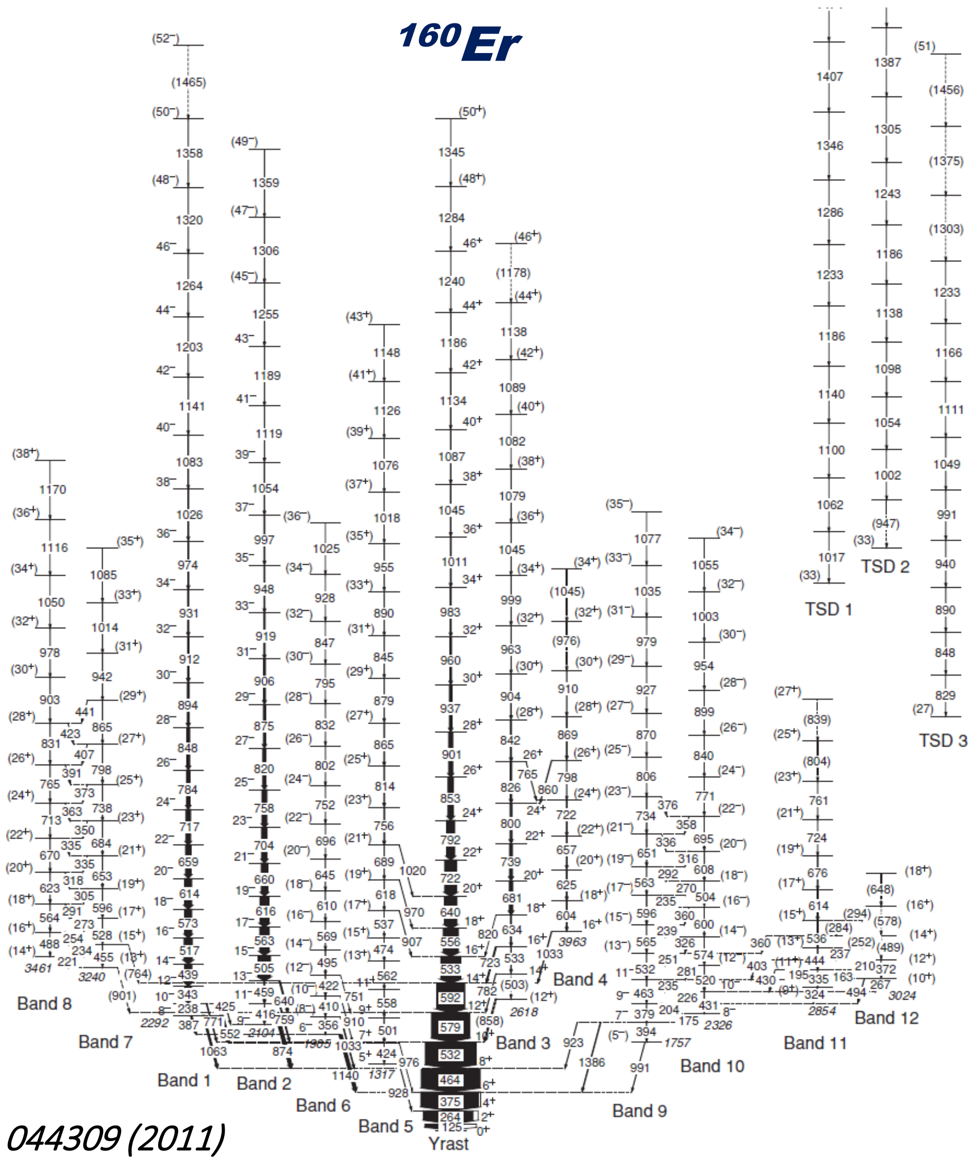
NSDD Texas A&M University Evaluation Center

Data-Based Research Project:
How to build a Level Scheme?

N. Nica

Data-based Physics Research
Level Scheme Re-Concept

- ***Question:*** ***What a Level Scheme is?***
  - ***An energy scale used to conventionally represent the decay paths of a nucleus from highest excited states to the ground state***
  - ***Levels: horizontal lines showing excitation energy, spin and parity***
  - ***Transitions: vertical arrows between levels indicating energy and intensity***
  - ***Bidimensional figure with true correlations only on vertical axis, while the horizontal direction is conventional***
  - ***Level (decay) Schemes: of explicit technical interest only for Nuclear Data Evaluation Community.***
- ***Shall we review the LS concept?***
- ***Can LS still be of fundamental interest, e.g. by correlating bands on both directions?***



J. Ollier *et al., Phys.Rev. C 83, 044309 (2011)*

Case study: ^{171}Yb nucleus high spin rotational bands

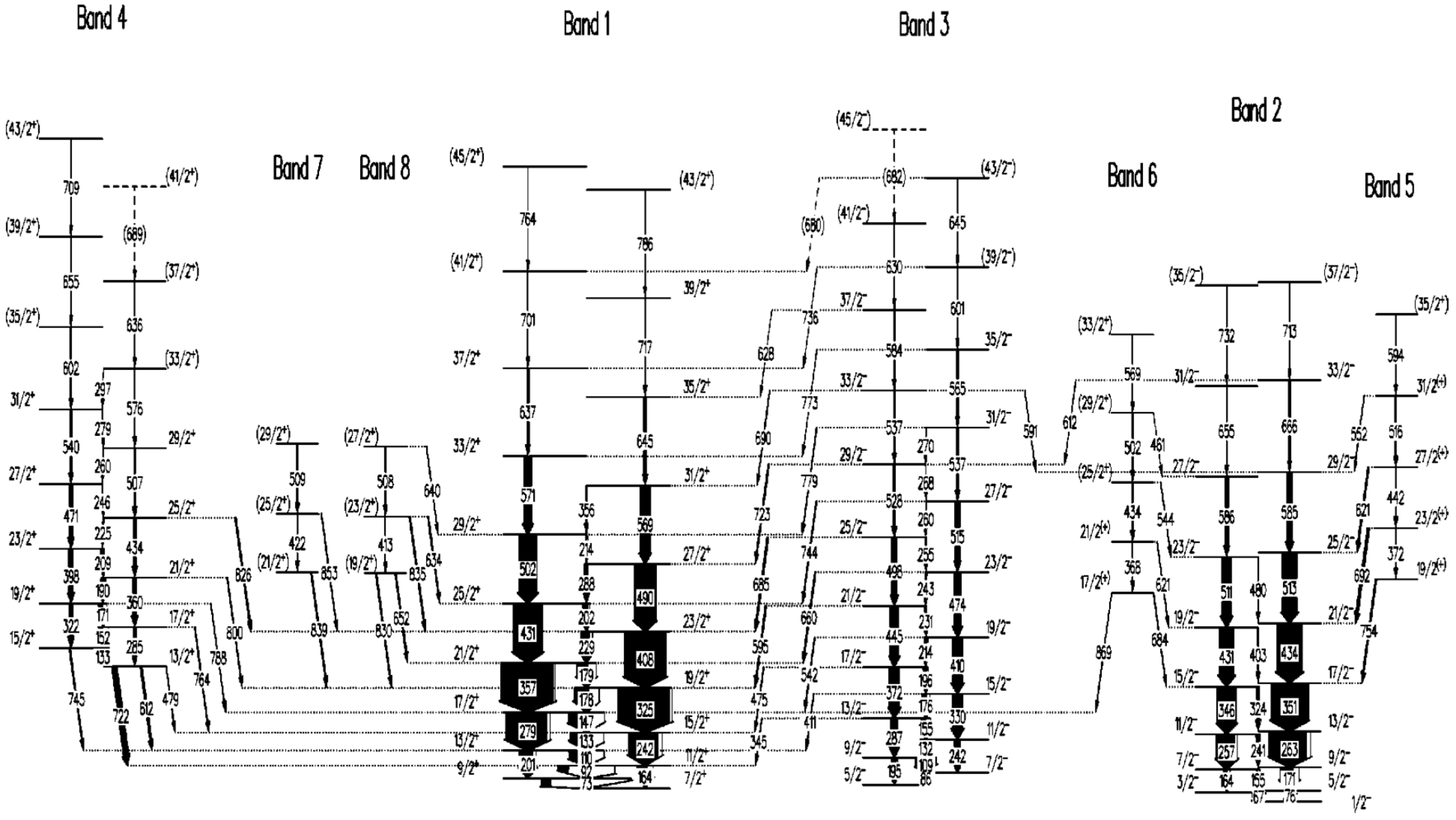


FIG. 5. Level scheme for ^{171}Yb.

D.E.Archer *et al*, Phys.Rev. C57, 2924 (1998)

How the bands are described?

Bohr-Mottelson Collective Rotor

$$E(I) = \frac{\hbar^2}{2\Im} I(I+1), \quad c = \frac{\hbar^2}{2\Im}$$

$$E_\gamma = E(I) - E(I-2) = \frac{\hbar^2}{2\Im}(4I-2) = 2c(2I-1)$$

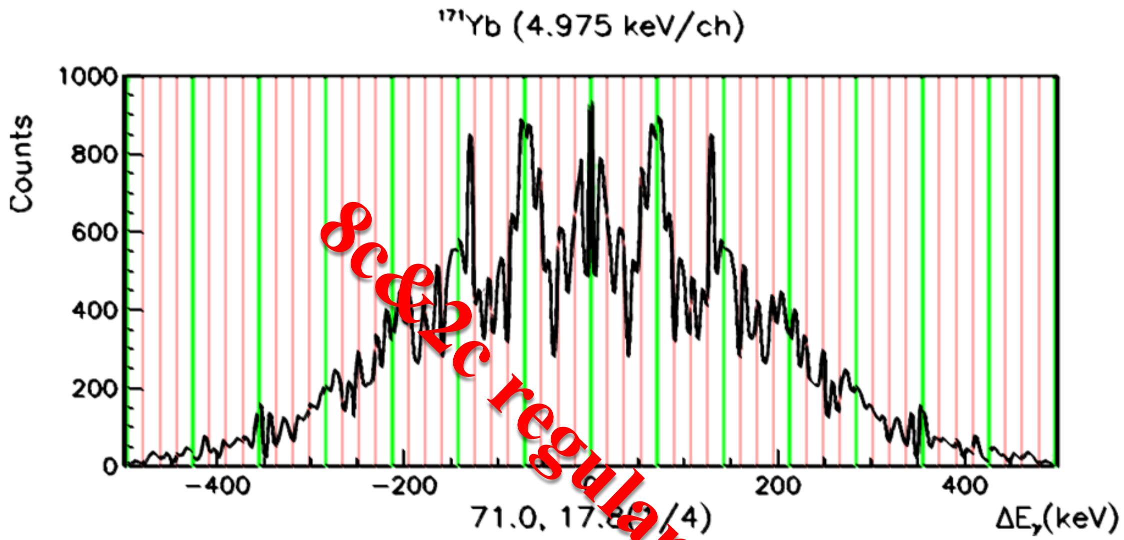
$$\Delta E_\gamma = E_\gamma(I) - E_\gamma(I-2) = 8\frac{\hbar^2}{2\Im} = 8c$$

New Parametrization (average behavior)

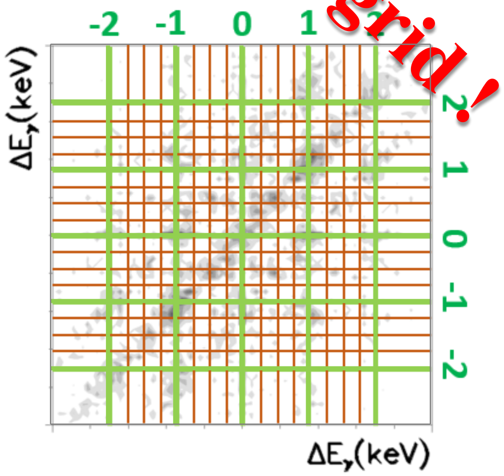
$$\boldsymbol{E_\gamma = 2c(2I + k - 1),\ k\ integer}$$

- *2c Moment of Inertia, Real*
- *(2I+k-1) Angular Momentum, Integer*

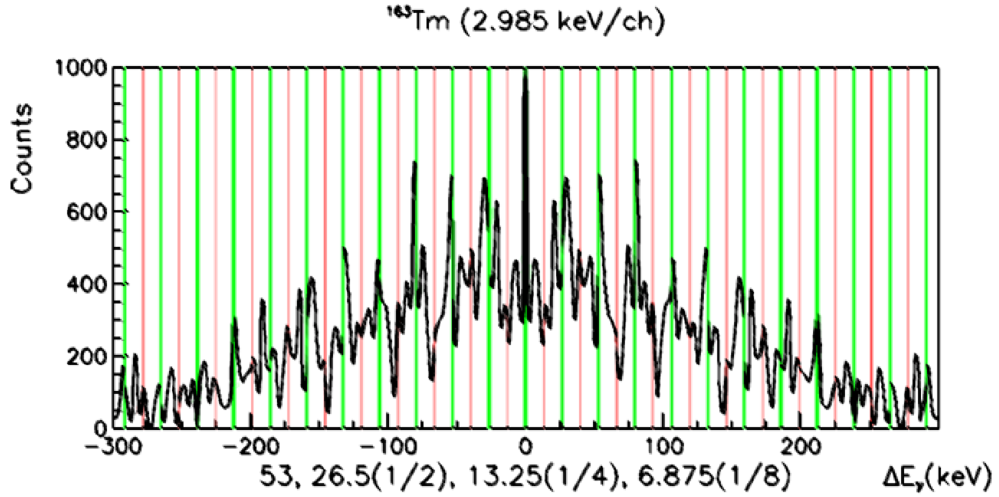
(ΔE_γ^x, ΔE_γ^y) Differential Coincidence Matrix Bitmap

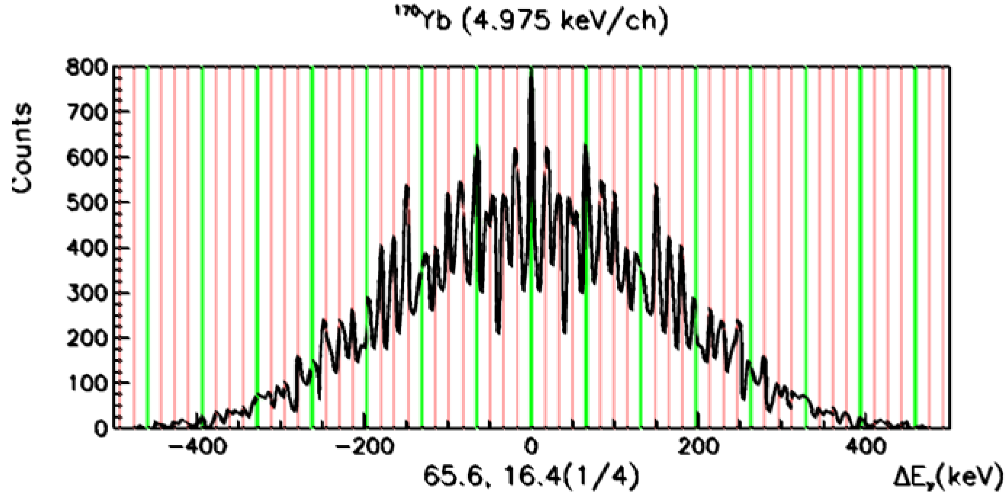


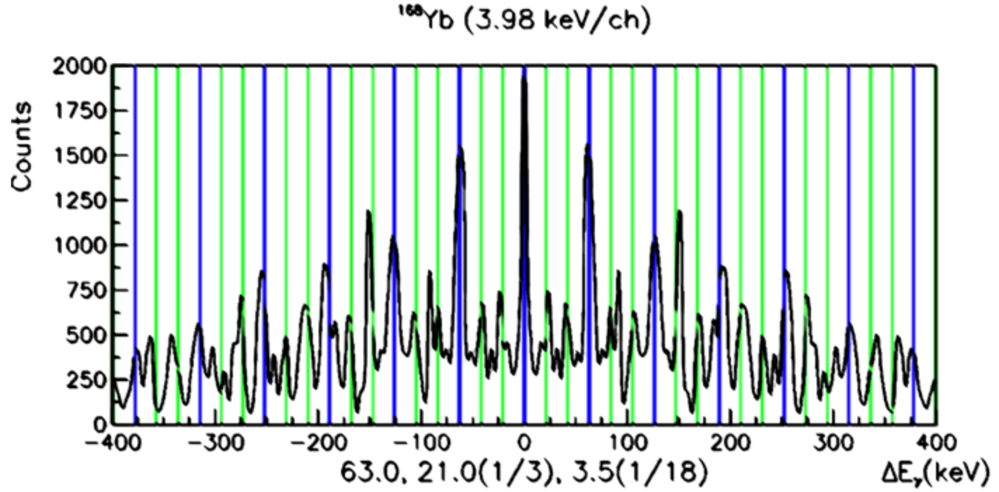
8c&2c regular grid !

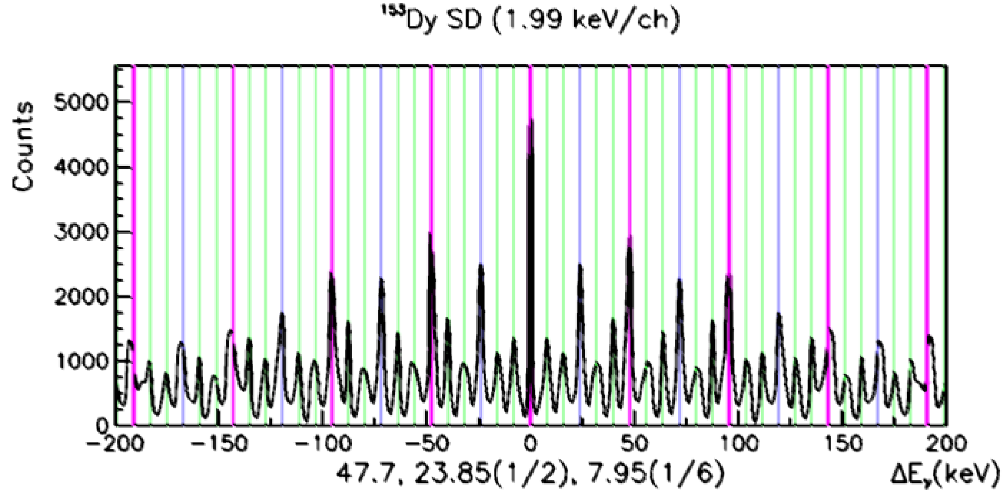


$(\Delta E_\gamma^x, \Delta E_\gamma^y)$ Differential Coincidence Matrix Bitmap (projection)









How the bands can be described?

Bohr-Mottelson Collective Rotor

$$E(I) = \frac{\hbar^2}{2\Im} I(I+1), \quad c = \frac{\hbar^2}{2\Im}$$

$$E_\gamma = E(I) - E(I-2) = \frac{\hbar^2}{2\Im}(4I-2) = 2c(2I-1)$$

$$\Delta E_\gamma = E_\gamma(I) - E_\gamma(I-2) = 8\frac{\hbar^2}{2\Im} = 8c$$

New Parametrization (average behavior)

$$E_\gamma = 2c(2I + k - 1),\ k\ \textit{integer}$$

- *2c Moment of Inertia, Real*
- (2*I*+*k*-1) *Angular Momentum, Integer*

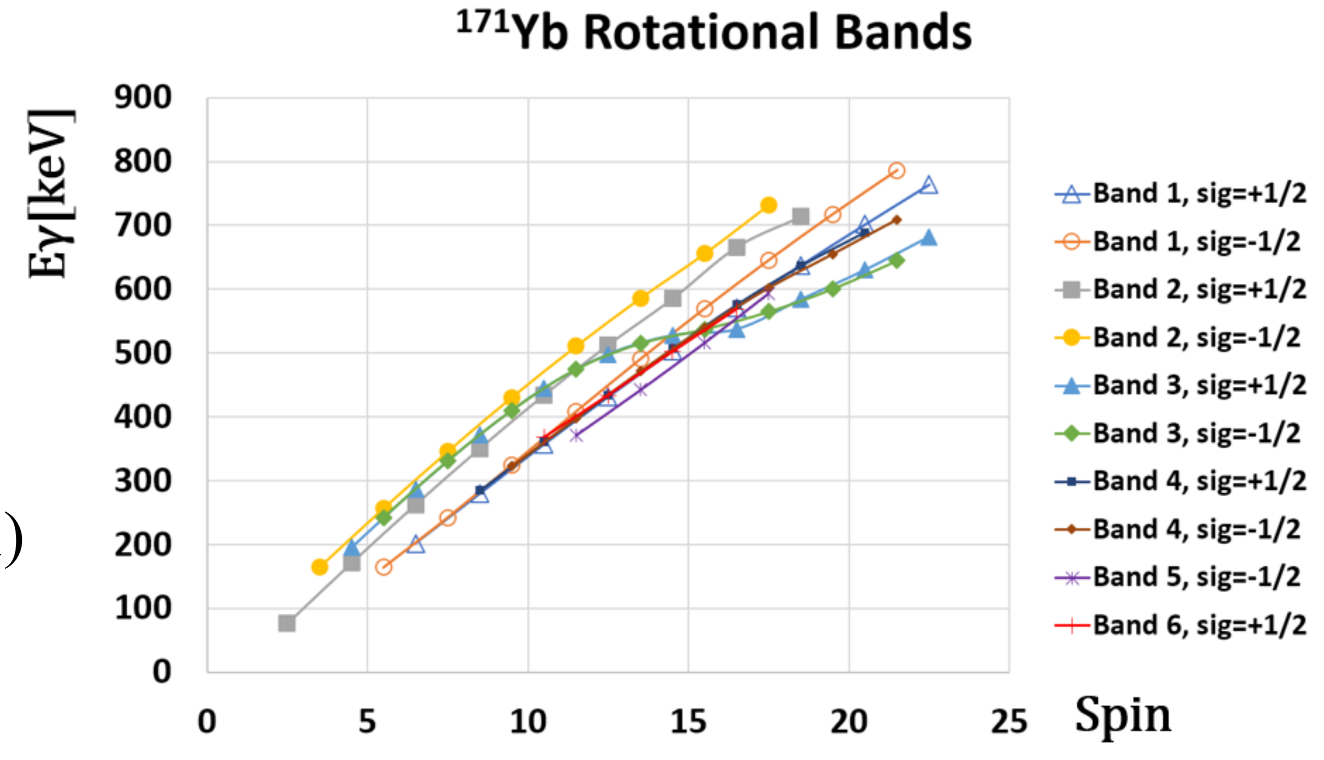


Fig. (Eγ's versus spins)

- *Quasi-linear beam almost parallel and equidistant*
- *Average behavior:* $2c(2I+k-1)$, $2c$ *Real*, $(2I+k-1)$ *Integer*
- *Determine from fit:* $2c$, k's $\Sigma(E\gamma(I)/2c-(2I+k-1))^2 = \min$
- *All k-bands have the same* $\Delta E_\gamma = 8c$ *and thus same* $\mathcal{J}^{(2)}_{eff}$

Case study: ^{171}Yb nucleus high spin rotational bands

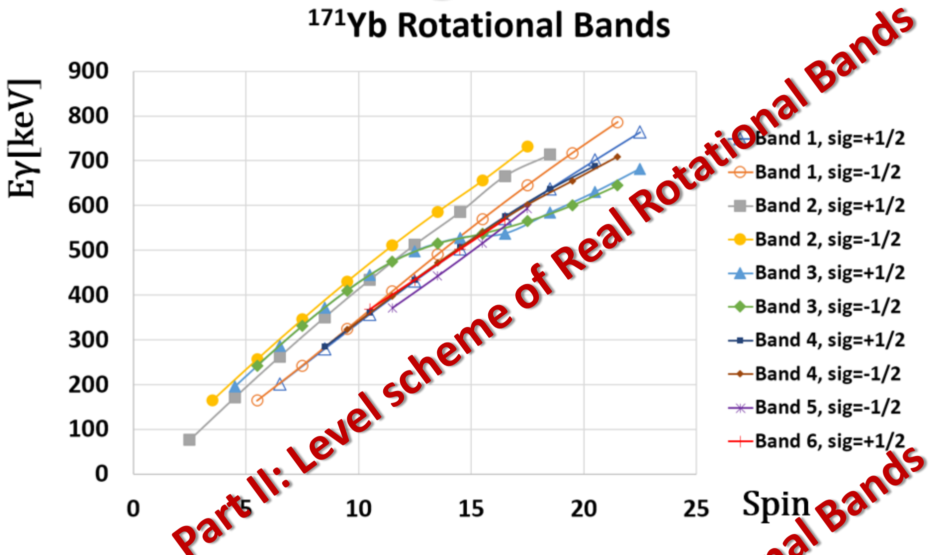


Fig. (Eγ's versus spins)

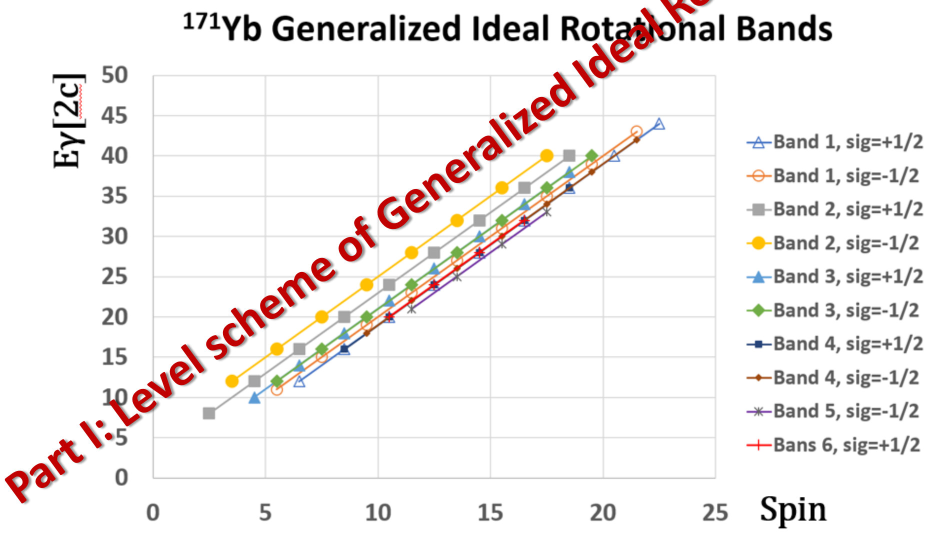


Fig. (Generalized Ideal Rotation Bands)

^{171}Yb band fits using $E_\gamma=2c(2I+k-1)$ parametrization, $\Sigma(E\gamma(I)/2c-(2I+k-1))^2$ =min

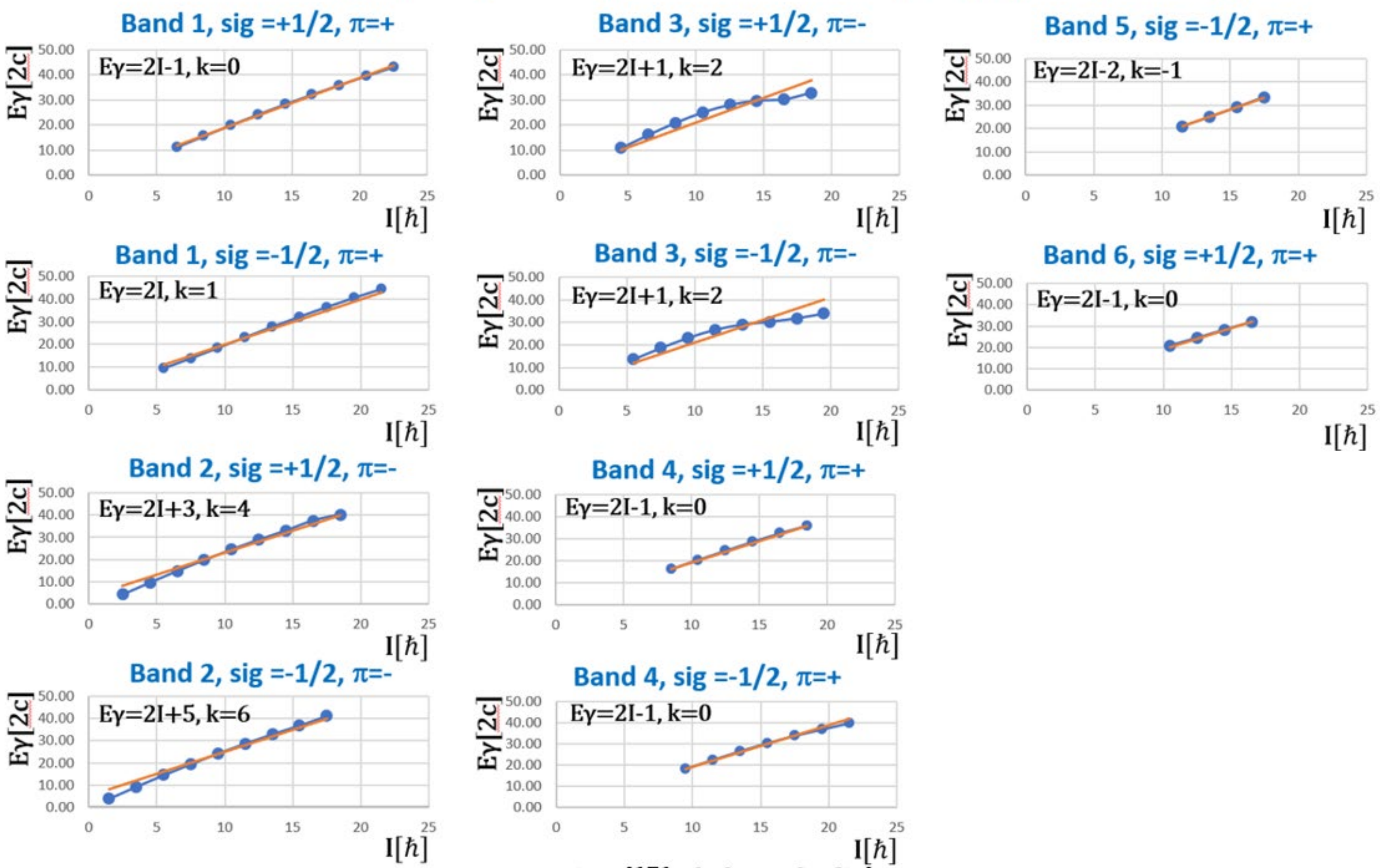


Fig. (^{171}Yb bands fit)

$$2c = 17.75 \text{ keV}$$

$$\mathcal{J}^{(2)}_{eff} = 56.34 \text{ ħ}^2/\text{MeV}$$

What we got for the average description of ^{171}Yb bands?

k-Generalized Ideal Rotor bands:

For **k=0**, **Bohr-Mottelson Ideal Rotor** bands: described by the 2cI(I+1) rule for even and odd spins

For **k≠0**, **k-Generalized Ideal Rotor** bands: have the same $\mathcal{J}^{(2)}_{eff}$ (same 8c!) but are no longer described by the 2cI(I+1) rule.

How to place the k-generalized ideal rotor bands in the level scheme?

By adding "stairs" of 2c levels to the k=0 band!

One gets a "***parabolic 2D building***"

- with the "0" floors of the k=0 Bohr-Mottelson I(I+1) levels vertically connected as by an elevator cabin,
- as well as by "fire escape" stairways for all k ≠0, of 2I+1 stairs for each floor, one for even spins and one for odd spins.
- In general, the energy levels can be indexed by three **integer numbers**, **(I,m,n)**, where I is the nuclear spin, m is the position of the "stair" level relative to the spin "floor", and n the energy of the level, which is a natural number in units of 2c.
- k≠0 bands are represented as **tilted paths on the Parabolic Level Scheme**

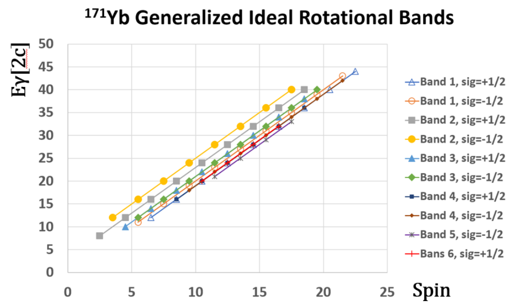


Fig. (Generalized Ideal Rotation Bands)

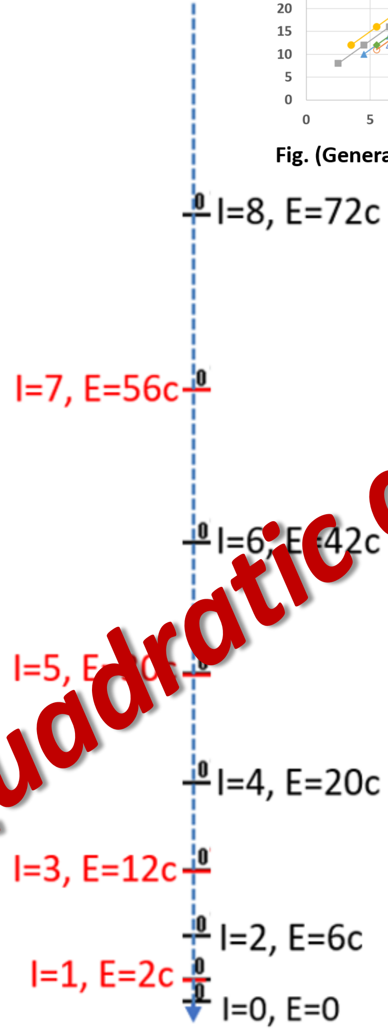


Fig. (Ideal rotor)

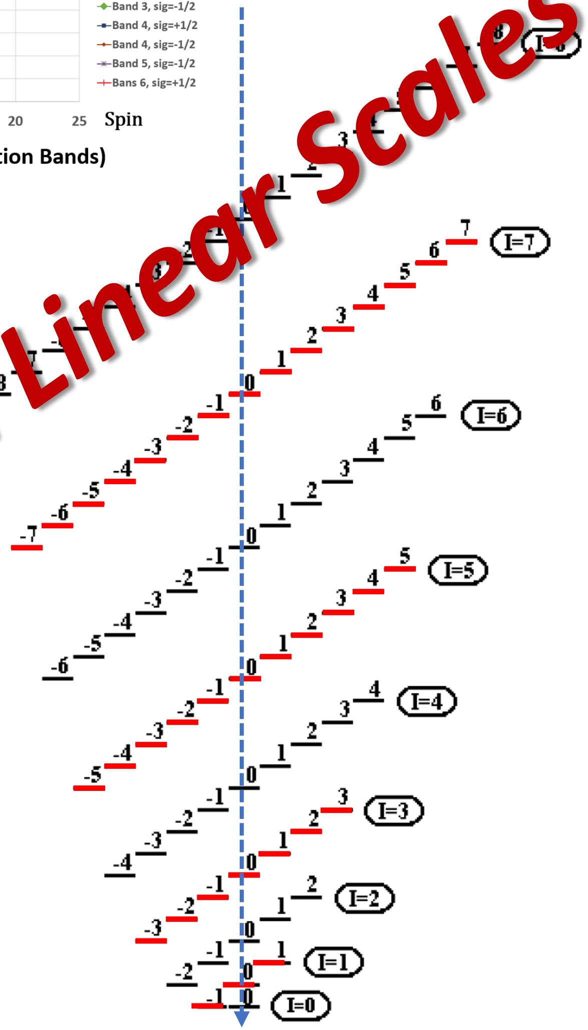


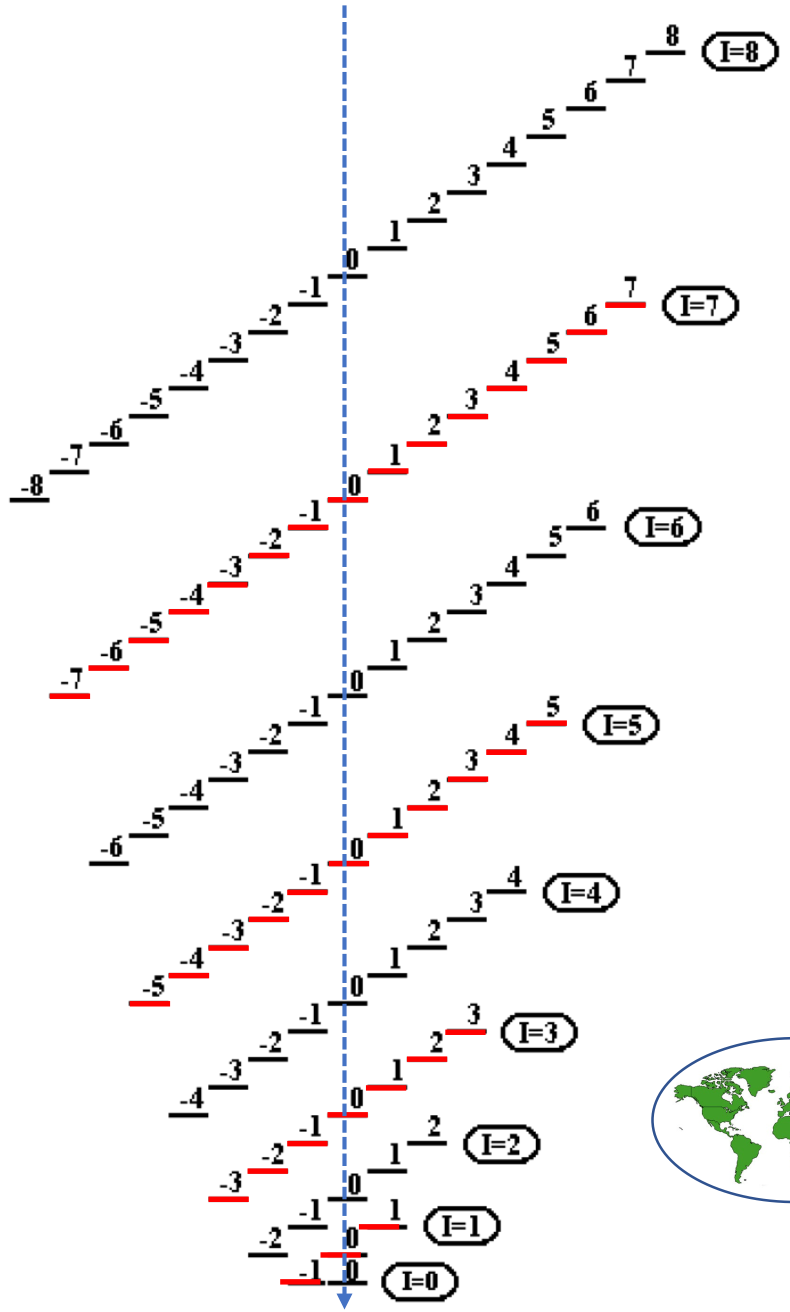
Fig. (Opened generalized ideal rotor)

Quadratic and Linear Scales

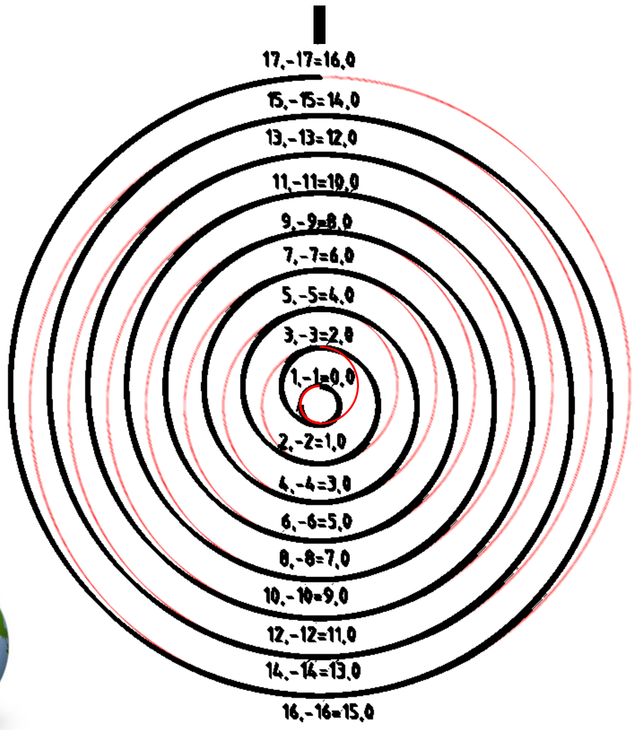
Parabolic Level Scheme for k bands

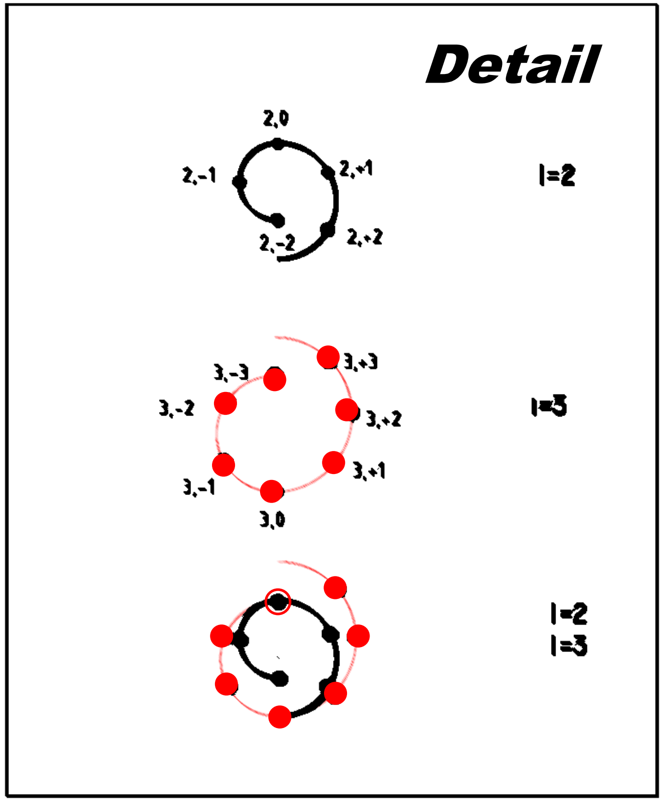
$$E_{\gamma}^{k}(I) = 2c\,(2I + k - 1),\ k = \pm 1, \pm 2, \ldots, I$$

DOUBLE HELIX



“Mercator-like” 2D view

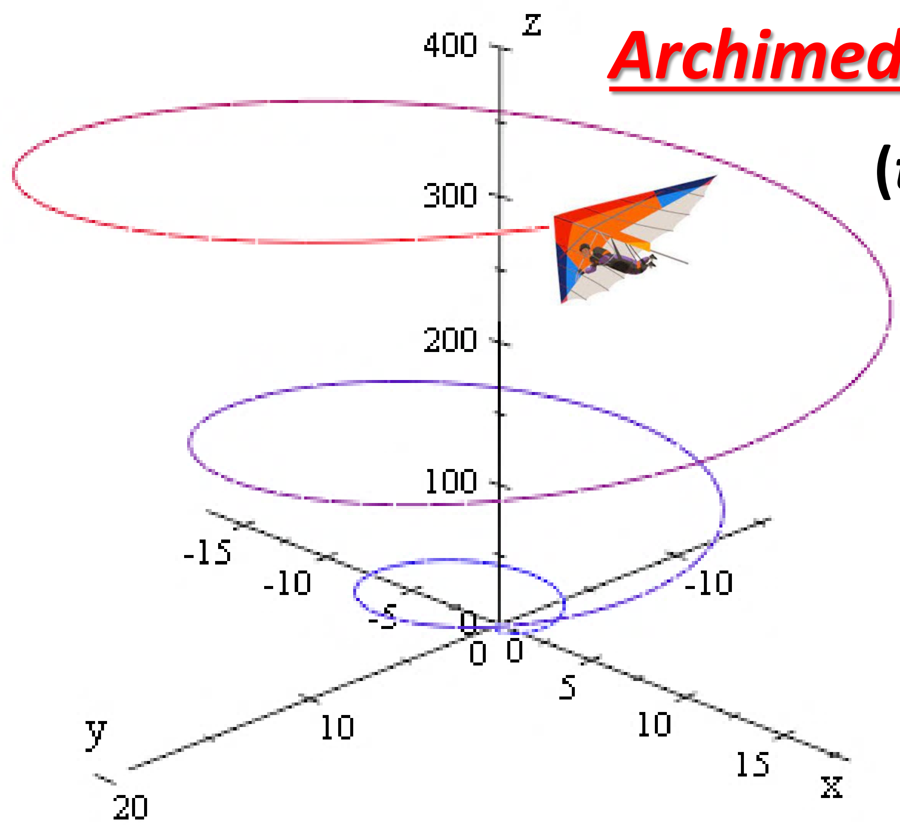




“Globe-like” 3D view (from above)

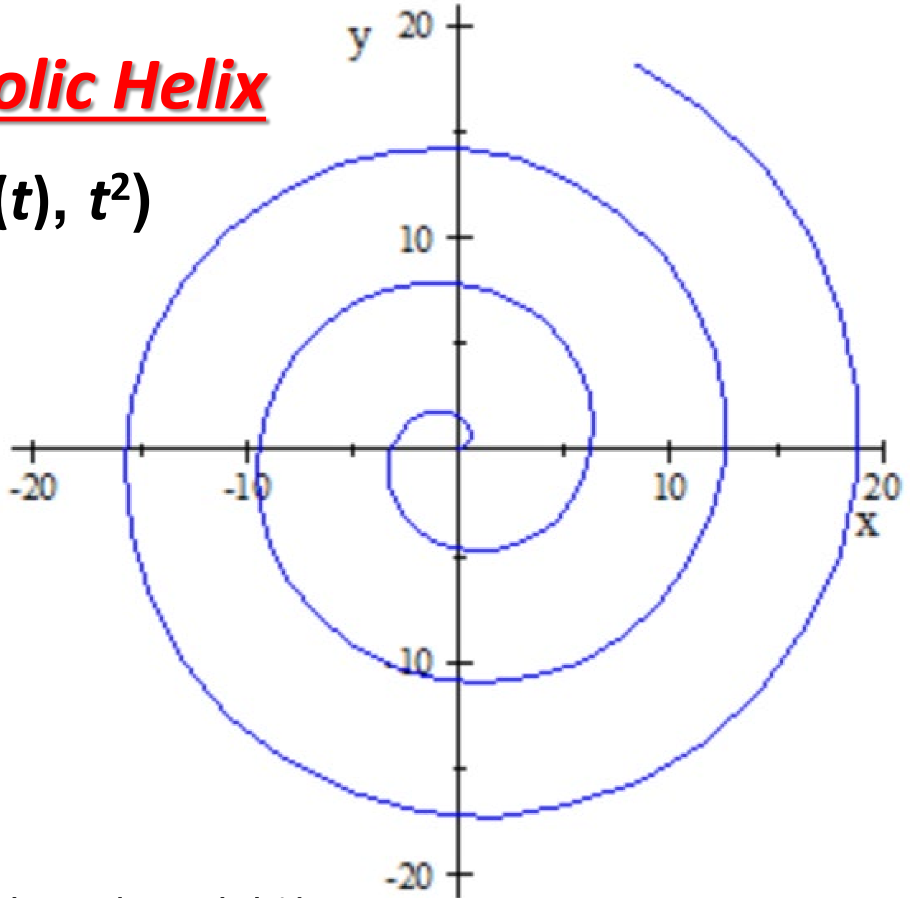
Correspondence Principle: Classical to Quantum Mechanics

a) Classical: Attenuated Rotational Motion (progressive decrease of energy and ang. momentum)



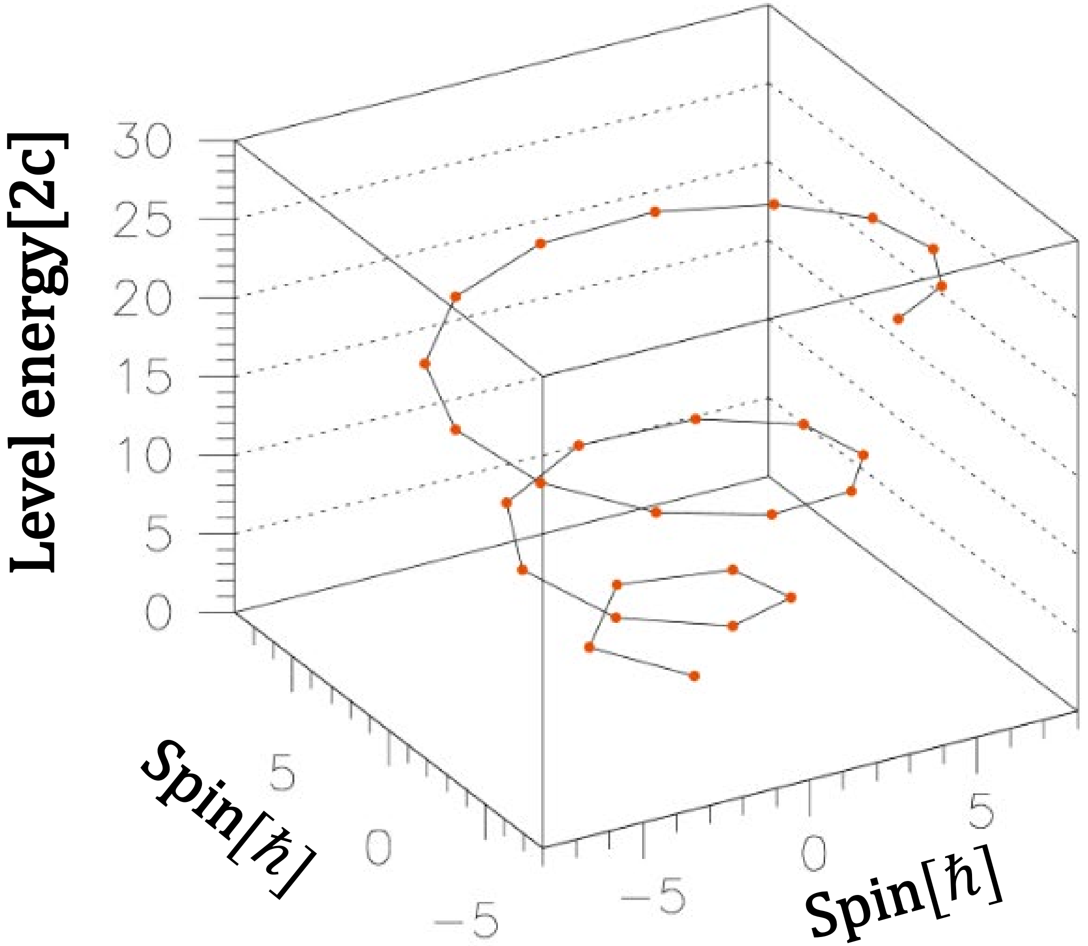
Archimedean Parabolic Helix

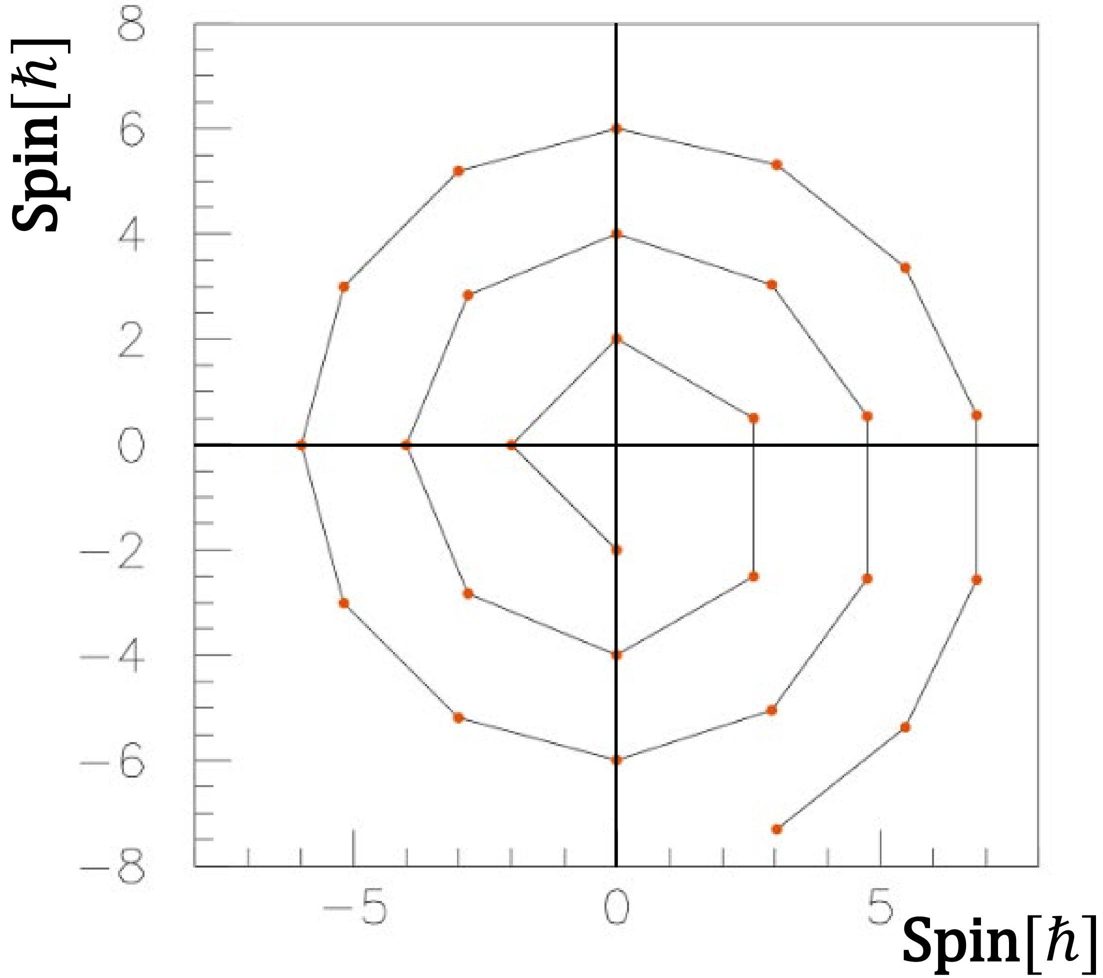
$(t\cos(t), t\sin(t), t^2)$



From "Interactive Mathematics": https://www.intmath.com/blog/mathematics/arc-length-of-a-spiral-around-a-paraboloid-8751

Correspondence Principle:
Classical to Quantum Mechanics
b) Quantum: Attenuated Rotational Motion
(progressive decrease of excitation energy and spin)

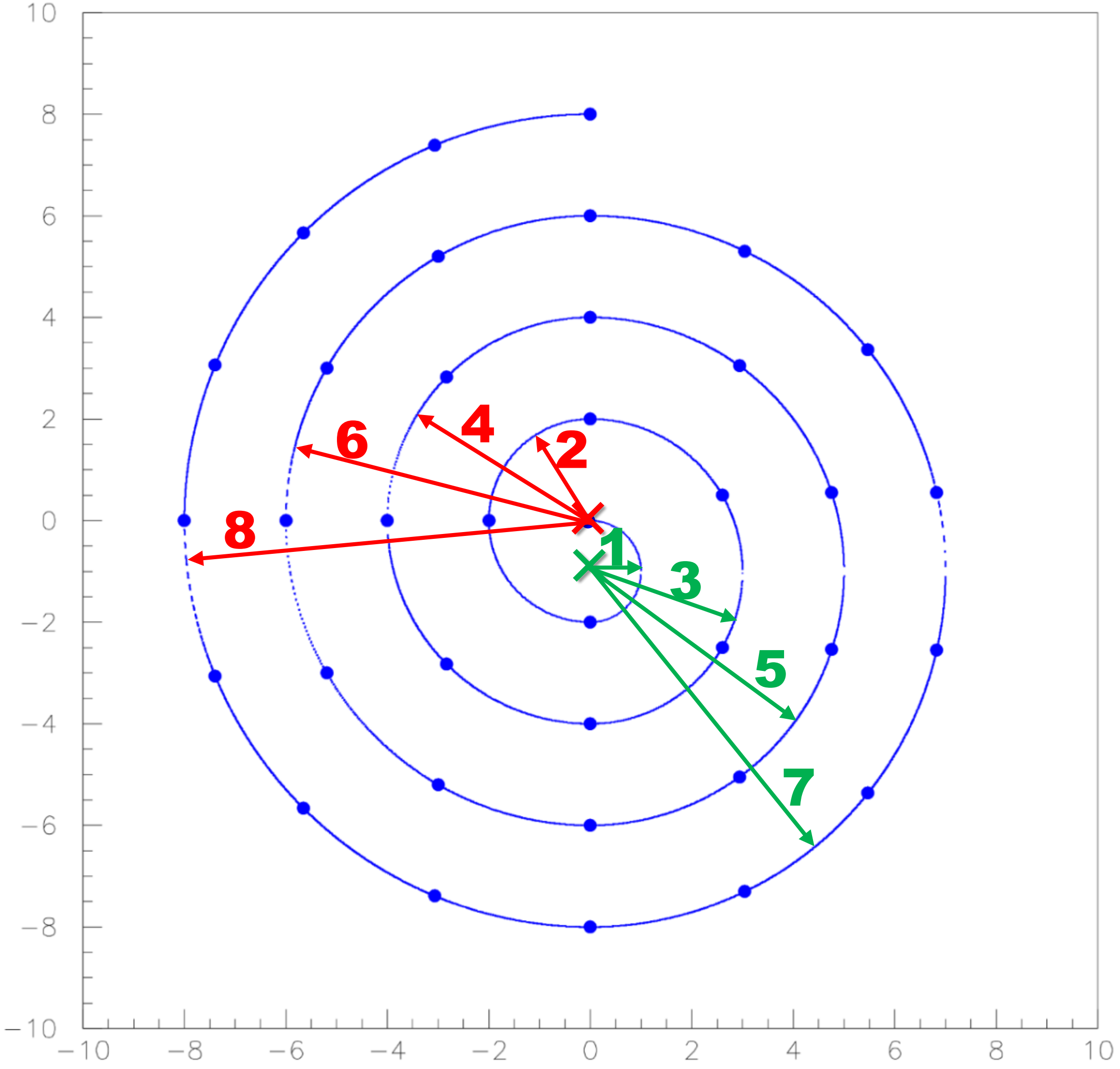




Quantum Helix: Attenuated Rotational Motion
(progressive decrease of excitation energy and spin)

- *The fact that quantum angular momentum is expressed by integer values implies that the radii of the quantum helix are also integers*
- *This implies that the quantum helix is composed of a series of alternating odd-even radii semicircles*
- *This also implies that unlike the classical helix, the quantum helix has two centers:*

1. *(0,0) center for even integer radii for even-l semicircles on the left*
2. *(0,-1) center for odd integer radii for odd-l semicircles on the right*



Helicoid Degrees of Freedom

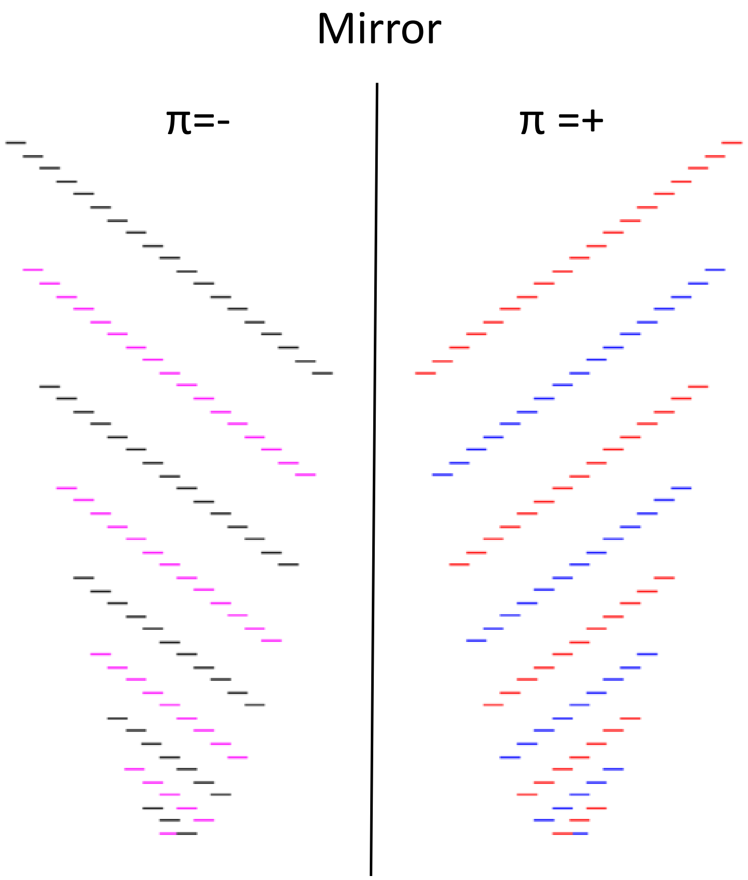


Fig. (Parity)

- One has **3 degrees of freedom** for helicoids:
  - **2 helicoids for spins**: sig=0,1 for integer spins; (and separately for sig=+1/2,-1/2 for half-integer spins)
  - **2 helicoids for direction of rotation**: clockwise and counterclockwise
  - **2 helicoids for parity**: positive and negative
- Therefore there are **2^3=8 helicoids** per even-A and odd-A nuclei, respectively.
- However, two combinations of spin-parity can be hosted by a same helicoid for both even-A and odd-A nuclei, which reduces to **2 helicoids** their number for all spin-parity combinations per direction of rotation (4 helicoids for both directions)
- We **cannot distinguish between the directions of rotation** at this stage, so we'll conventionally use only the set of **2 counterclockwise helicoids** for all representations
- Important to stress: **quantum double-helix can naturally support the direction of rotation** degree of freedom – "helicity" quantum observable

Double-Helix for even-A Nuclei

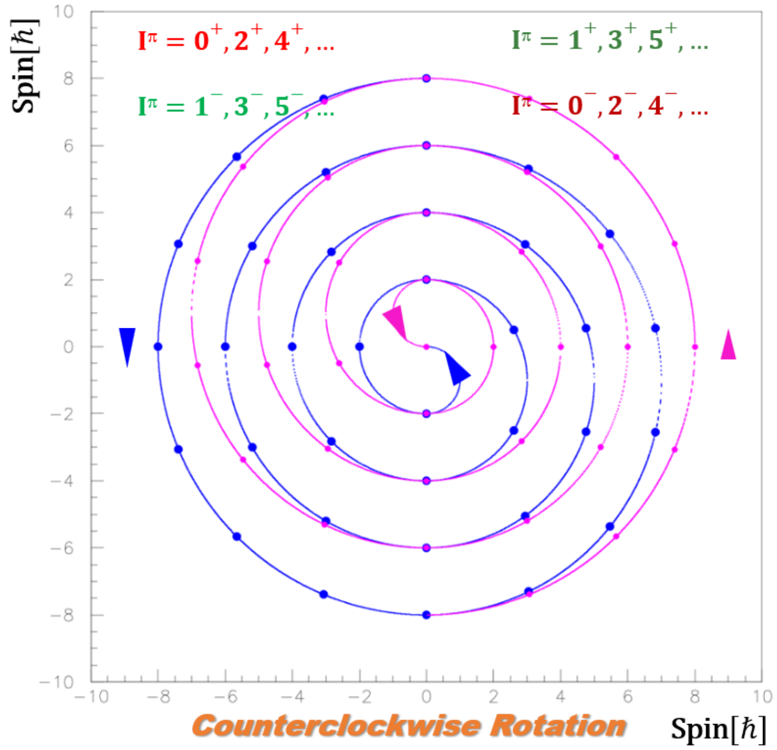


Fig. (Set of 2 helixes for integer spins)

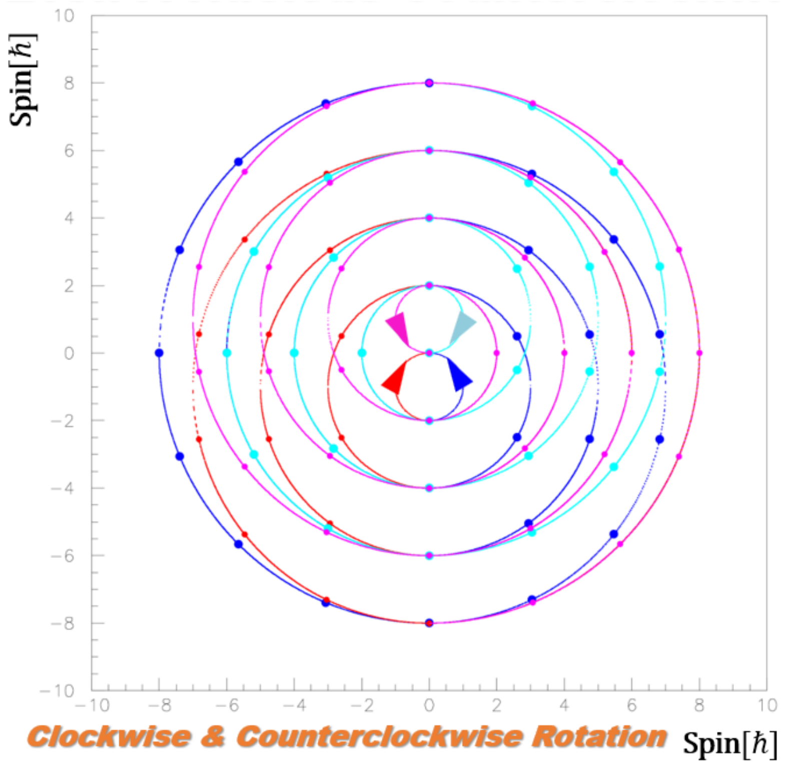


Fig. (Complete set of 4 helixes for integer spins)

Double-Helix for odd-A Nuclei

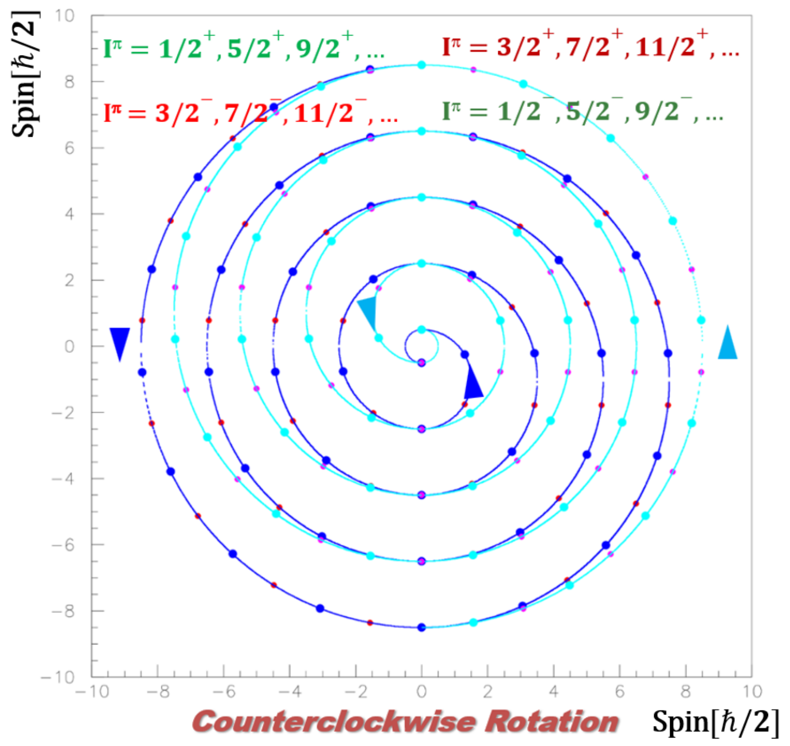


Fig. (Set of 2 helixes for integer half-spins)

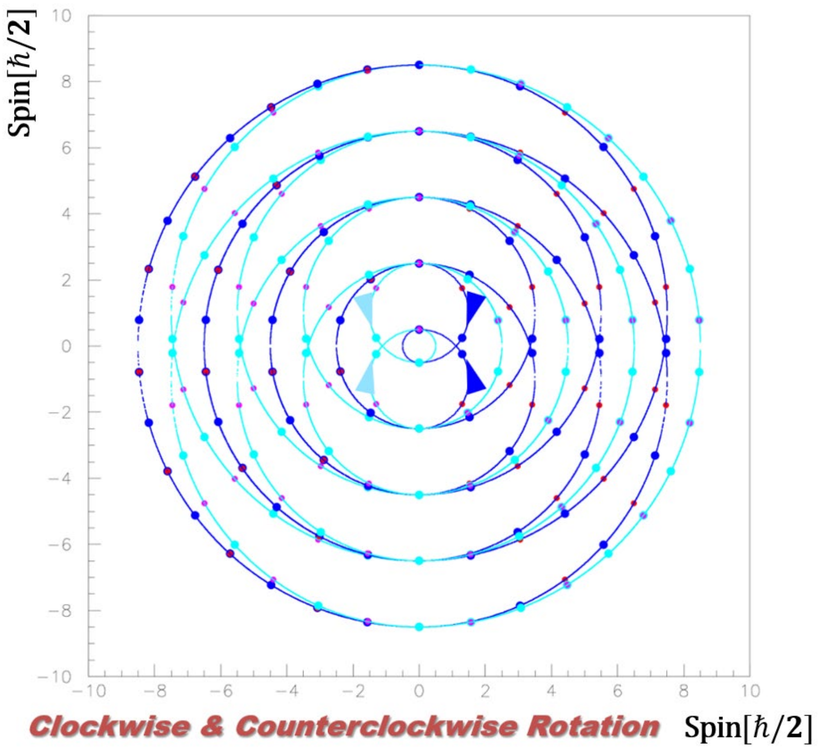


Fig. (Complete set of 4 helixes for half-integer spins)

D

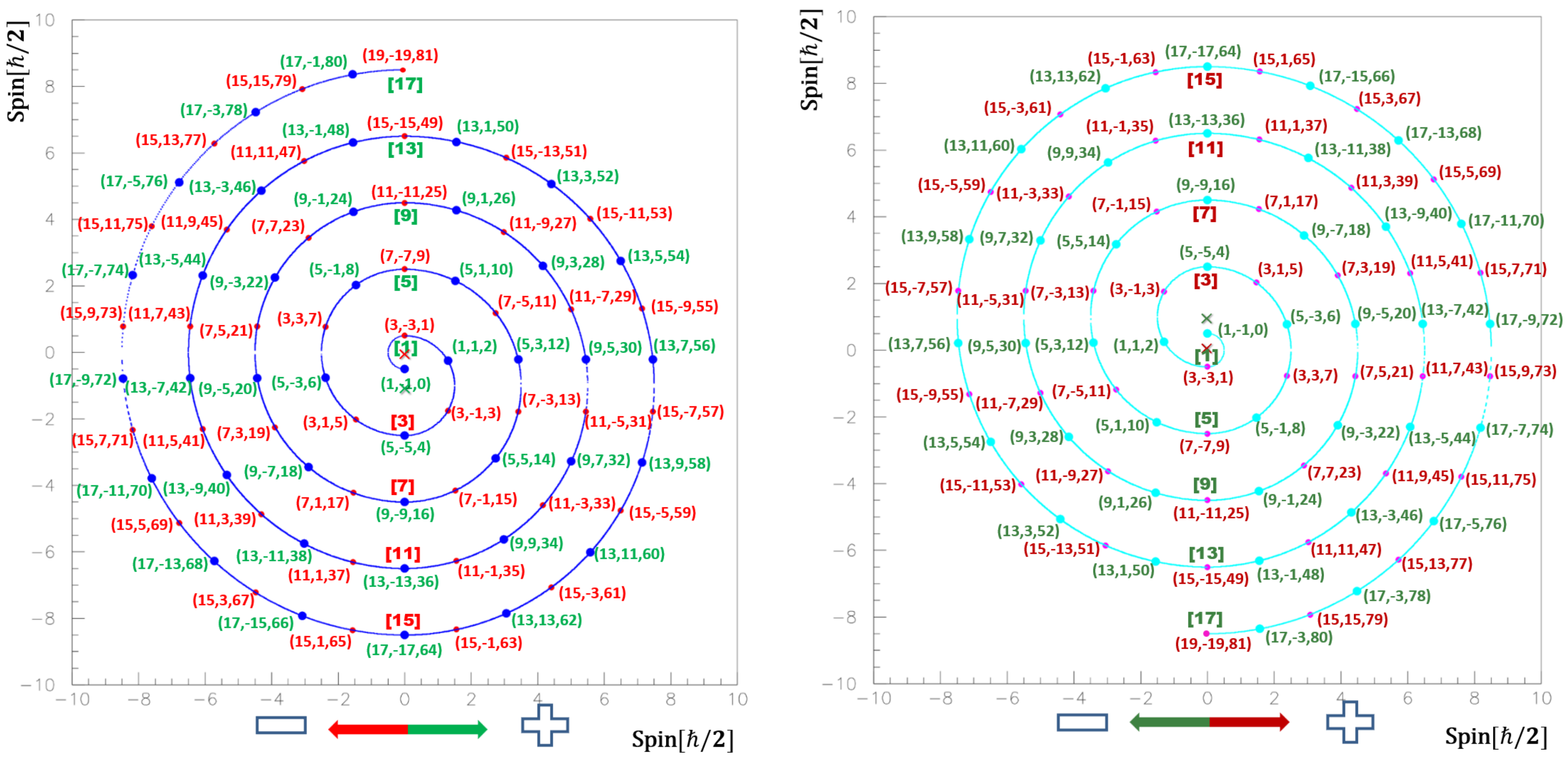


Fig. (Complete description of double helix for odd A)

k=0 ideal rotor signature partners bands on the double helix for even-A nuclei

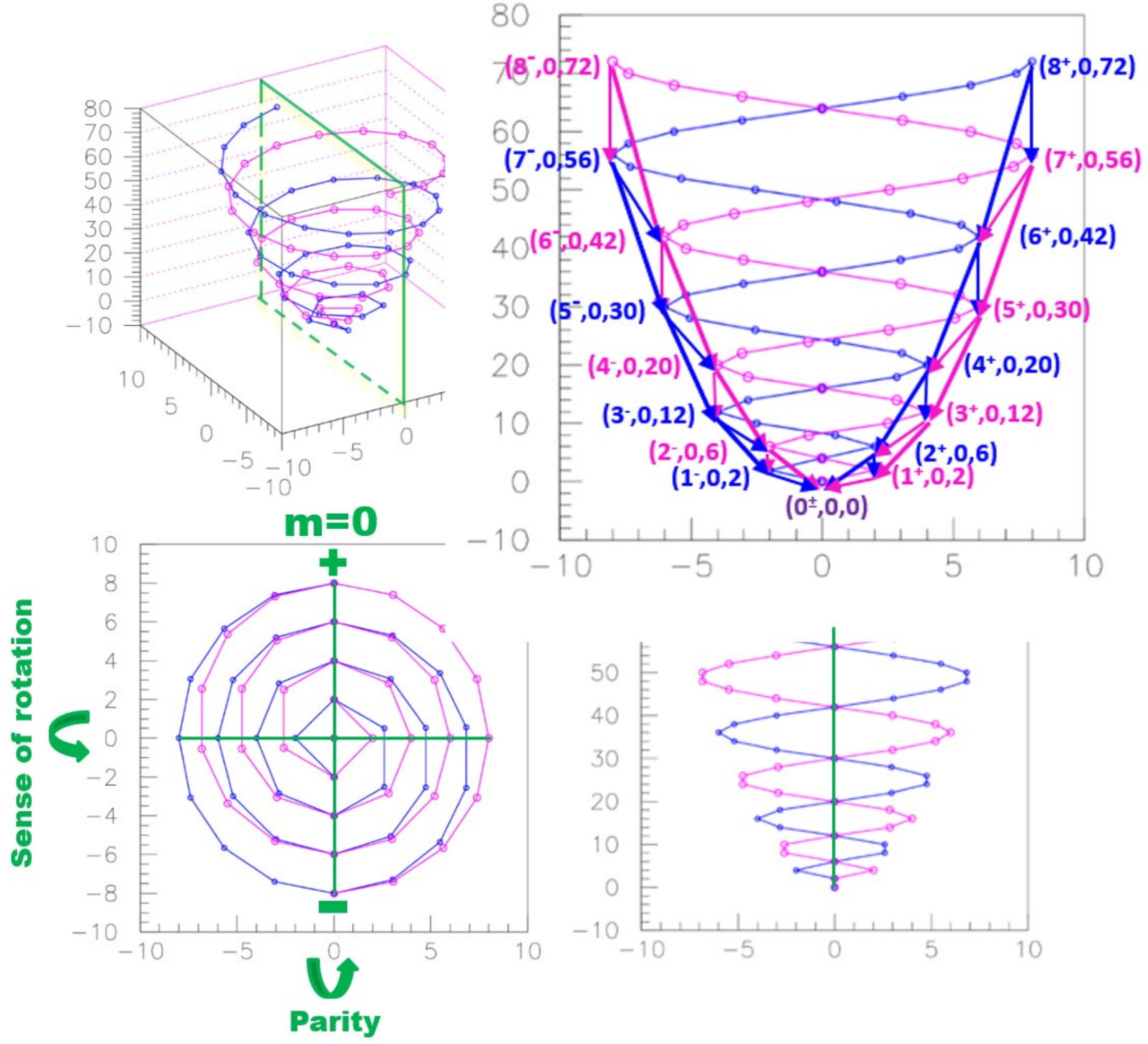


Fig. (Ideal signature partner bands on the double helicoid)

Positions of 2I+k-1 generalized rotor bands on the helicoid for even-A and odd-A nuclei

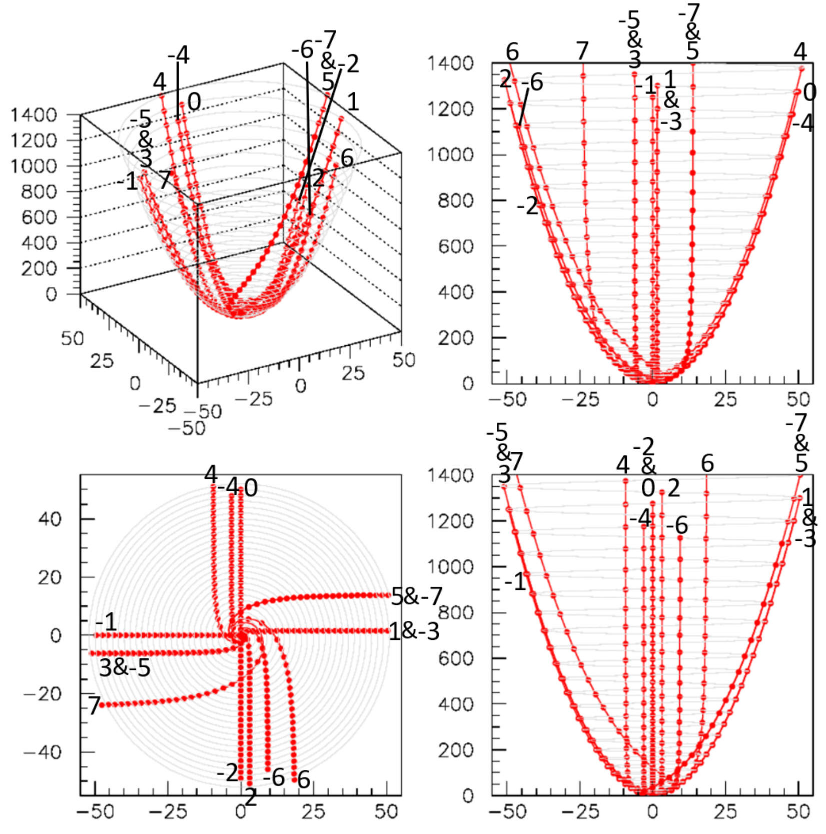


Fig. (Generalized (2I+k-1) bands on helix for even A)

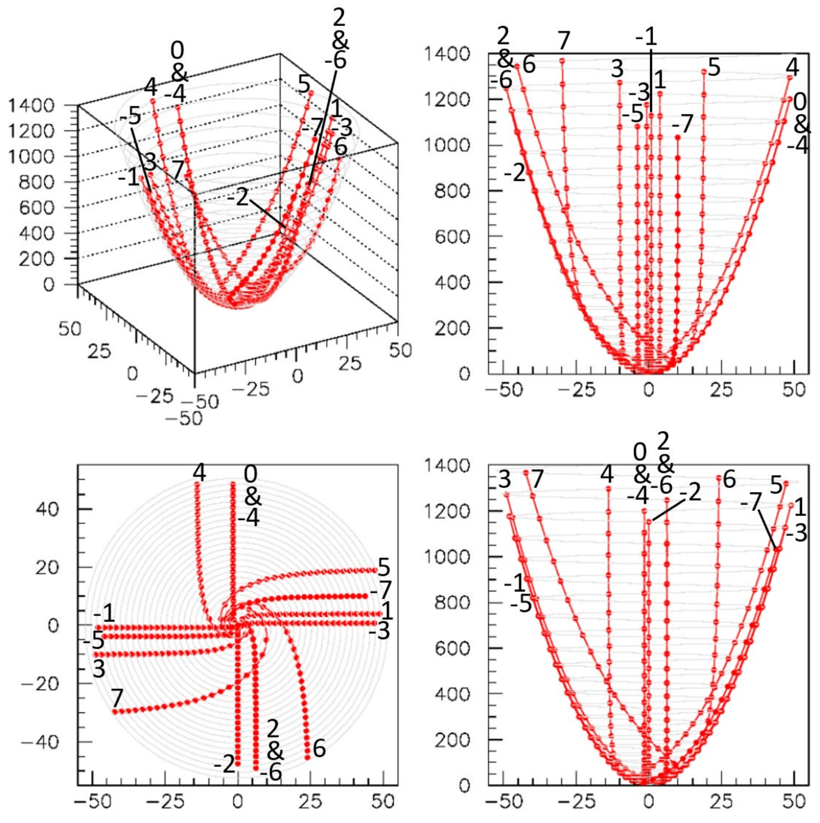
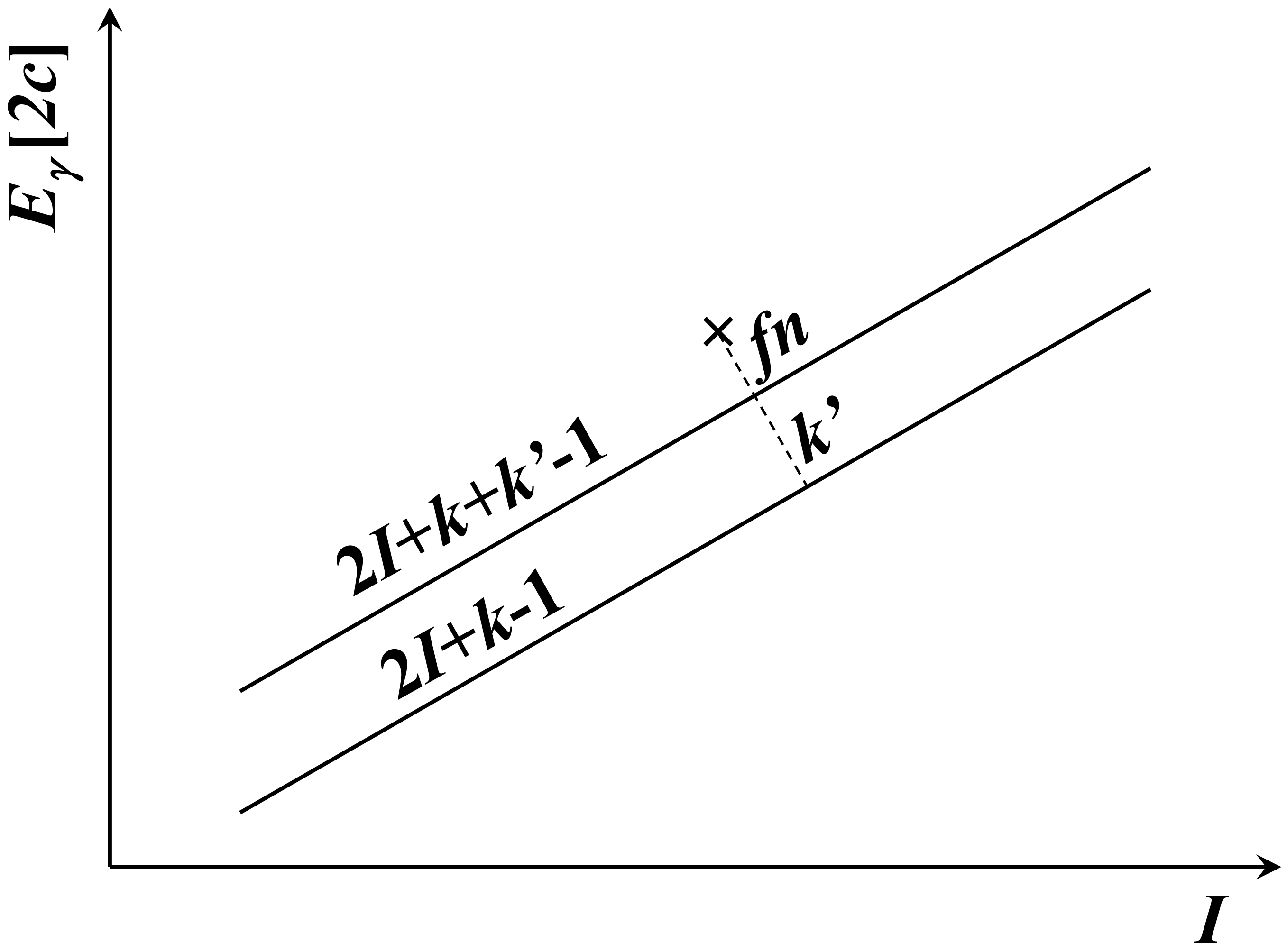


Fig. (Generalized (2I+k-1) bands on helix for odd A)

Double-Helix Level Scheme of ^{171}Yb nucleus



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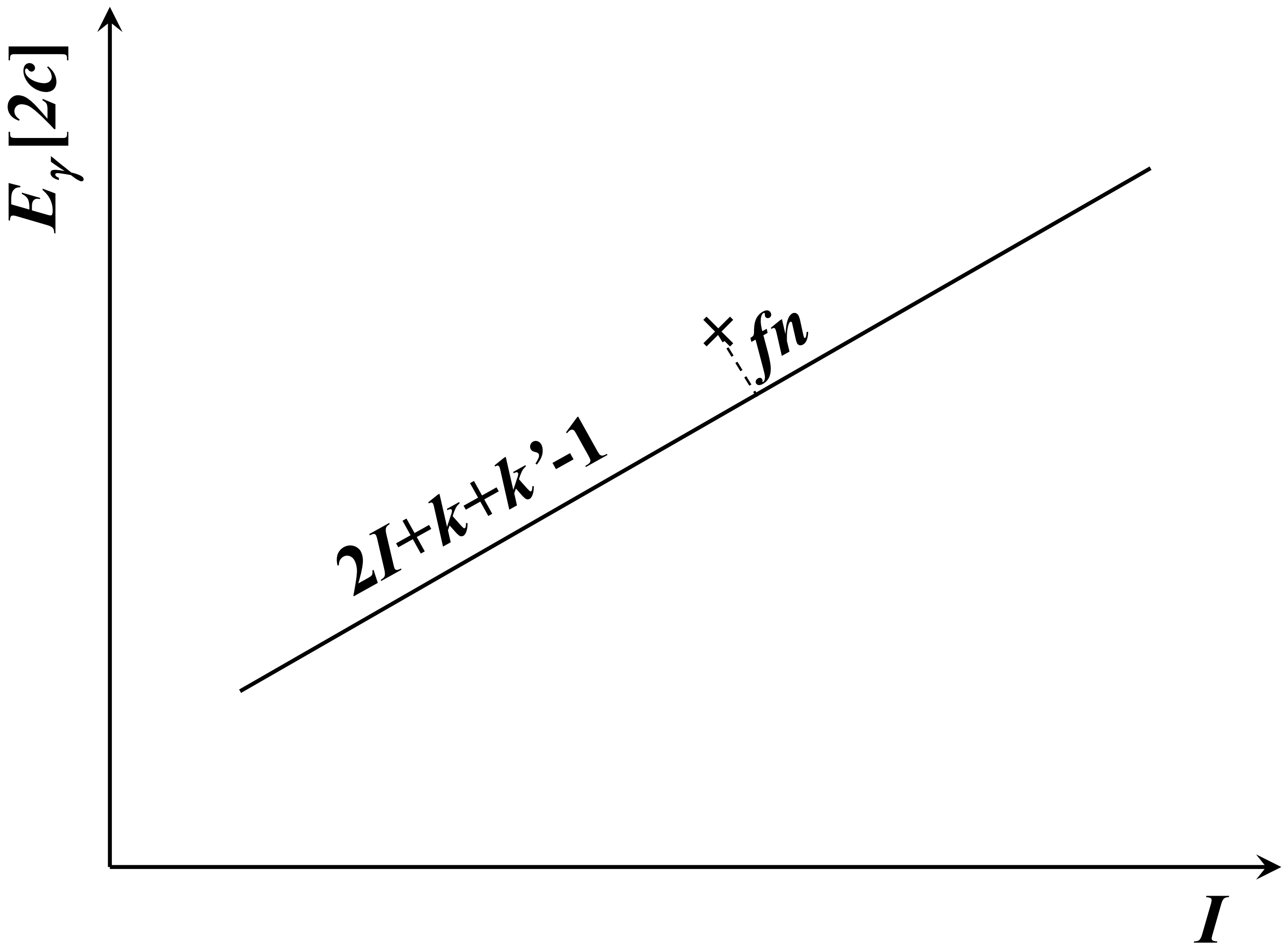


Double-Helix Level Scheme of ^{171}Yb nucleus

- Decomposition of experimental band values as

  $E\gamma$=2c[(2I+k-1)+(k'+fn)],

where **(2*I*+*k*-1)** is the ***average*** *part*, and **(*k*'+*fn*)** is the ***deviation*** from average, with ***k' integer*** and ***fn fractional*** numbers, respectively.

- Rewrite **$E\gamma$=2c(2I+k+k'-1+fn)** as

  $E\gamma$=2c[1+fn/(2I+k+k'-1)]×(2I+k+k'-1), with **2c_{band}=2c[1+fn/(2I+k+k'-1)]:**

  - **$E\gamma$= 2c_{band}×(2I+k+k'-1)**
  - ***Spin I*** describes ***macroscopic rotation, i.e. 2π helix loops***
  - ***k+k'*** extra ang. momenta analog to ***aligned intrinsic angular momenta*** coming from ***microscopic intrinsic rotation, i.e. variations of 2π helix loops***
  - One gets ***Bands Moment of Inertia***, $\mathcal{J}^{(2)}_{band}$=ħ²/2c_{band}

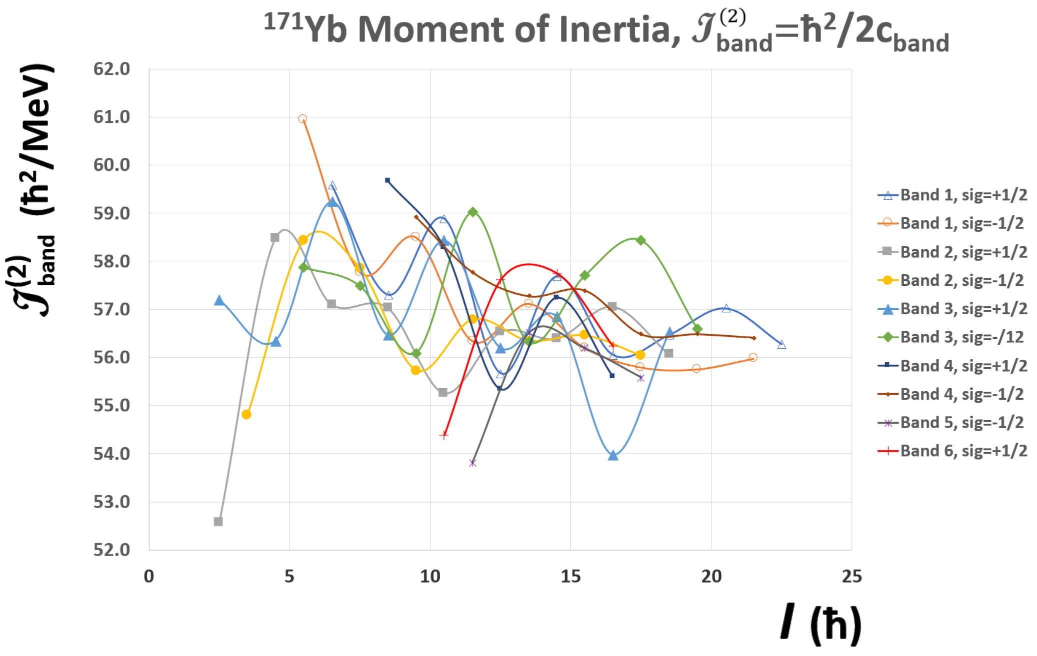


Table 1 ^{171}Yb data for the Band 2 sig=+1/2 (ground state band)

I	Eγ(keV)	Eγ/2c	(2I+k+k’-1)	$\mathcal{J}^{(2)}_{band}$ ($\hbar^2$/MeV)	$\sum_{levels}$ (Eγ/2c)	$\sum_{levels}$ (2I+k+k’−1)
$5/2^-$	76.1	4.29	4	52.6	4.29	4.0
$9/2^-$	171.0	9.63	10	58.5	13.92	14.0
$13/2^-$	262.7	14.80	15	57.1	28.72	29.0
$17/2^-$	350.7	19.76	20	57.0	48.48	49.0
$21/2^-$	434.3	24.47	24	55.3	72.95	73.0
$25/2^-$	513.0	28.90	29	56.5	101.85	102.0
$29/2^-$	585.2	32.97	33	56.4	134.82	135.0
$33/2^-$	666.0	37.53	38	57.1	172.35	173.0
$37/2^-$	713.5	40.19	40	56.1	212.54	213.0

Table 7 ^{171}Yb data for the Band 4 sig=+1/2

I	Eγ(keV)	Eγ/2c	(2I+k+k’-1)	$\mathcal{J}^{(2)}_{band}$ ($\hbar^2$/MeV)	$\sum_{levels}$ (Eγ/2c)	$\sum_{levels}$ (2I+k+k’−1)
$13/2^+$					**55.27**	**23.0**
$17/2^+$	284.9	16.05	17	59.7	71.32	40.0
$21/2^+$	360.3	20.30	21	58.3	91.62	61.0
$25/2^+$	433.7	24.44	24	55.3	116.06	85.0
$29/2^+$	506.6	28.54	29	57.2	144.60	114.0
$33/2^+$	575.6	32.43	32	55.6	177.03	146.0
$37/2^+$	636.4	35.85	36	56.6	212.88	182.0

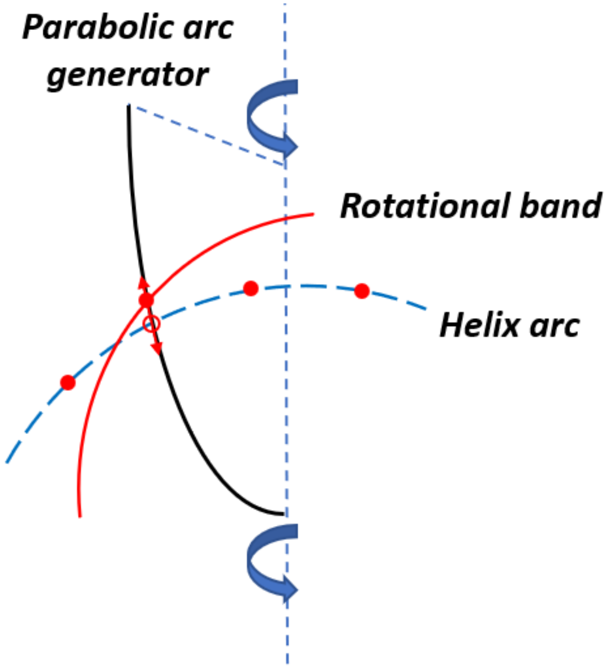


Table 13 ^{171}Yb linking transitions
Band 4 sig=+1/2 to Band 1 sig=-1/2

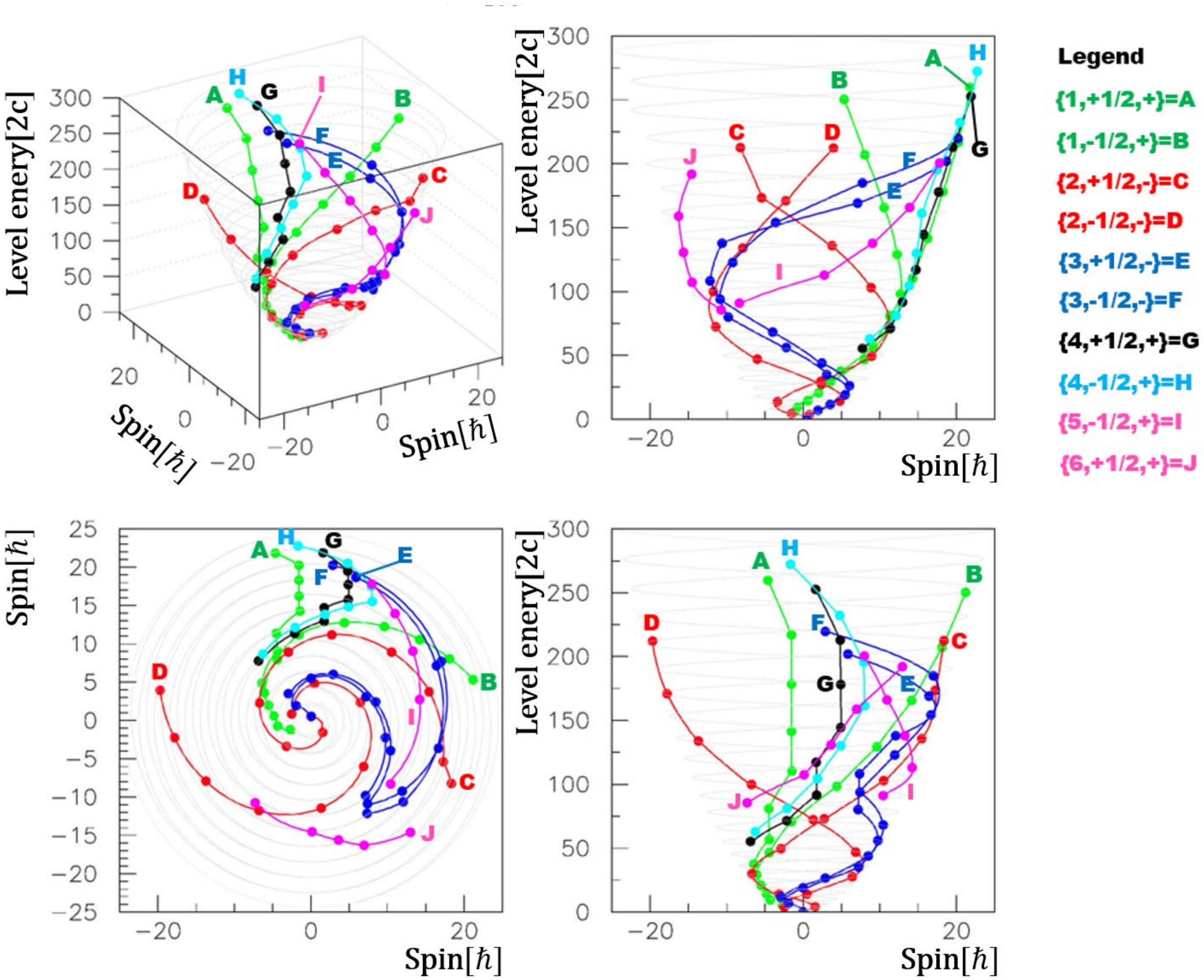
I	Eγ(keV)	Eγ/2c	(I+k+k’)
$13/2^+$	721.8	40.66	7.5
$17/2^+$	764.5	43.07	10.5*
$21/2^+$	800.2	45.08	12.5*
$25/2^+$	825.6	46.51	13.5

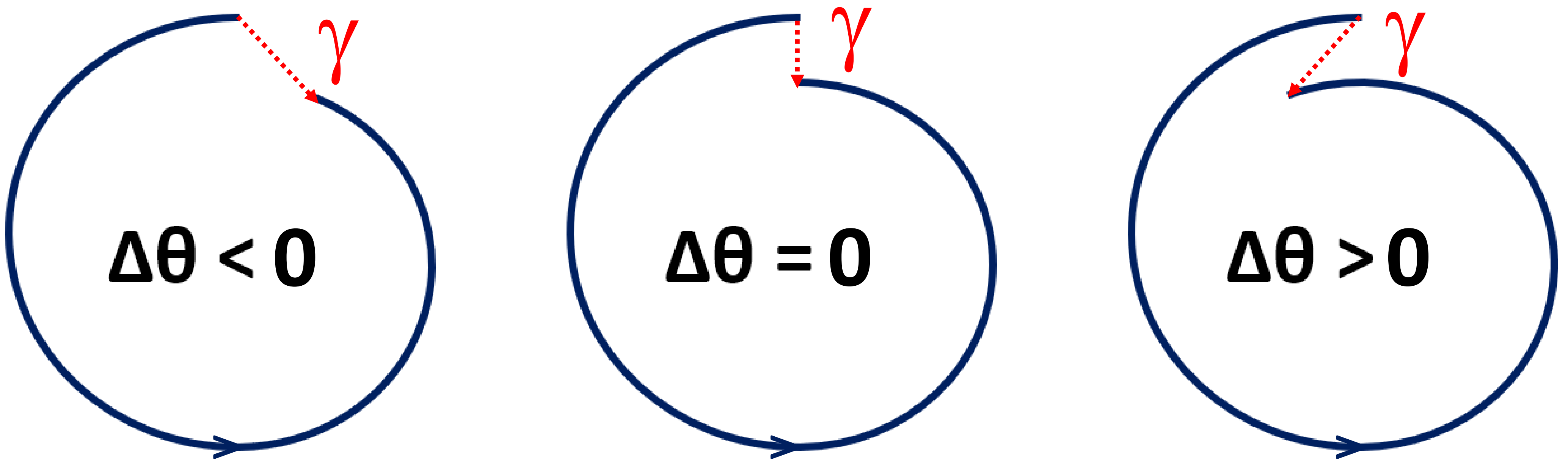
Table 14 ^{171}Yb linking transitions
Band 4 sig=-1/2 to Band 1 sig=+1/2

I	Eγ(keV)	Eγ/2c	(I+k+k’)
$15/2^+$	745.2	41.98	8.5
$19/2^+$	788.3	44.41	11.5*

Fig. (Helix arc and parabolic arc generator crossing)

Double Helix Level Scheme of ^{171}Yb nucleus





$$\theta(I, m) = \sum_{I,m}\left(I + \frac{m}{I}\right)\pi$$

$$\Delta\theta(I) = \theta(I) - \theta(I-2) - 2\pi$$

Fig. (Δθ apparent band rotation on the helicoid)

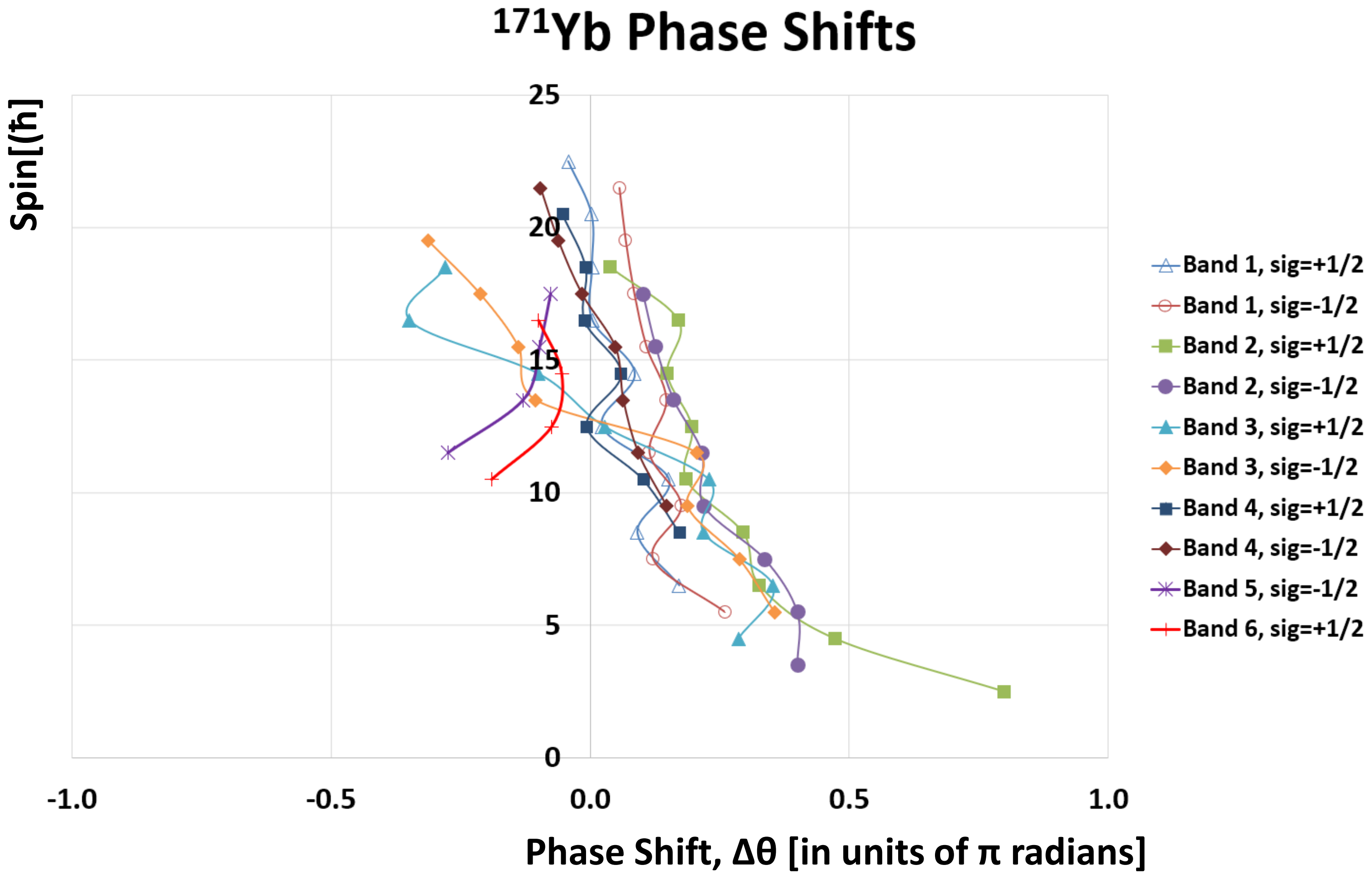


Fig. (^{171}Yb Phase Shifts)

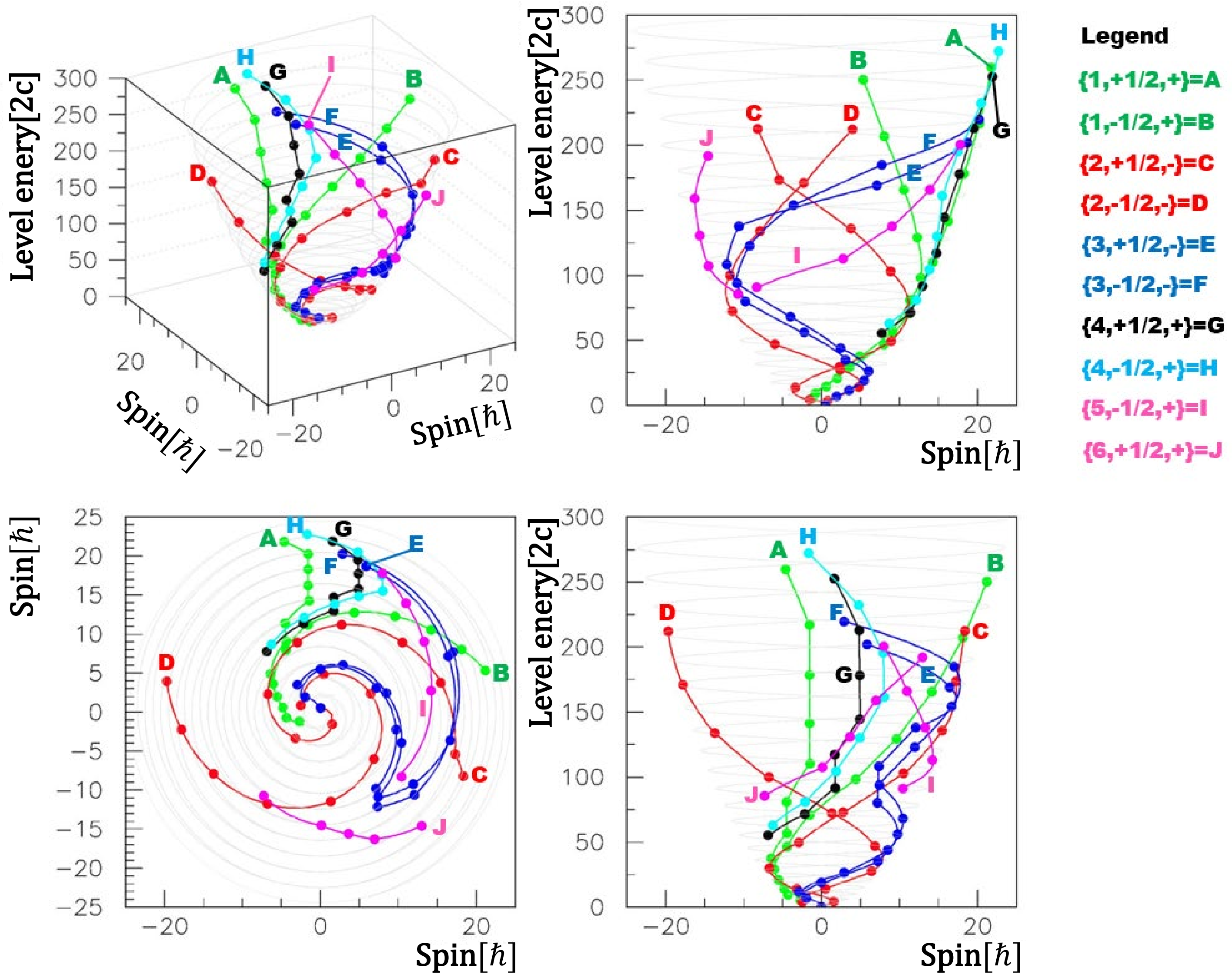
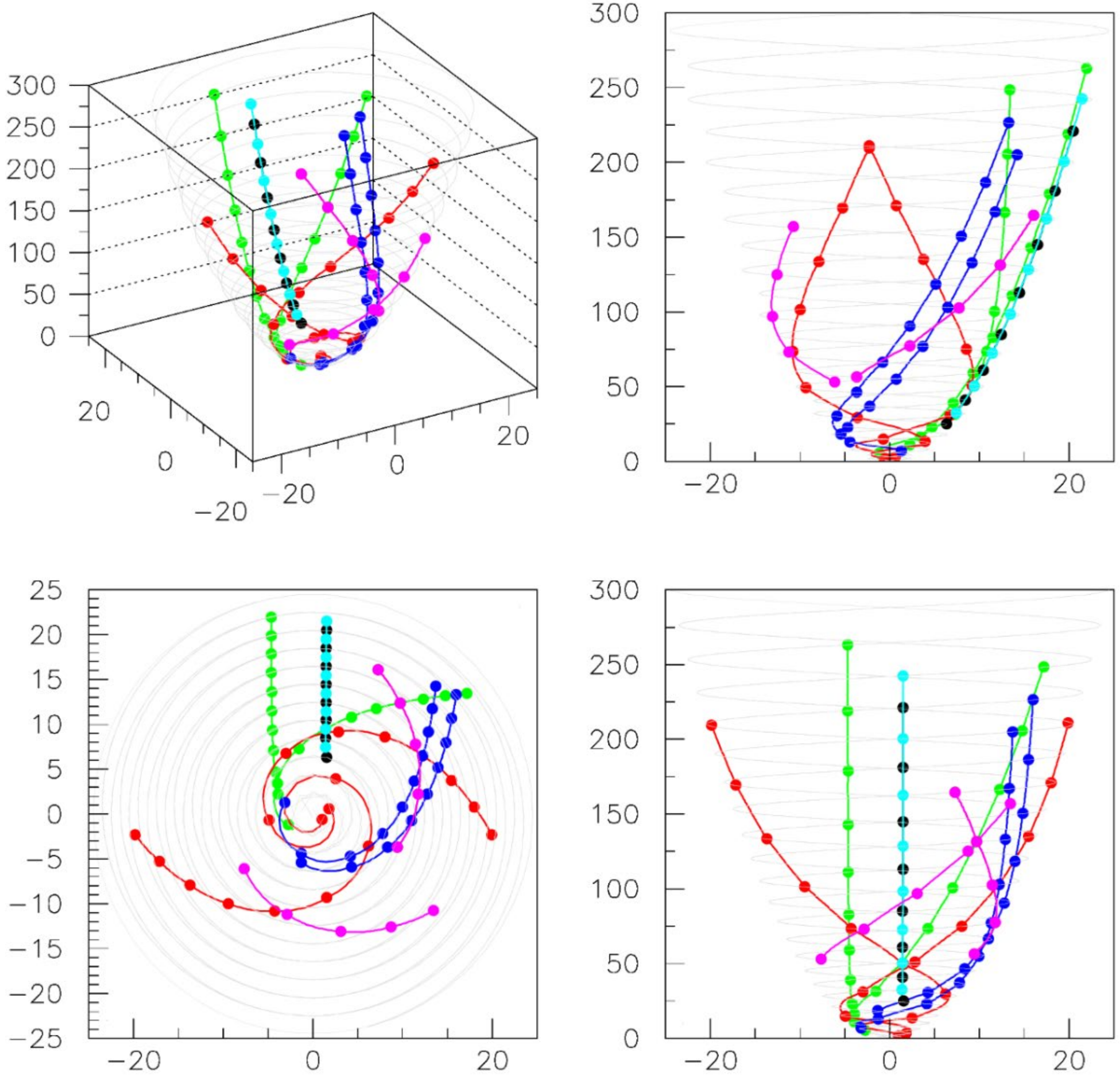


Fig. (Double helix of ^{171}Yb nucleus)



300
250
200
150
100
50
0
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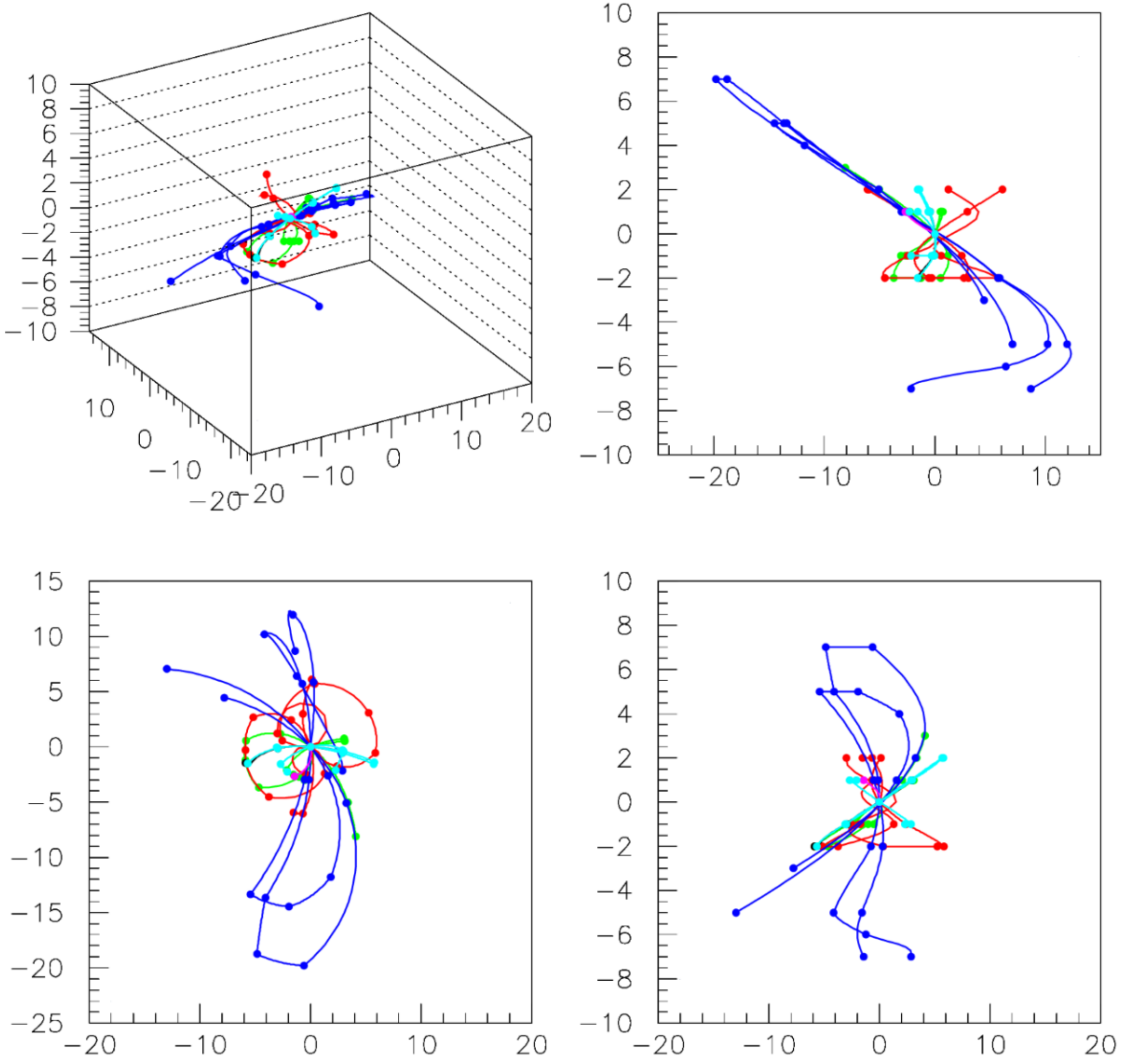

10 8 6 4 2 0 −2 −4 −6 −8 −10
10 0 −10 −20
−20 −10 0 10 20
10 8 6 4 2 0 −2 −4 −6 −8 −10
−20 −10 0 10
15 10 5 0 −5 −10 −15 −20 −25
−20 −10 0 10 20
10 8 6 4 2 0 −2 −4 −6 −8 −10
−20 −10 0 10 20

^{171}Yb nucleus Double Helix Level Scheme

Part I: Level scheme of Generalized Ideal Rotational Bands

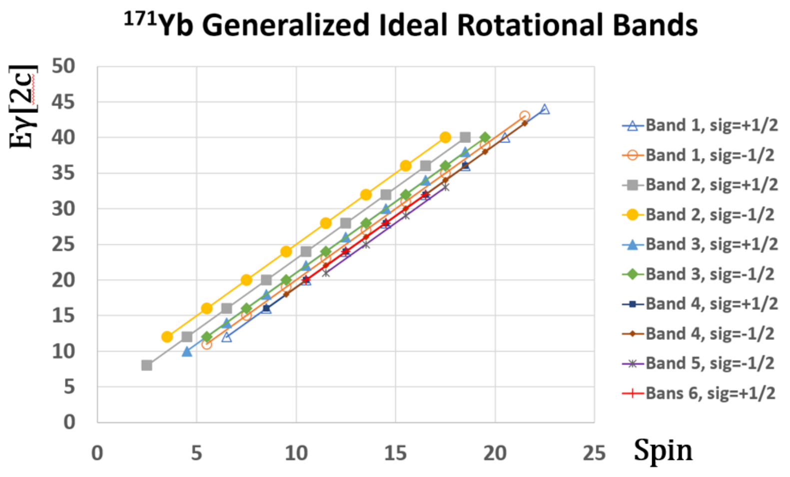
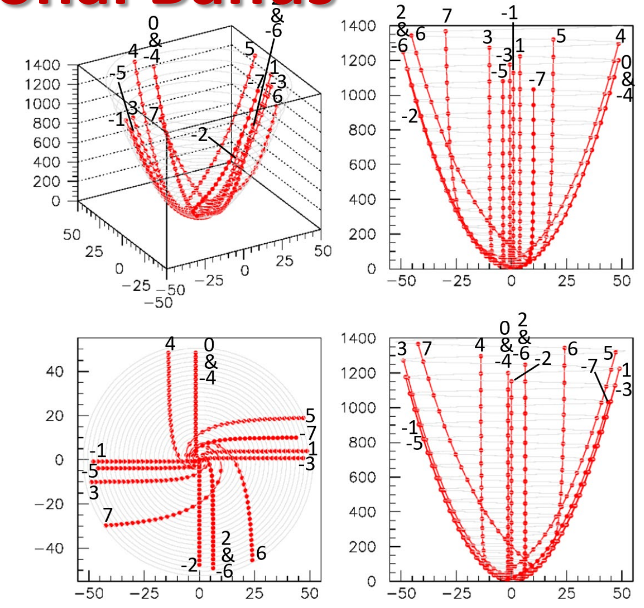


Fig. (Generalized Ideal Rotation Bands)



Part II: Level scheme of Real Rotational Bands

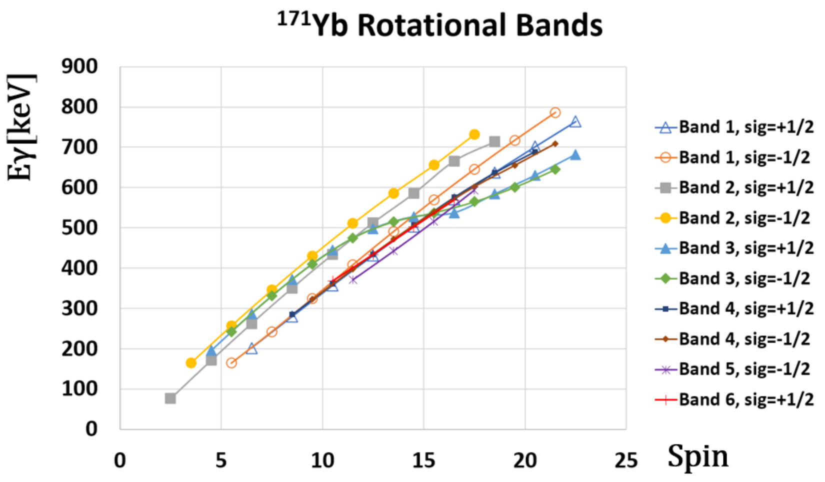
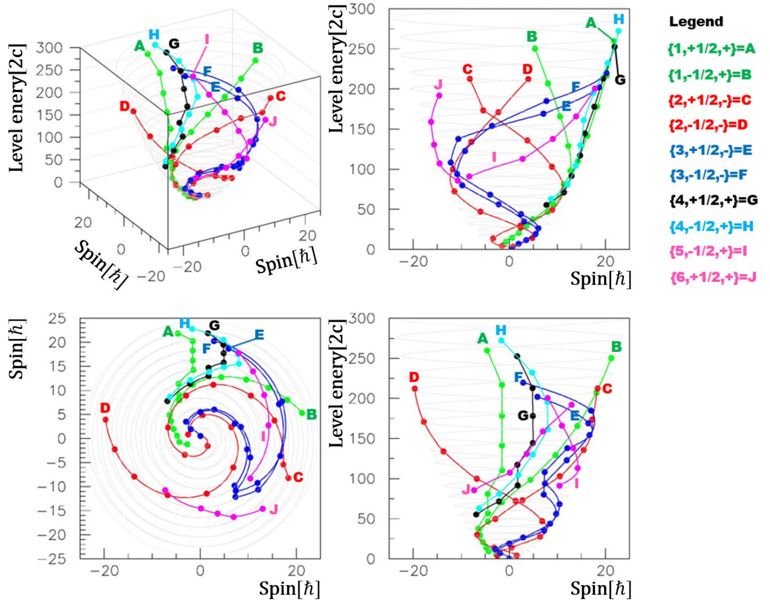


Fig. (Eγ's versus spins)



Double Helix - Conclusions

- ✓ *Do we have a re-concept of a Level Scheme?*
- ✓ *Yes. The new concept of a level scheme is probed by the construction of* ***Double Helix Level Scheme*** *of ^{171}Yb.*
- ✓ *Do we have new physical insight?*
- ✓ *Yes. Double Helix is:*
  - ***semiclassical description of nuclear macroscopic rotation***
  - ***with apparent clockwise and counterclockwise rotations of the γ-rays rotational bands exhibiting the microscopic rotation.***
- ✓ *By combining* ***linear and quadratic scales, Double Helix*** *eliminates arbitrariness and* ***transform the Level Scheme into an integrating imagistic technique of nuclear motion.***
- ✓ *Double Helix, a new class of the* ***general vortex motions in nature.***

DOUBLE HELICOID LEVELS SCHEME (DHLS)